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Surface tension and strength of local nonhomogeneous cylinder in the process of heating

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Within the framework of local gradient approach the stressed state of a cylinder is examined during the process of its heating. The obtained relations are used for the study of surface tension and dependence of tensile strength of cylinder on temperature and size of the body. It is shown that dependence of tensile strength breaking point and surface tension on the uniform temperature of the body is linear. The values of nearsurface stresses are changing in the process of heating causing the change of parameters of strength and surface tension. The results agreement with those known in literature is shown.

Keywords: interconnected thermomechanical processes, local gradient approach in thermomechanics, surface tension, strength, size effects, cylinder.

Introduction. The elements of real constructions and devices are usually found in the complex conditions of interaction with surroundings which to a great extent can change their operating characteristics, including strength parameters. Therefore it is important to develop models with account for materials structure and adequate describing of the behavior of the real bodies. Local gradient approach in thermomechanics [1, 2, 4, 6] allows description of tensile strength dependence on temperature and admixtures. Within the framework of this approach [1, 2, 4] the dependence of tensile strength on a steady over time temperature is studied and the matching of obtained results with the known experimental data is shown. There exists a considerable practical and scientific interest to research of the influence of variable over time temperature on the stressed-strained state and strength properties of bodies, including thin films and fibers that are widely used in modern mechanical engineering. It is known, that in such elements the contribution of surface and volume factors to internal energy is rateable and they feature size effects. It is practically impossible to conduct the proper experimental researches in the case of variable temperature.

The paper considers the local gradient approach in thermomechanics of the stressed-strained state of an infinite solid cylinder in the process of its heating. On this basis the influence of temperature on surface tension, tensile strength breaking point and proper size effects is investigated.

1. Problem formulation

Let us consider an isotropic thermoelastic cylinder (domain $r \le R$ in the cylindrical coordinates $\{r, \varphi, z\}$). At infinity $z \to \pm\infty$ cylinder is loaded by effort $\vec{p} = (0, 0, \pi R^2 p)$. Suppose that at the free of force load surface $r = R$ the value of chemical potential $\eta_a \ne 0$ is stated. In the initial moment of time $\tau = 0$ the temperature of the cylinder is uniform and equal T_*. For time $\tau > 0$ at the body surface $r = R$ there is stated the temperature value T_a different from the initial ($T_a \ne T_*$).

For considered external action the one-dimensional over spatial coordinate r situation in the body is realized

$$\hat{\sigma} = \hat{\sigma}(r,\tau), \qquad \eta = \eta(r,\tau), \qquad \theta = \theta(r,\tau),$$

where $\hat{\sigma}$ is the stress tensor, $\theta = T - T_*$ is the temperature T disturbance with respect to the initial value T_*.

Accept as wanted functions the nonzero components of stress tensor $\hat{\sigma}$, chemical potential η and temperature θ. The complete set of linearized equations for a thermoelastic cylinder at local gradient approach describing quasistatical situation in the body has the form [2]

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\varphi}{r} = 0, \qquad \frac{\partial \sigma_z}{\partial r} = \frac{\partial}{\partial r}\left(\frac{\lambda\sigma}{3\lambda + 2\mu} + 2\mu\alpha_m \eta - 2\mu\alpha_t \theta \right),$$
$$r\frac{\partial^2 \sigma_\varphi}{\partial r^2} + 2\frac{\partial \sigma_\varphi}{\partial r} - \frac{\partial \sigma_r}{\partial r} = \frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\left(\frac{\lambda\sigma}{3\lambda + 2\mu} + 2\mu\alpha_m \eta - 2\mu\alpha_t \theta \right)\right],$$
$$\frac{\partial^2 \eta}{\partial r^2} + \frac{1}{r}\frac{\partial \eta}{\partial r} - \kappa_1^2 \eta - \kappa_2^2 \sigma - \kappa_3^2 \theta = 0, \qquad \frac{\partial \theta}{\partial \tau} = a\left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} \right), \tag{1}$$

where σ_r, σ_φ, σ_z are stresses; $\sigma = \hat{\sigma} : \hat{I}$, $\hat{I}$ is identity tensor; λ, μ, α_m, α_t, κ_j are the constants $\left(j = \overline{1,3}\right)$. Denote neglecting of the stresses and chemical potential effects on temperature in the last equation of (1).

