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On nilpotent Chernikov 2-groups with elementary tops

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Abstract. We give an explicit description of nilpotent Chernikov 2-groups with elementary top and basis of rank 2.

1. Introduction

Recall that a *Chernikov p-group* [1,8] G is an extension of a finite direct sum M of *quasi-cyclic* p-groups, or, the same, the groups of type p^∞, by a finite p-group H. Note that M is the biggest abelian divisible subgroup of G, so both M and H are defined by G up to isomorphism. We call H and M, respectively, the *top* and the *bottom* of G. We denote by $M^{(m)}$ a direct sum of m copies M_k $(1 \leqslant k \leqslant m)$ of quasi-cyclic p-groups and fix elements $a_k \in M_k$ of order p. The group G is nilpotent if and only if the induced action of H on M is trivial [1, Theorem 1.9].

In the papers [2, 10] the classification of nilpotent Chernikov p-groups with elementary tops was related to the classification of tuples of skew-symmatric matrices over the filed $\mathbb{F}_p$. Namely, given an m-tuple of $n \times n$ skew-symmetric matrices $\boldsymbol{A} = (A_1, A_2, \dots, A_m)$, where $A_k = (a_{ij}^{(k)})$, we define the Chernikov p-group $G(\boldsymbol{A})$, which is an extension of $M^{(m)}$ by the elementary p-group $H_n = \langle\, h_1, h_2, \dots, h_n \mid h_i^p = 1,\ h_i h_j = h_j h_i \,\rangle$ such that $[h_i, a] = 1$ for each $a \in M^{(m)}$ and $[h_i, h_j] = \sum_k a_{ij}^{(k)} a_k$. Every nilpotent Chernikov p-group is of this kind and two m-tuples $\boldsymbol{A} = (A_1, A_2, \dots, A_m)$ and $\boldsymbol{B} = (B_1, B_2, \dots, B_m)$ define isomorphic groups if and only if there

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are invertible matrices $S \in \mathrm{GL}(n, \mathbb{F}_p)$ and $Q = (q_{kl}) \in \mathrm{GL}(m, \mathbb{F}_p)$ such that $B_k = \sum_l q_{lk}(SA_lS^\top)$ for all k. In this case we write $\boldsymbol{B} = S \circ \boldsymbol{A} \circ Q$ and call the m-tuples $\boldsymbol{A}$ and $\boldsymbol{B}$ *weakly equivalent*. Recall that the pairs $\boldsymbol{A}$ and $S \circ \boldsymbol{A}$ are called *congruent*.

If $m > 2$, a classification of m-tuples of skew-symmetric matrices is a *wild problem* in the sense of the representation theory, i.e. it contains a classification of representations of any finitely generated algebra[2]. So, there is no hope to obtain a "good" classification of Chernikov p-groups with the bottom $M^{(m)}$ for $m > 2$. Using the results of [9], we gave in the paper [2] a classification of Chernikov p-groups with elementary tops and the bottom $M^{(2)}$ for $p \neq 2$. Unfortunately, if $p = 2$, the technique of [9] does not work. In this paper we use instead the results of [11] to obtain an analogous classification for Chernikov 2-groups.

2. Alternating pairs

From now on $\mathbb{k}$ is a field of characteristic 2. We consider pairs (A, B) of alternating bilinear forms in a finite dimensional vector space over $\mathbb{k}$ or, the same, pairs of skew-symmetric matrices over $\mathbb{k}$, calling them *alternating pairs*. Let $\boldsymbol{R} = \mathbb{k}[t]$, the polynomial ring, $\boldsymbol{E} = \mathbb{k}(t)/\mathbb{k}[t]$ and $\mathrm{res} = \mathrm{res}_\infty : \boldsymbol{E} \to \mathbb{k}$ be the residue at infinity. Let M be a finite dimensional (over $\mathbb{k}$) $\boldsymbol{R}$-module and $F : M \times M \to \boldsymbol{E}$ be an $\boldsymbol{R}$-bilinear map. We call F *strongly alternating* if $\mathrm{res}\, F(u, u) = \mathrm{res}\, F(tu, u) = 0$ for all $u \in M$. Then also $F(u, v) = F(v, u)$ and $F(tu, v) = F(tv, u)$. Given a strongly alternating map F we set $A_F(u, v) = \mathrm{res}\, F(u, v)$ and $B_F(u, v) = \mathrm{res}\, F(tu, v)$. Obviously, (A_F, B_F) is a pair of alternating bilinear forms on M. We use the following facts from [11].

