УДК 539.3

Elastic Buckling and Optimization of Asymmetric I-Cross Sections of Cold-Formed Thin-Walled Beams

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Subject of the paper includes two open asymmetric I-cross sections of cold-formed thin-walled beams. The both beams are simply supported and are subjects to pure bending. Every I-section is separately described by dimensionless parameters. Geometric properties with warping functions and inertia moments are determined. The strength, global and local stability conditions are defined for both beams. Mathematical solution of elastic local buckling problem for the flange of I-thin-walled beam is experimentally verified. Optimal open cross section shapes of both beams are determined. The optimization criterion is formulated on the basis of a dimensionless objective function. Optimal open cross sections of both beams are compared with a classical I-section beam.

Key words: cold-formed thin-walled beam, optimal design, objective function, local buckling.

Introduction. Manufacturing of arbitrary open cross sections of cold-formed thin-walled beams is widespread and well-known in engineering. Contemporary studies of strength and stability problems of these beams are of very extensive character. Selected results of these studies are presented for example by: Bradford and Ge (1997), Bradford (1998), Chen (2000), Davies (2000), Pi and Trahair (2000), Rasmussen (2001), Hancock (2003), Mohri, Brouki and Roth (2003), Hsu and Chi (2003), Bambach and Rasmussen (2004), Corte *et al*. (2004), Di Lorenzo *et al*. (2004), Dinis, Camotim and Silvestre (2004), Magnucki, Szyc and Stasiewicz (2004), Trahair and Hancock (2004). Optimal designing of thin-walled beams under strength and stability constraints belongs also to contemporary studies. Magnucki and Monczak (2000), Magnucki (2002), Magnucka-Blandzi and Magnucki (2004), Magnucki and Ostwald (2005) determined optimal shapes of selected mono-symmetrical open cross sections of cold-formed thin-walled beams.

Subject of the optimization includes two cold-formed thin-walled beams with open cross sections, which was presented by Magnucki and Ostwald [15]. The first cross section is I-cross section of a cold-formed beam, and the second one is a generalized case of the first cross section. The both beams of the length L, depth H, and wall thickness t are in a pure bending state.

1. Geometric properties of an asymmetric I-section cold-formed beam

The first cross section of the asymmetric I-section cold-formed beam is shown in fig. 1.

This cross section of the beam is described by two following dimensionless parameters

$$x_1 = b/a, \quad x_3 = t/b. \tag{1}$$

The centroid of the cross section (the point O) and the shear center (the point C) are located in the center of the coordinate system *yz* — the principal axes. The total area and the geometric stiffness for Saint-Venant torsion of the cross section

$$A = 2at \cdot f_0, \quad J_t = \frac{2}{3} at^3 \cdot f_0, \tag{2}$$

where $f_0 = 1 + 2x_1$.

Moments of inertia of the cross section area with respect to the *y* and *z* axes and the warping moment of inertia are as follows

$$J_y = 2a^3 t \cdot f_2, \quad J_z = 2a^3 \cdot f_3, \quad J_\omega = 2a^5 t \cdot f_5, \tag{3}$$

where

$$f_2 = \frac{1}{6} x_1^3, \; f_3 = x_1 + (1 - x_1 x_3)^2 \left[x_1 + \frac{1}{3}(1 - x_1 x_3) \right],$$

$$f_5 = (1 - x_1 x_3)\tilde{\omega}_1^2 + \frac{1}{6}\left[\tilde{\omega}_1^2 + \tilde{\omega}_1\tilde{\omega}_2 + \tilde{\omega}_2^2 + 2\left(\tilde{\omega}_3^2 + \tilde{\omega}_3\tilde{\omega}_4 + \tilde{\omega}_4^2\right) + \tilde{\omega}_5^2 + \tilde{\omega}_5\tilde{\omega}_6 + \tilde{\omega}_6^2\right] x_1.$$