The initial and boundary conditions for the temperature are written in the form

$$\theta(x,0) = 0, \qquad \theta(R,\tau) = \theta_a \equiv T_a - T_*. \tag{2}$$

Chemical potential and stresses should satisfy the conditions

$$\eta\big|_{r=R} = \eta_a, \quad \vec{n}\cdot\hat{\sigma}\big|_{r=R} = 0 \tag{3}$$

at the body surface $r = R$ with external normal vector $\vec{n}$ and the condition

$$\frac{1}{\pi R^2}\iint\limits_{r<R} \sigma_z \, r dr d\varphi = p \tag{4}$$

in the arbitrary cross-section $z = const$ of the cylinder.

2. Problem solution

From (1) it is easy to see that at the first stage the temperature can be found with following determining of the stresses and chemical potential.

According to [3] temperature in the cylinder is described by formula

$$\theta(r,\tau)=\theta_a\left(1-\sum_{n=1}^{\infty}\frac{2J_0(\nu_n r)}{\nu_n R\,J_1(\nu_n R)}\exp(-\nu_n^2 a\tau)\right), \tag{5}$$

where ν_n are real zeros of equation $J_0(R\nu)=0$, $n=1, 2, ...$, J_0 and J_1 are Bessel functions of the first kind.

Taking temperature in the form (5) the chemical potentials and nonzero stresses find from first four equations of (1) and conditions (3), (4)

$$\eta(r,\tau)=\eta_a+\frac{2A_c}{b_m}\left[\frac{I_0(\xi r)}{I_0(\xi R)}-1\right]+\theta_a\sum_{n=1}^{\infty}\left(c_n-\frac{\alpha_t}{\alpha_m}\right)\frac{2J_0(\nu_n r)}{\nu_n R J_1(\nu_n R)}e^{-\nu_n^2 a\tau},$$

$$\sigma_r(r,\tau)=A_c\left[\frac{I_1(\xi r)}{\xi r I_0(\xi R)}-\frac{I_1(\xi R)}{\xi R I_0(\xi R)}\right]+\theta_a\sum_{n=1}^{\infty}\frac{b_m c_n}{\nu_n R}\left[\frac{J_1(\nu_n r)}{\nu_n r J_1(\nu_n R)}-\frac{1}{\nu_n R}\right]e^{-\nu_n^2 a\tau},$$

$$\sigma_\varphi(r,\tau)=A_c\left[\frac{I_0(\xi r)}{I_0(\xi R)}-\frac{I_1(\xi r)}{\xi r I_0(\xi R)}-\frac{I_1(\xi R)}{\xi R I_0(\xi R)}\right]+$$

$$+\theta_a\sum_{n=1}^{\infty}\frac{b_m c_n}{\nu_n R}\left[\frac{J_0(\nu_n r)}{J_1(\nu_n R)}-\frac{J_1(\nu_n r)}{\nu_n r J_1(\nu_n R)}-\frac{1}{\nu_n R}\right]e^{-\nu_n^2 a\tau},$$

$$\sigma_z(r,\tau)=p+A_c\left[\frac{I_0(\xi r)}{I_0(\xi R)}-\frac{2I_1(\xi R)}{\xi R I_0(\xi R)}\right]+\theta_a\sum_{n=1}^{\infty}\frac{b_m c_n}{\nu_n R}\left[\frac{J_0(\nu_n r)}{J_1(\nu_n R)}-\frac{2}{\nu_n R}\right]e^{-\nu_n^2 a\tau}. \tag{6}$$