Fact 1. The map $F \mapsto (A_F, B_F)$ induces a one-to-one correspondence between isomorphism classes of non-degenerated strongly alternating maps and isomorphism classes of pairs of alternating forms (A, B) such that A is non-degenrated.

Fact 2. Isomorphism classes of indecomposable non-degenerated strongly alternating maps $F : M \times M \to \boldsymbol{E}$ are in one-to-one correspondence with powers $f^n(t)$ of irreducible polynomials $f(t) \in \mathbb{k}[t]$. Namely $f^n(t)$ corresponds to the strongly alternating map $F_{f,n} : M_{f,n} \to \boldsymbol{E}$, where $M_{f,n} = (\boldsymbol{R}/f^n\boldsymbol{R})^2 = \langle\, u, v \mid f^n u = f^n v = 0\,\rangle$, such that $F_{f,n}(u, v) = 1/f^n (mod \mathbb{k}[t])$, while $F_{f,n}(u, u) = F_{f,n}(v, v) = 0$.

We denote the alternating pair corresponding to the map $F_{f,n}$ by $\boldsymbol{A}_{f,n} = (A_{f,n}, B_{f,n})$.

Consider the matrices of size $n \times (n+1)$

$$I_{n+} = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}, \quad I_{n-} = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix}$$

and alternating pairs

$$\boldsymbol{A}_{\infty,n} = (A_{\infty,n}, B_{\infty,n}), \quad \boldsymbol{A}_{+,n} = (A_{+,n}, B_{+,n}),$$

where

$$A_{\infty,n} = \begin{pmatrix} 0 & J_n \\ J_n^\top & 0 \end{pmatrix}, \quad B_{\infty,n} = \begin{pmatrix} 0 & I_n \\ I_n & 0 \end{pmatrix}$$
$$A_{+,n} = \begin{pmatrix} 0 & I_{n+} \\ I_{n+}^\top & 0 \end{pmatrix}, \quad B_{+,n} = \begin{pmatrix} 0 & I_{n-} \\ I_{n-}^\top & 0 \end{pmatrix},$$

I_n is the $n \times n$ unit matrix and J_n is the $n \times n$ nilpotent Jordan block.

Fact 3. Every indecomposable alternating pair (A, B) with the degenerated form A is isomorphic to one of the pairs $(A_{\infty,n}, B_{\infty,n}), (A_{+,n}, B_{+,n}),$.

Fact 4. Every alternating pair decomposes into an orthogonal direct sum of indecomposable pairs. This decomposition is unique up to isomorphism and permutation of summands.

Lemma 2.1. *There is a* $\Bbbk$*-basis in* $M_{f,n}$ *such that the forms* $A_{f,n}$ *and* $B_{f,n}$ *are given by the matrices* $A_{f,n} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$ *a and* $B_{f,n} = \begin{pmatrix} 0 & \Phi \\ \Phi^\top & 0 \end{pmatrix}$, *where* Φ *is the Frobenius matrix with the characteristical polynomial* $f^n(t)$.

Note that $(A_{\infty,n}, B_{\infty,n}) = (B_{t,n}, A_{t,n})$.

Proof. We include $\Bbbk[t]$ into the ring $\Bbbk[[t]]$ of formal power series and into the field $\Bbbk((t))$ of Laurent series. If $\deg g = d$ and $g(0) \neq 0$, we set $g^*(t) = t^d g(1/t)$ and choose a polynomial $\tilde{g}(t)$ of degree d such that $g^*(t)\tilde{g}(t) \equiv 1 \pmod{t^{d+1}}$. It exists and is unique since $g^*(t)$ is invertible in $\Bbbk[[t]]$.

Let $f(t) \neq t$, $g(t) = f^n(t)$, $d = \deg g(t)$ and $g(t) = t^d + \alpha_1 t^{d-1} + \dots + \alpha_d$. Then $g^*(t) = 1 + \alpha_1 t + \dots + \alpha_d t^d$ and $\tilde{g}(t) = 1 + \beta_1 t + \dots + \beta_d t^d$, where, for every $m \leqslant d$,

$$\alpha_m + \alpha_{m-1}\beta_1 + \alpha_{m-2}\beta_2 + \dots + \alpha_1\beta_{m-1} + \beta_m = 0 \qquad (2.1)$$