The warping functions — values of the functions in points of the profile (fig. 1)

$$\omega_i = \tilde{\omega}_i \cdot a^2, \quad i = \overline{1,6}, \tag{4}$$

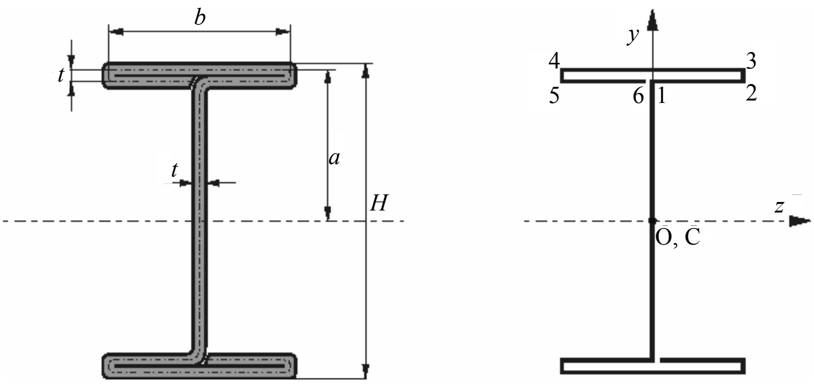

Fig. 1. Scheme of the asymmetric I-section beam

where

$$\tilde{\omega}_1 = -\frac{2x_1^3 x_3}{1+(2-x_3)x_1}, \ \tilde{\omega}_2 = \tilde{\omega}_1 - \frac{1}{2}(1-x_1x_3)x_1, \ \tilde{\omega}_3 = \tilde{\omega}_1 - \frac{1}{2}(1-2x_1x_3)x_1,$$

$$\tilde{\omega}_4 = \tilde{\omega}_1 + \frac{1}{2}(1+2x_1x_3)x_1, \ \tilde{\omega}_5 = \tilde{\omega}_1 + \frac{1}{2}(1+3x_1x_3)x_1, \ \tilde{\omega}_6 = \tilde{\omega}_1 + 2x_1^2 x_3.$$

2. Geometric properties of an asymmetric lipped I-section cold-formed beam

The second cross section of an asymmetric lipped I-section cold-formed beam is shown in fig. 2.

This cross section of the beam is described by three following dimensionless parameters

$$x_1 = b/a, \quad x_2 = c/b, \quad x_3 = t/b. \tag{5}$$

The product of inertia of the cross section is zero for the principal axes yz $(J_{yz} = 0)$, that

$$x_2 = \frac{1}{x_1}\left(1 - \sqrt{1 - kx_1}\right), \tag{6}$$

where $k = (1 - x_1x_3)/2 + (2 - x_1x_3)x_3$.

The total area, the geometric stiffness for Saint-Venant torsion and moments of inertia of the cross section are as follows

$$A = 2at \cdot f_0, \quad J_t = \frac{2}{3}at^3 \cdot f_0, \tag{7}$$

$$J_y = 2a^3 t \cdot f_2, \quad J_z = 2a^3 \cdot f_3, \quad J_\omega = 2a^5 t \cdot f_5, \tag{8}$$

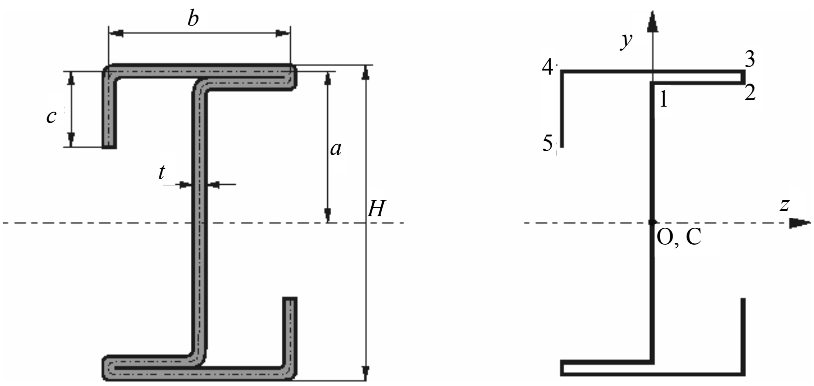

Fig. 2. Scheme of the asymmetric lipped I-section beam

where

$$f_0 = 1 + \left(\frac{3}{2} + x_2 - x_3\right)x_1, \quad f_2 = \frac{1}{8}(1 + 2x_2)x_1^3,$$

$$f_3 = x_1 + (1 - x_1x_3)^2\left[\frac{1}{2}x_1 + \frac{1}{3}(1 - x_1x_3)\right] + \frac{1}{3}\left[1 - (1 - x_1x_2)^3\right],$$

$$f_5 = (1 - x_1x_3)\tilde{\omega}_1^2 + \frac{1}{3}\left[\frac{1}{2}\left(\tilde{\omega}_1^2 + \tilde{\omega}_1\tilde{\omega}_2 + \tilde{\omega}_2^2\right) + \tilde{\omega}_3^2 + \right.$$

$$\left. + \tilde{\omega}_3\tilde{\omega}_4 + \tilde{\omega}_4^2 + \left(\tilde{\omega}_4^2 + \tilde{\omega}_4\tilde{\omega}_5 + \tilde{\omega}_5^2\right)x_2\right] x_1 .$$