Here J_k and I_k are Bessel and modified Bessel functions of the first kind ($k=0, 1$),

$$A_c=\frac{b_m}{2\chi(\xi R)}\left(\frac{\kappa_1^2\eta_a+\kappa_2^2 p+\kappa_3^2\theta_a}{\xi^2}-4D\theta_a\sum_{n=1}^{\infty}\frac{c_n\exp(-\nu_n^2 a\tau)}{(\nu_n R)^2}\right),$$

$$b_m=4\mu\frac{3\lambda+2\mu}{\lambda+2\mu}\alpha_m,\quad D=\frac{b_m\kappa_2^2}{\xi^2},\quad \chi(\xi R)=1-D\left(1-\frac{2I_1(\xi R)}{\xi R I_0(\xi R)}\right),$$

$$b_t=4\mu\frac{3\lambda+2\mu}{\lambda+2\mu}\alpha_t,\quad c_n=\frac{\alpha_t}{\alpha_m}+\frac{\xi_t^2}{\xi^2+\nu_n^2},\quad \xi^2=\kappa_1^2+b_m\kappa_2^2,\quad \xi_t^2=\kappa_3^2-b_t\kappa_2^2.$$

With increase of the radius R for «thick» cylinders ($\xi R >> 1$), considering that $I_1(\xi R)/(\xi R I_0(\xi R))$ vanishes as $(\xi R)^{-1}$, for stresses we obtain

$$\sigma_r(r,\tau)=\frac{A_h I_1(\xi r)}{\xi r I_0(\xi R)}+\theta_a\sum_{n=1}^{\infty}\frac{b_m c_n}{\nu_n R}\left[\frac{J_1(\nu_n r)}{\nu_n r J_1(\nu_n R)}-\frac{1}{\nu_n R}\right]e^{-\nu_n^2 a\tau},$$

$$\sigma_\varphi(r,\tau) = A_h\left[\frac{I_0(\xi r)}{I_0(\xi R)} - \frac{I_1(\xi r)}{\xi r I_0(\xi R)}\right] +$$

$$+\theta_a\sum_{n=1}^{\infty}\frac{b_m c_n}{\nu_n R}\left[\frac{J_0(\nu_n r)}{J_1(\nu_n R)} - \frac{J_1(\nu_n r)}{\nu_n r J_1(\nu_n R)} - \frac{1}{\nu_n R}\right]e^{-\nu_n^2 a\tau},$$

$$\sigma_z(r,\tau) = p + A_h\frac{I_0(\xi r)}{I_0(\xi R)} + \theta_a\sum_{n=1}^{\infty}\frac{b_m c_n}{\nu_n R}\left[\frac{J_0(\nu_n r)}{J_1(\nu_n R)} - \frac{2}{\nu_n R}\right]e^{-\nu_n^2 a\tau}, \qquad (7)$$

where $A_h = \dfrac{b_m}{2}\left(\dfrac{\kappa_1^2\eta_a + \kappa_2^2 p + \kappa_3^2\theta_a}{\kappa_1^2} - \dfrac{4D\theta_a}{1-D}\sum_{n=1}^{\infty}\dfrac{c_n e^{-\nu_n^2 a\tau}}{(\nu_n R)^2}\right)$.

For $\tau \to \infty$ cylinder approaches equilibrium state, in which stresses are expressed by formulas

$$\sigma_r(r,\infty) = \frac{b_m}{2}\frac{\kappa_1^2\eta_a + \kappa_2^2 p + \kappa_3^2\theta_a}{\xi^2\chi(\xi R)}\left[\frac{I_1(\xi r)}{\xi r I_0(\xi R)} - \frac{I_1(\xi R)}{\xi R I_0(\xi R)}\right],$$

$$\sigma_\varphi(r,\infty) = \frac{b_m}{2}\frac{\kappa_1^2\eta_a + \kappa_2^2 p + \kappa_3^2\theta_a}{\xi^2\chi(\xi R)}\left[\frac{I_0(\xi r)}{I_0(\xi R)} - \frac{I_1(\xi r)}{\xi r I_0(\xi R)} - \frac{I_1(\xi R)}{\xi R I_0(\xi R)}\right],$$

$$\sigma_z(r,\infty) = p + \frac{b_m}{2}\frac{\kappa_1^2\eta_a + \kappa_2^2 p + \kappa_3^2\theta_a}{\xi^2\chi(\xi R)}\left[\frac{I_0(\xi r)}{I_0(\xi R)} - \frac{2I_1(\xi R)}{\xi R I_0(\xi R)}\right]. \qquad (8)$$