(we set $\alpha_0 = \beta_0 = 1$). Consider the basis $\{\, u_k, v_k \mid 0 \leqslant k < d\,\}$ of $M_{f,n}$, where $v_k = t^k v, u_k = t^{d-k-1}u$. Then $F_{f,n}(u_k, u_l) = F_{f,n}(v_k, v_l) = 0$ for all k, l, while $F_{f,n}(u_l, v_k) = h_{k,l} = t^{d+k-l-1}/g(t) \pmod{\Bbbk[[t]]}$. Denote by $\operatorname{co}_1 h$ the coefficient by t^{-1} in the Laurent series h. Recall that $\operatorname{res}_\infty h$, where $h \in \Bbbk((t))$, equals $\operatorname{co}_1 t^{-2}h(1/t)$. Therefore,

$$\begin{aligned}
A_{f,n} &= \operatorname{co}_1 t^{-2}h_{k,l}(1/t)\\
&= \operatorname{co}_1 \frac{t^{l-k-1}}{t^d g(1/t)} = \operatorname{co}_1 t^{l-k-1}\tilde{g}(t) = \begin{cases}\beta_{k-l} & \text{if } k \geqslant l,\\ 0 & \text{if } k < l;\end{cases}\\
B_{f,n} &= \operatorname{co}_1 t^{-3}h_{k,l}(1/t)\\
&= \operatorname{co}_1 \frac{t^{l-k-2}}{t^d g(1/t)} = \operatorname{co}_1 t^{l-k-2}\tilde{g}(t) = \begin{cases}\beta_{k-l+1} & \text{if } k \geqslant l-1,\\ 0 & \text{if } k < l-1.\end{cases}
\end{aligned}$$

So the matrices of the forms $A_{f,n}$ and $B_{f,n}$ in this basis are, respectively,

$$\begin{pmatrix} 0 & A \\ A^\top & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & B \\ B^\top & 0 \end{pmatrix}, \tag{2.2}$$

where

$$A = \begin{pmatrix} 1 & \beta_1 & \beta_2 & \dots & \beta_{d-1} \\ 0 & 1 & \beta_1 & \dots & \beta_{d-2} \\ 0 & 0 & 1 & \dots & \beta_{d-3} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix},$$

$$B = \begin{pmatrix} \beta_1 & \beta_2 & \beta_3 & \dots & \beta_{d-1} & \beta_d \\ 1 & \beta_1 & \beta_2 & \dots & \beta_{d-2} & \beta_{d-1} \\ 0 & 1 & \beta_1 & \dots & \beta_{d-3} & \beta_{d-2} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & \beta_1 \end{pmatrix}.$$

The relations (2.1) imply that

$$A^{-1} = \begin{pmatrix} 1 & \alpha_1 & \alpha_2 & \dots & \alpha_{d-1} \\ 0 & 1 & \alpha_1 & \dots & \alpha_{d-2} \\ 0 & 0 & 1 & \dots & \alpha_{d-3} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix},$$

and $A^{-1}B = \Phi$, the Frobenius matrix with the characteristical polynomial $g(t) = f^n(t)$. Thus, multiplying the matrices of bilinear forms $A_{f,n}$ and $B_{f,n}$ from (2.2) by the matrix

$$\begin{pmatrix} A^{-1} & 0 \\ 0 & I \end{pmatrix}$$

on the left and by the transposed matrix on the right, we accomplish the proof of the lemma in this case.

If $f(t) = t$, we obtain the necessary form of the matrices directly in the basis $\{u_k, v_k\}$ as above. □

Now we resume the above considerations.

Theorem 2.2. *Every indecomposable alternating pair is isomorphic to one of the pairs*

$$\boldsymbol{A}_{f,n} = (A_{f,n}, B_{f,n}),\ \boldsymbol{A}_{\infty,n} = (A_{\infty,n}, B_{\infty,n}),\ \boldsymbol{A}_{+,n} = (A_{+,n}, B_{+,n})$$

given by Fact 3 and Lemma 2.1. Every alternating pair decomposes uniquely (up to permutation of summands) into an orthogonal sum of indecomposable strongly alternating pairs from this list.