The warping functions are (fig. 2)

$$\omega_i = \tilde{\omega}_i \cdot a^2, \quad i = \overline{1,5}, \tag{9}$$

where

$$\tilde{\omega}_1 = \frac{1 - 9x_1x_3 - 2(2 + 4x_1x_3 + x_1x_2)x_2}{4[2 + x_1(3 + 2x_2 - 2x_3)]}x_1^2, \quad \tilde{\omega}_2 = \tilde{\omega}_1 - \frac{1}{2}(1 - x_1x_3)x_1,$$

$$\tilde{\omega}_3 = \tilde{\omega}_1 - \frac{1}{2}(1 - 2x_1x_3)x_1, \quad \tilde{\omega}_4 = \tilde{\omega}_1 + \frac{1}{2}(1 + 2x_1x_3)x_1,$$

$$\tilde{\omega}_5 = \tilde{\omega}_1 + \frac{1}{2}(1 + x_1x_2 + 2x_1x_3)x_1.$$

3. Elastic buckling of the flange of the I-section cold formed beam

3.1. Analytical solution. Mathematical model for local buckling of the upper flange of the beam is assumed in the form of a beam on an elastic foundation [15]. Scheme of the deformation of the cross section of the beam is shown in fig. 3.

The differential equation for the beam on an elastic foundation is in the following form

$$\frac{d^4w}{dx^4} + k^2\frac{d^2w}{dx^2} + \beta \cdot w(x) = 0, \tag{10}$$

where $k^2 = F/EJ_{z,f}$, $\beta = c/EJ_{z,f}$, $J_{z,f} = 2bt^3/3$, $c = 8E(t/b)^3$ are the module of the elastic foundation, F is the longitudinal compression force of the upper flange.

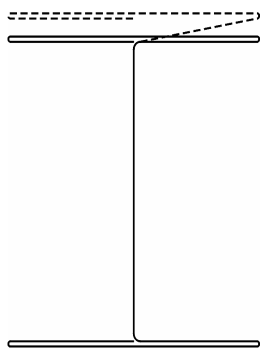

Fig. 3. Deformed cross section of the beam under pure bending

The web is rigid as compared to the flange of the beam. In consequences, the deflection function determining the buckling shape is assumed in the following form

$$w(x) = w_a \cdot \sin^2(\pi m\, x/L), \tag{11}$$

індекс *M* означає атом поверхні
where w_a is amplitude, m is natural number.

The differential equation (10) is solved with the Galerkin method. The critical force is obtained in the following form

$$F_{CR} = 8E\frac{t^3}{b}\min_Y\left[\frac{2}{3}Y^2 + \frac{3}{4}\frac{1}{Y^2}\right] = 8\sqrt{2}E\frac{t^3}{b}, \tag{12}$$

where $Y = m\pi b/L$.

The critical stress for the compressed flange of the I-section of cold-formed beam will be

$$\sigma_{CR}^{(anal)} = \frac{F_{CR}}{2bt} = 4\sqrt{2}E\left(\frac{t}{b}\right)^2. \tag{13}$$

For the beam: $L = 900$ mm is the length, $H = 162$ mm is the depth, $b = 80$ mm is the width, $t = 0{,}6$ mm is the wall thickness, the critical stress $\sigma_{CR}^{(anal)} = 65{,}2\ \mathrm{MPa}$.

3.2. Experimental tests. The mathematical model of the flange of the beam includes some simplifications, and the analytical solution of the buckling problem is only approximate. Experimental research was carried out for two cold-formed thin-walled beams (the first case of beam — the asymmetric I-section, fig. 1). Sizes of the beams: $L_c = 1000$ mm is the total length of the beam, $H = 162$ mm is the depth, $b = 80$ mm is the width, $t = 0{,}6$ mm is the wall thickness, $L = 900$ mm is the length of the beam under pure bending. The beam with elastically buckled upper flange is shown in fig. 4.

In results of the experimental tests the following critical stresses are obtained: $\sigma_{CR}^{(eks1)} = 62{,}8\ \mathrm{MPa}$, $\sigma_{CR}^{(eks2)} = 83{,}7\ \mathrm{MPa}$. The value of the critical stress (13) calculated analytically is contained between these values.

