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# The first theorem of Andrey Roiter

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Abstract. The article contains memories about A.V. Roiter.

I first met Andrey in the Fall of 1959, in Uzhgorod, at a conference organized by I.R. Shafarevich and D.K. Faddeev. Most of the talks at the conference were on algebraic geometry, homological algebra and algebraic topology, but there were also two survey talks on representation theory–by D.K. Faddeev on unimodular representations, and by S.D. Berman on modular representations of finite groups.

In his talk Faddeev quoted a number of important theorems on unimodular representations-the classical Jordan-Zassenhaus theorem, the theorems of Maranda on $p$-adic representations, and described in detail the recent problem which arose in connection with the results of Diederichsen on unimodular representations of cyclic groups.

In fact, Diederichsen proved in 1938 (see [1]) that if $G$ is a cyclic group of prime order $p$ then the number of indecomposable representation is finite and it is equal to $2h + 1$ where $h$ is the number of the ideal classes of the $p^{\text{th}}$ cyclotomic field $Z(\epsilon)$; another proof of this result was given by Reiner in [2]. The situation for cyclic groups of higher order was considerably more complicated, by this time the only fact that was known was that the paper of Diederichsen contained an infinite series of indecomposable representations for the cyclic group of order 4.

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Faddeev announced that his student, A.V. Roiter, proved that the cyclic group of order 4 had only 9 indecomposable representations, so if this was true then Diederichsen's result was incorrect.

Roiter's paper did not appear yet, at this time it was very difficult to get a preprint because the Soviet government believed that copy machines were a threat for the existence of the "dictatorship of the proletariat", so we could not read the proof of Roiter's theorem; this proof could not be presented in one hour talk because it was based on quite tedious computations. Faddeev wrote on the blackboard the nine matrices which gave all the indecomposable representations of the cyclic group of order 4:

$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

$$\begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},$$

$$\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}, \quad \begin{pmatrix} -1 \end{pmatrix}, \quad \begin{pmatrix} 1 \end{pmatrix}.$$

Everyone, including Faddeev, was quite surprised by this result. Faddeev said that he understood that seven of Roiter's representations were obtained in the ideals of the integral group ring, but there were two mysterious representations ("tainstvennie predstavleniya").

Andrey Roiter's paper appeared a year later, in 1960 in [3]. His proof was correct and he pointed out that Diederichsen's paper contained a mistake. Andrey did not mention in his paper that while working on his theorem he did not know about Diederichsen's paper, it was Faddeev who said that Andrey learned about the existence of this paper after he had already proven his theorem.

We all highly value every mathematical fact which expands our knowledge or changes our view. However when I think about Roiter's first theorem I value not only its content and importance but I remember also that he proved it when he was an undegraduate student who just began his research activity and that at the time no one in the Soviet Union worked on these types of problems.

This was really a remarkable achievement!

## References

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