3. Weak equivalence and Chernikov groups

We denote by $\mathfrak{A}$ the set of all pairs $\boldsymbol{A}$, where $\boldsymbol{A} \in \{\boldsymbol{A}_{f,n}, \boldsymbol{A}_{\infty,n}, \boldsymbol{A}_{+,n}\}$, and by $\mathfrak{F}$ the set of functions $\kappa : \mathfrak{A} \to \mathbb{Z}_{\geqslant 0}$ such that $\kappa(\boldsymbol{A}) = 0$ for almost all $\boldsymbol{A}$. For any function $\kappa \in \mathfrak{F}$ we set $\mathfrak{A}^\kappa = \bigoplus_{\boldsymbol{A} \in \mathfrak{A}} \boldsymbol{A}^{\kappa(\boldsymbol{A})}$. For the classification of Chernikov 2-groups we have to answer the question:

Given two functions with finite supports $\kappa, \kappa' : \mathfrak{A} \to \mathbb{Z}_{\geqslant 0}$, when are the pairs $\mathfrak{A}^\kappa$ and $\mathfrak{A}^{\kappa'}$ weakly congruent?

Evidently, $(\boldsymbol{A}_1 \oplus \boldsymbol{A}_2) \circ Q = (\boldsymbol{A}_1 \circ Q) \oplus (\boldsymbol{A}_2 \circ Q)$, so the pairs $\boldsymbol{A}$ and $\boldsymbol{A} \circ Q$ are indecomposable simultaneously. For every pair $\boldsymbol{A} \in \mathfrak{A}$ we denote by $\boldsymbol{A} * Q$ the unique pair from $\mathfrak{A}$ which is congruent to $\boldsymbol{A} \circ Q$. The map $\boldsymbol{A} \mapsto \boldsymbol{A} * Q$ defines an action of the group $\mathfrak{g} = \mathrm{GL}(2, \Bbbk)$ on the set $\mathfrak{A}$, hence on the set $\mathfrak{F}$ of functions $\kappa : \mathfrak{A} \to \mathbb{Z}_{\geqslant 0}$: $(Q * \kappa)(\boldsymbol{A}) = \kappa(\boldsymbol{A} * Q)$.

Corollary 3.1. *The pairs $\mathfrak{A}^\kappa$ and $\mathfrak{A}^{\kappa'}$ are weakly congruent if and only if the functions κ and κ' belong to the same orbit of the group $\mathfrak{g}$.*

$(A_{+,n}, B_{+,n})$ is a unique indecomposable couple of dimension $2n+1$. For every other pair $\boldsymbol{A} = (A, B)$ the polynomial $\det(xA + yB)$ is a square: $\det(x_1 A + x_2 B) = \Delta_{\boldsymbol{A}}(x_1, x_2)^2$ for some $\Delta_{\boldsymbol{A}}(x_1, x_2)$ (the *Pfaffian* of $x_1 A + x_2 B$, see [7]). Namely,

$$\Delta_{\boldsymbol{A}}(x, y) = \begin{cases} x_2^n & \text{if } \boldsymbol{A} = \boldsymbol{A}_{\infty,n}, \\ x_2^{dn} f(x_1/x_2) & \text{if } \boldsymbol{A} = \boldsymbol{A}_{f,n} \text{ and } \deg f = d. \end{cases}$$

If $(A', B') = (A, B) \circ Q$, where $Q = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix}$, then $\Delta_{(A',B')}(x_1, x_2) = \Delta_{(A,B)}((x_1, x_2)Q) = \Delta_{(A,B)}(q_{11}x_1 + q_{21}x_2, q_{12}x_1 + q_{22}x_2)$. So now we can repeat the considerations of [2], obtaining analogous results for the fields of characteristic 2 and Chernikov 2-groups.

We say that an irreducible homogeneous polynomial $g \in \Bbbk[x_1, x_2]$ is *unital* if either $g = x_2$ or its leading coefficient with respect to x_1 equals 1. Let $\mathbb{P} = \mathbb{P}(\Bbbk)$ be the set of unital homogeneous irreducible polynomials from $\Bbbk[x_1, x_2]$ and $\tilde{\mathbb{P}} = \tilde{\mathbb{P}}(\Bbbk) = \mathbb{P} \cup \{\varepsilon\}$. Note that $\mathbb{P}$ actually coincides with the set of the closed points of the projective line $\mathbb{P}^1_{\Bbbk} = \operatorname{Proj} \Bbbk[x_1, x_2]$ [6]. For $g \in \mathbb{P}$ and $Q \in \mathfrak{g}$, let $Q * g$ be the unique polynomial $g' \in \mathbb{P}$ such that $g((x, y)Q) = \lambda g'$ for some non-zero $\lambda \in \Bbbk$. (It is the natural action of $\mathfrak{g}$ on $\mathbb{P}^1_{\Bbbk}$.) We also set $Q * \varepsilon = \varepsilon$ for any Q. It defines an action of $\mathfrak{g}$ on $\tilde{\mathbb{P}}$. Denote by $\tilde{\mathfrak{F}} = \tilde{\mathfrak{F}}(\Bbbk)$ the set of all functions $\rho : \tilde{\mathbb{P}} \times \mathbb{N} \to \mathbb{Z}_{\geqslant 0}$ such that $\rho(g, n) = 0$ for almost all pairs (g, n). Define the actions of the group $\mathfrak{g}$ on $\tilde{\mathfrak{F}}$ setting $(\rho * Q)(g, n) = \rho(Q * g, n)$. For every pair $(g, n) \in \tilde{\mathbb{P}} \times \mathbb{N}$ we define a pair of skew-symmetric forms $\boldsymbol{A}(g, n)$:

$$\boldsymbol{A}(g, n) = \begin{cases} (A_{\infty,n}, B_{\infty,n}) & \text{if } g = x_2, \\ (A_{+,n}, B_{+,n}) & \text{if } g = \varepsilon, \\ (A_{f,n}, B_{f,n}) & \text{where } f = g(x, 1) \text{ otherwise.} \end{cases}$$

Let $\tilde{\mathfrak{A}} = \tilde{\mathfrak{A}}(\Bbbk) = \{\boldsymbol{A}(g, n) \mid (g, n) \in \tilde{\mathbb{P}} \times \mathbb{N}\}$. For every function $\rho \in \tilde{\mathfrak{F}}$ we set $\tilde{\mathfrak{A}}^\rho = \bigoplus_{(g,n) \in \tilde{\mathbb{P}} \times \mathbb{N}} \boldsymbol{A}(g, n)^{\rho(g,n)}$. The preceding considerations imply the following theorem.

Theorem 3.2. 1) *Every pair of skew-symmetric bilinear forms over the field* $\Bbbk$ *is weakly congruent to* $\tilde{\mathfrak{A}}^\rho$ *for some function* $\rho \in \tilde{\mathfrak{F}}(\Bbbk)$.

2) *The pairs* $\tilde{\mathfrak{A}}^\rho$ *and* $\tilde{\mathfrak{A}}^{\rho'}$ *are weakly congruent if and only if the functions* ρ *and* ρ' *belong to the same orbit of the group* $\mathfrak{g} = \mathrm{GL}(2, \Bbbk)$.

For every function $\rho \in \tilde{\mathfrak{F}}(\mathbb{F}_2)$ set $G(\rho) = G(\tilde{\mathfrak{A}}^\rho)$.

Theorem 3.3. *Let $\mathfrak{R}$ be a set of representatives of orbits of the group $\mathfrak{g} = \mathrm{GL}(2, \mathbb{F}_2)$ acting on the set of functions $\tilde{\mathfrak{F}}(\mathbb{F}_p)$. Then every nilpotent Chernikov 2-group with elementary top and the bottom $M^{(2)}$ is isomorphic to the group $G(\rho)$ for a uniquely defined function $\rho \in \mathfrak{R}$.*

The description of these groups in terms of generators and relations is also the same as in [2]. Note that all of them are of the form $G(\boldsymbol{A})$, where $\boldsymbol{A} = \bigoplus_{k=1}^{s} \boldsymbol{A}_k$ and all $\boldsymbol{A}_k$ belong to the set $\{\boldsymbol{A}_{\infty,n}, \boldsymbol{A}_{+,n}, \boldsymbol{A}_{f,n}\}$. Each term $\boldsymbol{A}_k$ corresponds to a subset $\{h_{ki}\}$ of generators of the group H and we have to precise the values of $[h_{ki}, h_{kj}]$ (all other commutators are zero). They are given in Table 1. Recall that a_1 and a_2 are generators of the subgroup $\{\, a \in M^{(2)} \mid 2a = 0 \,\}$.

Table 1.

$\boldsymbol{A}_k$	i, j	$[h_{ki}, h_{kj}]$
$\boldsymbol{A}_{+,n}$	$j = d + i$	a_1
	$j = d + i - 1$	a_2
	otherwise	0
$\boldsymbol{A}_{\infty,n}$	$j = d + i$	$a_2,$
	$j = d + i - 1$	$a_1,$
	otherwise	0
$\boldsymbol{A}_{f,n}$	$j = d + i < 2d$	a_1
	$j = d + i - 1$	a_2
	$i < d,\ j = 2d$	$\lambda_{d-i+1} a_2$
	$i = d,\ j = 2d$	$a_1 + \lambda_1 a_2$
	otherwise	0

where $f^n(x) = x^d + \lambda_1 x^{d-1} + \cdots + \lambda_d$

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