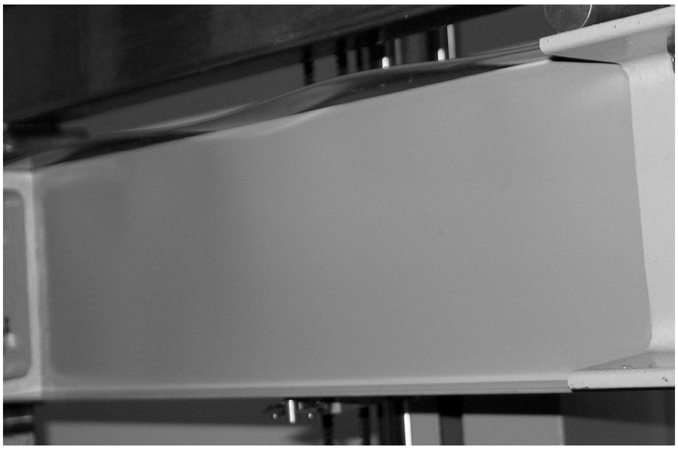

Fig. 4. General view of the beam with buckled flange (Fot. M. Ostwald)

4. Optimization of I-section cold-formed beams

The optimization criterion assumed based on the paper of Magnucka-Blandzi and Magnucki (2004) has the following form

$$\max_{x_i}\left[\Phi_{m1}(x_i)\cup\Phi_{m2}(x_1,\lambda)\right]=\Phi_{\max}, \tag{14}$$

where $\Phi_{mj}=M_j/EA^{3/2}$ is the dimensionless objective functions, $\lambda=L/H$ is the relative length of the beam, j = 1 is the strength condition, j = 2 is the lateral buckling condition.

Maximal value of the dimensionless objective function Φ_{mj} is equivalent to a maximum of moment M as a load or minimal value of the area A of the cross section of the beam. Hence, the assumed criterion includes two optimization problems: maximization of the load and minimization of the area of the beam cross section.

Constraints of allowable solutions are:

- the strength condition

$$M\le M_1, \tag{15}$$

where $M_1=2J_z\sigma_a/H$, $H=2a+t$ is depth of the beam, σ_a is the allowable stress, M is the bending moment as the load;

- the lateral buckling condition

$$M\le M_2, \tag{16}$$

where

$$M_2=\frac{\pi E}{\sqrt{2(1+\nu)}c_sL}\sqrt{J_yJ_t\left[1+2(1+\nu)\frac{\pi^2}{L^2}\frac{J_\omega}{J_t}\right]},$$

and $L=\lambda\cdot H$ is the length of the beam, c_s is the coefficient of safety, E, v are the material constants;

- the local buckling condition

$$\sigma_m\le\frac{\sigma_{CR}}{c_{sl}}, \tag{17}$$

where σ_m is the maximal stresses in the flange or in the web of the beam, $c_{sl}=1{,}5c_s$ is the coefficient of safety for the local buckling [15, 16], $\sigma_{CR}^{(flange)}=4\sqrt{2}E(t/b)^2$ is the critical stress for the flange of I-section cold-formed beam (13), $\sigma_{CR}^{(web)}=\dfrac{\pi^2}{2(1-\nu^2)}E\left(\dfrac{t}{a}\right)^2$ is the critical stress for the flat web of thin-walled beam

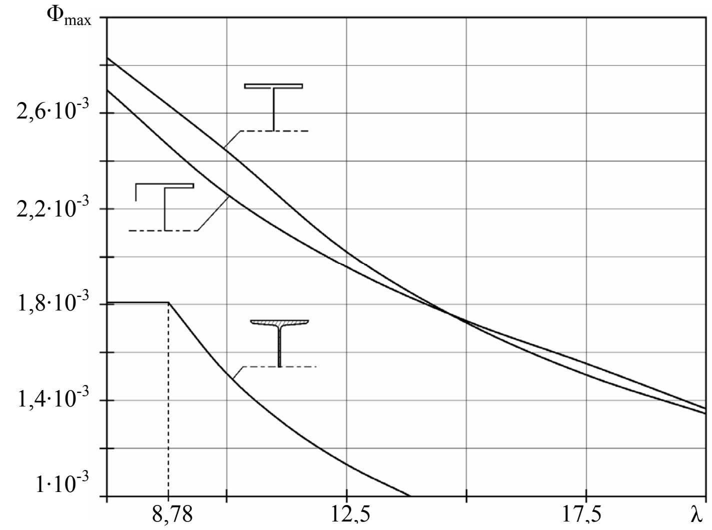

Fig. 5. Maximal values of the dimensionless objective function for two beams

[15, 16], $\sigma_{CR}^{(bent)} = \dfrac{2}{(1+\nu)(4-3c/a)} E\left(\dfrac{t}{c}\right)^2$ is the critical stress for the bent flange of the beam [15, 16, 20].