Nonzero stressed-strained state of the considered body at equilibrium without external force load is brought about by nearsurface nonhomogeneity caused by different particles interaction conditions the in inner and nearsurface regions. In this case parameter ξ^{-1} is characteristic size of the nearsurface nonhomoheneity region. Note also that relations (8) describe the stressed state in the initial moment of time $\tau = 0$, if replaced by $\theta_a = 0$. Specific values $\sigma_w^* = 2\sigma_w \big/ \left(\eta^{(x)} b_m\right)$, $w \in \{r, \varphi\}$, $\sigma_z^* = 2(\sigma_z - p)\big/\left(\eta^{(x)} b_m\right)$ of stresses distribution, where $\eta^{(x)} = \left(\kappa_1^2\eta_a + \kappa_2^2 p + \kappa_3^2\theta_a\right)\big/\xi^2$, for parameters $\alpha_t\theta_a/\alpha_m\eta^{(x)} = 0{,}2$, $\xi l = 10$, $D = 0{,}25$, $\alpha_m\kappa_3^2 R^2/\alpha_t = 1$, are shown in Fig. 1 for relative time $\tau^* = a\tau/R^2 = 1$.

From above formulas we arrive to conclusion that temperature does not have an influence on the characteristic size of the nearsurface nonhomogeneity region, however can substantially change the quantitative values of chemical potential and stresses.

3. Surface tension

Use the obtained solution for determination of surface tension that is the integral characteristic of the nearsurface stretching stresses in the unloaded cylinder. The nearsurface stresses σ_φ, σ_z in the cylinder are stretching. They decrease with distance from surface $r = R$ and vanish to zero at $r = r_a^\varphi$ and $r = r_a^z$, correspondingly. In the inner regions $r < r_a^\varphi$ and $r < r_a^z$ of the body stresses are negative (see Fig. 1).

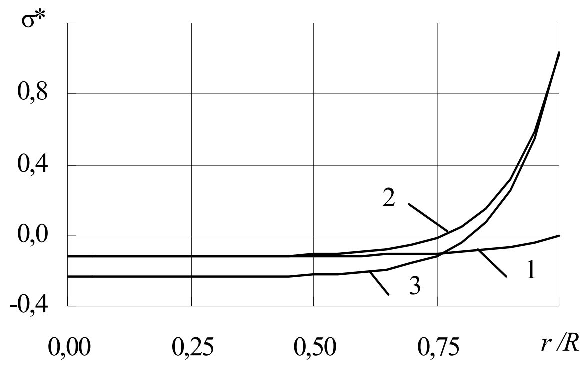

Fig. 1. Specific stresses distribution in cylinder, curves 1-3 represent σ^*_r, σ^*_φ and σ^*_z

In [2] it is shown that the point in which stresses change their sign does not depend on the uniform body temperature. On this base for r_a^φ and r_a^z determining we get equations

$$I_0(\xi r_a^\varphi) - \frac{I_1(\xi r_a^\varphi)}{\xi r_a^\varphi} - \frac{I_1(\xi R)}{\xi R} = 0, \quad I_0(\xi r_a^z) - \frac{2}{\xi R} I_1(\xi R) = 0. \tag{9}$$

The second equation disregarding notation is the same as the one obtained in [2].

Solution of (9) we find approximately in the form

$$r_a^\varphi = R\left(1 - \frac{\ln(\xi R)}{\xi R}\right), \quad r_a^z = R\left(1 - \frac{\ln(\xi R)}{\xi R} + \frac{\ln(2)}{\xi R} - \frac{\ln(\xi R)}{2(\xi R)^2}\right). \tag{10}$$

Take into account that the point where stresses change the sign is close to the surface $r = R$ of the cylinder and the value of dimensionless radius ξR for the real bodies is considerably greater than 1

$$\left|\frac{r_a - R}{R}\right| << 1, \quad \xi R >> 1,$$

and use well-known asymptotic for the Bessel functions

$$I_0(r) = \frac{\exp(r)}{\sqrt{2\pi r}}\left[1 + \frac{1}{8r} + o\left(\frac{1}{r}\right)\right], \; I_1(r) = \frac{\exp(r)}{\sqrt{2\pi r}}\left[1 - \frac{3}{8r} + o\left(\frac{1}{r}\right)\right].$$

On the base of above approximations and solution (6) for surface tension we obtain relations

$$f_{\varphi}=f_{\kappa}\left(1-\Re-\left(\frac{5}{2}+\frac{2D}{1-D}\right)\frac{1}{\xi R}+\frac{1}{2}\Re^{2}+\frac{1+D}{1-D}\frac{\Re}{\xi R}\right)+$$

$$+\theta_{a}\sum_{n=1}^{\infty}\frac{b_{m}c_{n}}{\nu_{n}^{2}R}\left((1-\Re)\left(1-\frac{J_{1}[\nu_{n}R(1-\Re)]}{J_{1}(\nu_{n}R)}\right)+\frac{1}{2}\Re^{2}-\frac{J_{0}[\nu_{n}R(1-\Re)]}{\nu_{n}R\,J_{1}(\nu_{n}R)}\right)\exp(-\nu_{n}^{2}a\tau). \quad (11)$$

$$f_{z}=f_{\kappa}\left(1-2\Re-\left(\frac{5}{2}+\frac{2D}{1-D}+2\ln 2\right)\frac{1}{\xi R}+\frac{1+D}{1-D}\frac{\Re}{\xi R}-\frac{1}{2}\Re^{2}\right)+$$

$$+\theta_{a}\sum_{n=1}^{\infty}\frac{b_{m}c_{n}}{\nu_{n}^{2}R}(1-\Re)^{2}\left(1-\frac{J_{1}[\nu_{n}R(1-\Re)]}{(1-\Re)J_{1}(\nu_{n}R)}\right)\exp(-\nu_{n}^{2}a\tau). \quad (12)$$

where $f_{\kappa}=\frac{b_{m}}{2\xi(1-D)}\left(\eta_{a}\frac{\kappa_{1}^{2}}{\xi^{2}}+\theta_{a}\frac{\kappa_{3}^{2}}{\xi^{2}}-4D\theta_{a}\sum_{n=1}^{\infty}\frac{c_{n}\exp(-\nu_{n}^{2}a\tau)}{(\nu_{n}R)^{2}}\right)$, $\Re=\frac{\ln(\xi R)}{\xi R}$.

For surface tension in cylinders of sufficiently large radius we write

$$f_{\varphi}=\frac{b_{m}}{2\xi(1-D)}\left(\eta_{a}\frac{\kappa_{1}^{2}}{\xi^{2}}+\theta_{a}\frac{\kappa_{3}^{2}}{\xi^{2}}-4D\theta_{a}\sum_{n=1}^{\infty}\frac{c_{n}\exp(-\nu_{n}^{2}a\tau)}{(\nu_{n}R)^{2}}\right)\left(1-\frac{\ln(\xi R)}{\xi R}\right), \quad (13)$$

$$f_{z}=\frac{b_{m}}{2\xi(1-D)}\left(\eta_{a}\frac{\kappa_{1}^{2}}{\xi^{2}}+\theta_{a}\frac{\kappa_{3}^{2}}{\xi^{2}}-4D\theta_{a}\sum_{n=1}^{\infty}\frac{c_{n}\exp(-\nu_{n}^{2}a\tau)}{(\nu_{n}R)^{2}}\right)\left(1-\frac{2\ln(\xi R)}{\xi R}\right). \quad (14)$$

From formulas (13), (14) follow that surface tension f_{φ} is larger than f_z and the difference vanishes with cylinder radius increase. With $R\to\infty$ these tensions tend to the value of surface tension in the half-space. Note also, that dependence of f_{φ} on R is of same kind as the dependence of surface tension in a layer on its thickness [2].