5. Numerical calculations

The numerical calculations are carried out for a family of the beams of relative length $7{,}5 \le \lambda = 20$, and $\sigma_a / E = 0{,}0015$, $\nu = 0{,}3$, $c_s = 1{,}8$. Maximal values of the objective function $\Phi_{\max}$ (14) are numerically determined for each beam of the family (fig. 5). The plot shows that both investigated cross-sections of cold-formed thin-walled beams are practically equivalent. Maximal values $\Phi_{\max}$ of both beams approximate each other in the investigated interval $\lambda \in \langle 7{,}5;20{,}0 \rangle$. Moreover, $\Phi_{\max}$ for standard I-section beam is shown in fig. 5. Examples of optimal cross sections of these beams for depth H = 200 mm and λ = 7,5, λ = 12,5 are shown in fig. 6.

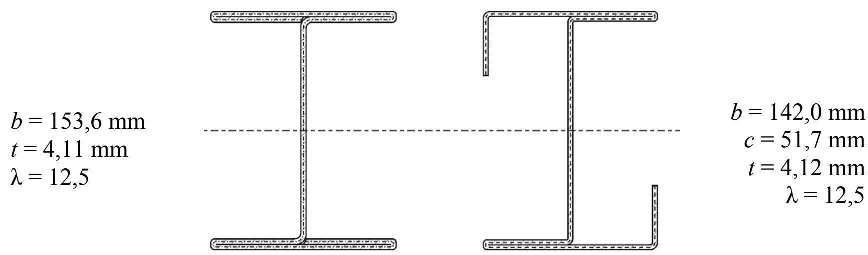

Fig. 6. Optimal cross sections of cold-formed thin-walled beams (H = 200 mm)

Conclusions. The optimization criterion (14) with dimensionless objective functions Φ_{mj} is a quality measure of cross sections of beams. This criterion enables sorting and comparing the beams with different shapes of cross sections. Values of the objective function for both cold-formed thin-walled beams are similar. The quality of the thin-walled beams subject to the study considerably exceeds the one of standard I-section beam. The maximal values $\Phi_{\max}$ of thin-walled beams exceed by 40 percent the ones of the standard I-section beam.

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Пружна втрата стійкості й оптимізація асиметричних поперечних перерізів двотаврових холодноформованих тонкостінних балок

Кшиштоф Маґнуцкі, Мар’ян Оствальд

У роботі розглядаються холодноформовані тонкостінні двотаврові балки з відкритими асиметричними поперечними перерізами двох типів. Балки вільно оперті та перебувають в умовах чистого згинання. Переріз кожної з балок описують безрозмірними параметрами, що враховують їх геометрію, моменти інерції та деформацію. Визначені умови міцності, локальної та глобальної стійкості для обох типів балок. Проведено експериментальну перевірку отриманого розв’язку задачі про локальну втрату стійкості полиці двотавра. На основі сформульованого критерію оптимізації, з використанням безрозмірної цільової функції, визначено оптимальні форми поперечних перерізів для обох балок. Отримані оптимальні параметри поперечних перерізів порівняно з класичним двотавровим перерізом.

Упругая потеря устойчивости и оптимизация асимметрических поперечных сечений двухтавровых холодноформированных тонкостенных балок

Кшиштоф Магнуцки, Марьян Оствальд

В работе рассматриваются холодноформированные тонкостенные двухтавровые балки открытых асимметричных сечений двух типов. Принимается, что балки свободно опертые и находятся в условиях чистого изгиба. Сечения балок описывают безразмерными параметрами, которые учитывают их геометрию, моменты инерции и деформацию. Исследуются параметры прочности, условия локальной и глобальной устойчивости балок. Проведена экспериментальная проверка полученного математического решения задачи о локальной потере устойчивости полки двухтавра. На основании сформулированного критерия оптимизации, с использованием безразмерной функции цели, определены оптимальные формы поперечных сечений балок. Проведено сравнение полученных оптимальных параметров поперечных сечений с классическим двухтавровым сечением.

Отримано 10.04.06