With time increase ($\tau\to\infty$) surface tension f_{φ}, f_z tends to

$$f_{\varphi}=\frac{b_{m}(\eta_{a}\kappa_{1}^{2}+\theta_{a}\kappa_{3}^{2})}{2(1-D)\xi^{3}}\left(1-\frac{\ln(\xi R)}{\xi R}\right),\quad f_{z}=\frac{b_{m}(\eta_{a}\kappa_{1}^{2}+\theta_{a}\kappa_{3}^{2})}{2(1-D)\xi^{3}}\left(1-\frac{2\ln(\xi R)}{\xi R}\right).$$

Here we see that in the stationary state the dependence of surface tension on temperature is linear.

Size effect of surface tension is illustrated in Fig. 2 as reduced surface tensions $f_{\varphi}^{*}=f_{\varphi}/f_{\kappa}^{0}$ and $f_{z}^{*}=f_{z}/f_{\kappa}^{0}$ dependence on specific dimensionless cylinder radius ξR for such parameters values $D = 0{,}25$, $\alpha_t b_m\theta_a/\alpha_m f_{\kappa} = 0{,}2$, $\alpha_m\kappa_3^2R^2/\alpha_t=1$, $\tau=1$. Here $f_{\kappa}^{0}=b_m\eta_a/(2\xi)$ is the value of tension f_{κ} in the initial moment of time.

In the process of heating due to temperature influence on stresses the surface tension changes. In Fig. 3 reduced surface tension dependence on specific time $t = a\tau/R^2$ for parameters $D = 0{,}25$, $\alpha_t b_m\theta_a/\alpha_m f_{\kappa} = 0{,}2$, $\alpha_m\kappa_3^2R^2/\alpha_t=1$, $\xi R = 50$ is shown.

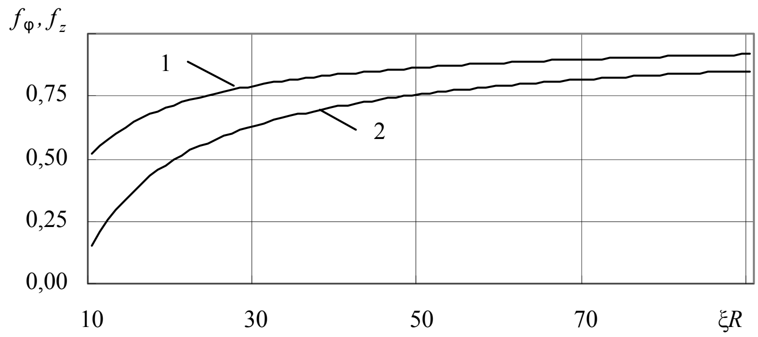

Fig. 2. Surface tension dependence on cylinder size (f_φ^* curve 1, f_z^* curve 2)

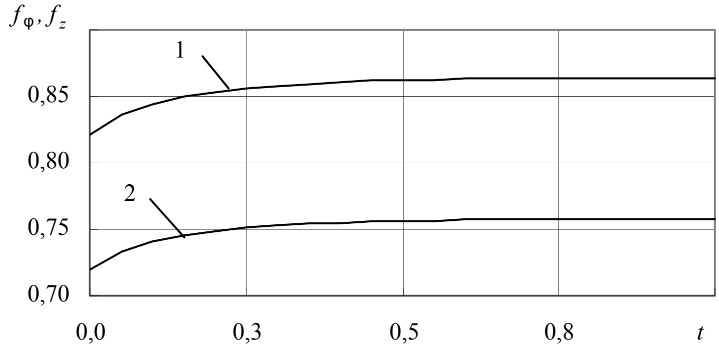

Fig. 3. Surface tension dependence on time (f_φ^* curve 1, f_z^* curve 2)

4. Strength of the cylinder during heating

In the case of stretching force load ($p > 0$) the surface value of stresses $\sigma_z(R, \tau)$ is the largest stresses in the cylinder. During heating process they are changing from value

$$\sigma_z(R,0) = p + \frac{b_m}{2}\frac{\kappa_1^2\eta_a + \kappa_2^2 p}{\xi^2\chi(\xi R)}\left[1 - \frac{2I_1(\xi R)}{\xi R I_0(\xi R)}\right],$$

at the initial moment of time to value

$$\sigma_z(R,\infty) = p + \frac{b_m}{2}\frac{\kappa_1^2\eta_a + \kappa_2^2 p + \kappa_3^2\theta_a}{\xi^2\chi(\xi R)}\left[1 - \frac{2I_1(\xi R)}{\xi R I_0(\xi R)}\right],$$

that is arrived at $\tau \to \infty$ according to formula

$$\sigma_z(R,\tau) = p + A_c\left[1 - \frac{2I_1(\xi R)}{\xi R I_0(\xi R)}\right] - \frac{b_m}{2}\theta_a \sum_{n=1}^{\infty} a_n c_n \frac{2J_1(\nu_n^c R)}{\nu_n^c R}.$$

The above relations will be used for investigation of strength of the cylinder during heating ($\theta_a > 0$). As a criterion we accept the criterion of the first classical theory of strength [5]. Suppose that the body will fracture instantly if at any point of the body maximum stresses reach critical for the body material value σ^{kr}. Using above formulas for critical value p^{kr} of the intensity of external force loading causing fracture we obtain

$$p^{kr} = \frac{2\chi(\xi R)}{1+\chi(\xi R)}\sigma^{kr} - \frac{1-\chi(\xi R)}{1+\chi(\xi R)}\frac{\kappa_1^2\eta_a + \kappa_3^2\theta_a}{\kappa_2^2} + \frac{4b_m\theta_a}{1+\chi(\xi R)}\sum_{n=1}^{\infty}\frac{c_n e^{-\nu_n^2 a\tau}}{(\nu_n R)^2}. \quad (15)$$

Denoting σ_+ the intensity of critical loading causing fracture of large radius cylinders ($\xi R >> 1$) at initial temperature ($\theta = 0$)

$$\sigma_+ = \frac{2(1-D)}{2-D}\sigma^{kr} - \frac{D}{2-D}\frac{\kappa_1^2\eta_a}{\kappa_2^2},$$

the formula (15) takes the form

$$p^{kr} = \frac{2-D}{1-D}\frac{\chi(\xi R)}{1+\chi(\xi R)}\sigma_+ + \left(\frac{D}{1-D}\frac{\chi(\xi R)}{1+\chi(\xi R)} - \frac{1-\chi(\xi R)}{1+\chi(\xi R)}\right)\frac{\kappa_1^2}{\kappa_2^2}\eta_a -$$

$$-\left(\frac{1-\chi(\xi R)}{1+\chi(\xi R)}\frac{\kappa_3^2}{\kappa_2^2} - \frac{4b_m}{1+\chi(\xi R)}\sum_{n=1}^{\infty}\frac{c_n e^{-\nu_n^2 a\tau}}{(\nu_n R)^2}\right)\theta_a. \quad (16)$$

For large radius cylinders neglecting the size effects at arbitrary temperature this relation becomes

$$p^{kr}(\tau) = \sigma_+ - \left(\frac{D}{2-D}\frac{\kappa_3^2}{\kappa_2^2} - \frac{4b_m}{2-D}\sum_{n=1}^{\infty}\frac{c_n e^{-\nu_n^2 a\tau}}{(\nu_n R)^2}\right)\theta_a. \quad (17)$$

Right hand side of formulas (16), (17) may be interpreted as relations describing cylinder strength during heating with and without account for size effect respectively. For $\tau \to \infty$ intensity of critical load for thin fibers (Fig. 4) tends to

$$p^{kr}(\infty) = \frac{2-D}{1-D}\frac{\chi(\xi R)}{1+\chi(\xi R)}\sigma_+ + \left(\frac{D}{1-D}\frac{\chi(\xi R)}{1+\chi(\xi R)} - \frac{1-\chi(\xi R)}{1+\chi(\xi R)}\right)\frac{\kappa_1^2}{\kappa_2^2}\eta_a - \frac{1-\chi(\xi R)}{1+\chi(\xi R)}\frac{\kappa_3^2}{\kappa_2^2}\theta_a,$$

and for cylinders with radius being far greater than the characteristic size of the near-surface nonhomogeneity region, the intensity of the critical load tends to

$$p^{kr}(\infty) = \sigma_+ - \frac{D}{2-D}\frac{\kappa_3^2}{\kappa_2^2}\theta_a.$$

Right hand side of the last two formulas may be interpreted as relations describing the strength of the cylinder under uniform temperature.

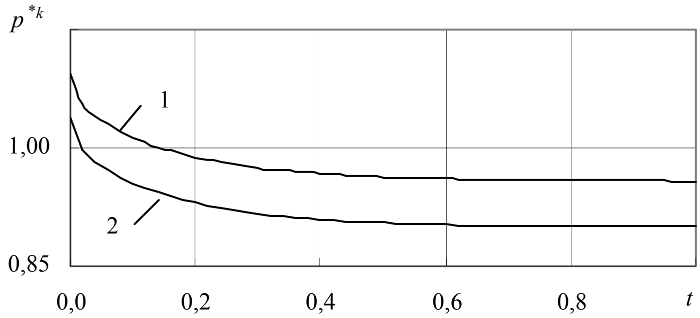

Fig. 4. Dependence on time of the force loading intensity causing cylinder fracture (ξR = 10 curve 1, ξR = 25 curve 2)

One can see that the force loading intensity causing brittle fracture of the cylinder nonmonotonously decreases with time. Relation (17) may be used for finding the time of body fracture for given intensity of loading in the case this would happen due to temperature factor. Accepting $p^{kr}(\tau) = p$ and using (17) we get the following transcendental equation for finding the time of cylinder fracture

$$\sigma_+ - \left(\frac{D}{2-D} \frac{\kappa_3^2}{\kappa_2^2} - \frac{4b_m}{2-D} \sum_{n=1}^{\infty} \frac{c_n e^{-\nu_n^2 a\tau}}{(\nu_n R)^2} \right) \theta_a = p \,.$$

If solution τ of this equation exists, it will be the moment of the time when cylinder fractures. In opposite case (the equation has no solution) breaking point will not be reached.

The dependence of some parameters of the body mechanical state on the temperature was considered above. The reason for its change over time lays in the change of temperature described by (5). So there exists a correlation of the processes of surface tension and breaking point limit change over time. This dependence may be written using (13), (14), (16).

Conclusions. On the basis of carried out investigations we can state the following:

- The value of force loading causing brittle fracture of the cylinder is determined by its physical-mechanical properties, surface values of chemical potential and temperature and quality of its change in time.
- Within the framework of the considered model the dependence of the breaking point and surface tension on the body homogeneous temperature is linear.
- The temperature change in the course of time (heating, cooling) causes the change of nearsurface stresses in a cylinder and thus affects strength parameters, surface tension and their size effects.

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Поверхневий натяг і міцність локально неоднорідного циліндра у процесі нагрівання

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У рамках локально градієнтного підходу досліджено напружений стан необмеженого циліндра у процесі його нагрівання. Отримані співвідношення використано для вивчення поверхневого натягу та залежності межі міцності циліндра від його розмірів і температури. Показано, що залежність межі міцності та поверхневого натягу від однорідної температури тіла є лінійною. У процесі нагрівання змінюються значення приповерхневих напружень, що призводить до зміни параметрів міцності та поверхневого натягу. Вказано на узгодженість одержаних результатів із відомими у літературі.

Поверхностное натяжение и прочность локально неоднородного цилиндра в процессе нагрева

Тарас Нагирный, Константин Червинка

В рамках локально градиентного подхода исследовано напряженное состояние бесконечного цилиндра в процессе его нагрева. Полученные соотношения использованы для изучения поверхностного натяжения и зависимости предела прочности цилиндра от его размеров и температуры. Показано, что зависимость предела прочности и поверхностного натяжения от однородной температуры тела является линейной. В процессе нагрева изменяются значения поверхностных напряжений, что ведет к изменению параметров прочности и поверхностного натяжения. Показано согласованность полученных результатов с известными в литературе.

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