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Love-like waves in the heterostructure consisting of superconducting $YBa_2Cu_3O_{6+x}$ and $La_{2-x}Sr_xCuO_4$

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The Love-like waves propagation solely in the magnetic vortex field existing in a superconducting heterostructure that consists of a layer applied on a half-space is investigated. The amplitude distributions of those waves have been presented.

Keywords: superconductivity, Abrikosov vortices, waves.

Introduction. Magnetic flux can penetrate a type-II superconductor in the form of Abrikosov vortices (also called flux lines, flux tubes or fluxons), each carrying a quantum of the magnetic flux. These tiny vortices of supercurrent tend to arrange themselves in a triangular or quadratic flux-line lattice. Since the vortices are formed by the applied magnetic field, around each of them the supercurrent flows. Moreover, there also exist some Lorentz force interactions among them. Those interactions are an origin of an additional mechanical (stress) field occurring in the type-II superconductor [1-6].

The vortex field near the lower critical magnetic field intensity limit H_{c1} is of the elastic character. However, if the density of the supercurrent is above its critical value and/or the temperature is sufficiently high, there occurs a flow of vortex lines in the superconducting body. In such a situation vortices behave as a fluid rather than as an elastic lattice. The «fluidity» of the vortex array is also observed when the applied magnetic field tends to its upper critical limit H_{c2}. In this way we encounter a very interesting situation in a type-II superconductor. We may say that two mechanical fields in the medium there coexist. One of them is of a pure elastic character coming from the mechanical properties of crystal lattice of the superconductor. The second one comes from the vortex array which, being of elastic character near the lower magnetic field strength limit H_{c1}, transfers smoothly into «fluid» form near the upper magnetic field strength limit H_{c2}. This way within the mixed state, the ordered and disordered states of the vortex field can anomalously coexist. The paper deals with an analysis of an elastic wave propagation solely along vortices in a heterostructure consisting of the superconducting layer applied on the superconducting substrate. Dispersion and amplitude distributions of such waves in the vortex field in that structure have been presented.

1. Formulation of the problem

The subject of our investigations is now the Love-like wave propagation along x_2 direction in a heterostructure that consists of two type-II superconductors in the form of a layer $-h < x_2 < 0$ ($YBa_2Cu_3O_{6+x}$ - YBCO) — «1» of thickness h and a half-space $x_1 > 0$ ($La_{2-x}Sr_xCuO_4$ - LSCO) — «2».

For the sake of simplicity our considerations are related to the lattice-like state of the vortex field ($\alpha = 1$, $\beta = 0$) [7, 10] as well as we have omitted the temperature influence on the vortex field (isothermal process).

The general, linearized equations describing the propagation of harmonic waves in the above heterostructure (solely in the vortex field, assuming completely depinned vortices) read as follows [7-10]

$$\mu u_{i,jj} + \eta \dot{u}_{i,jj} + (\lambda + \mu) u_{j,ij} + \frac{1}{3}\eta \dot{u}_{j,ij} + \mu_0 \left(h_{r,i} - h_{i,r}\right) H_r^0 - \rho \ddot{u}_i = 0 ,$$
$$\lambda_0^2 h_{i,kk} - h_i + u_{i,k} H_k^0 - u_{k,k} H_i^0 = 0 . \tag{1}$$

Since the viscosity coefficient η is very small the damping features in the vortex field in the sequel are neglected. The linearization has been done assuming the total magnetic field in the structure to be

$$\mathbf{H} = \mathbf{H}^0 + \mathbf{h}, \quad |\mathbf{h}| << \left|\mathbf{H}^0\right|, \quad \mathbf{H}^0 = \left[H_1^0, 0, 0\right], \quad H_1^0 = const , \tag{2}$$

where $\mathbf{h}$ is the small contribution to the total magnetic field $\mathbf{H}$ coupled with the displacement vector $\mathbf{u}$. Lame's constants, λ and μ, have been calculated from H_1^0 and H_{c1} [5, 11], μ_0 is the permeability of vacuum and λ_0 is the London penetration depth. Note that (1) are valid simultaneously for both arrays «1» and «2» in Fig. 1.

Now assuming the solutions of (1) in the following form (see Fig. 1)

$$f(x_1, x_2, t) = \overline{f}(x_1) \exp\left[i(\omega t - k x_2)\right], \tag{3}$$

where $f(x_1, x_2, t)$ stands for all fields in the set (1), that is,

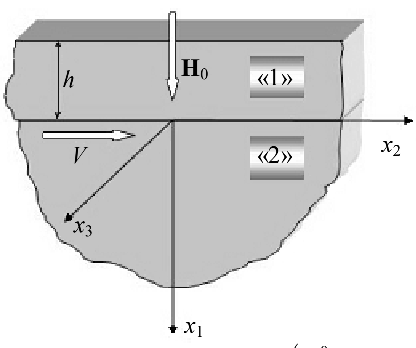

Fig. 1. Geometry of the problem $\left(\mathbf{H}^0 = [H_0, 0, 0]\right)$

$$f(x_1, x_2, t) = \{u_3, h_3\}(x_1, x_2, t) \tag{4}$$

and Love's mode concerns only u_3 component, (1) may be rewritten as follows [12-14]

$$\mu_K u^K_{3,jj} - \mu_0 h^K_{3,1} H^0_1 - \rho_K \ddot{u}^K_3 = 0 \text{ and } \lambda_0 h^0_{3,jj} - h^K_3 + u^K_{3,1} H^0_1 = 0 \text{ both with } j=1,2, \tag{5}$$

where $K = 1, 2$ distinguishes the layer «1» and the halfspace «2» (Fig. 1).

To facilitate investigations of (5) and analysis of its solutions the above formula are converted to dimensionless form (see [9]). Recasting the set (5) into dimensionless form and using (3), (4) we obtain

$$\begin{aligned}
&\tilde{\mu}_K \frac{d^2 u^K_z}{dx^2} + \frac{\Omega^2}{V^2}\left(V^2 \tilde{\rho}_K - \tilde{\mu}_K\right) u^K_z + \tilde{\mu}_0 H_0 \frac{dh^K_z}{dx} = 0, \\
&\tilde{\lambda}^2_{0K} \frac{d^2 h_z}{dx^2} - \left(\tilde{\lambda}^2_{0K} \frac{\Omega^2}{V^2} + 1\right) h^K_z + H_0 \frac{du^K_z}{dx} = 0,
\end{aligned} \tag{6}$$

$\tilde{\mu}_K$ denotes Lame-like shear elastic coefficient, u^K_z is the component of elastic displacement, Ω is the wave frequency, V is the phase velocity, $\tilde{\rho}_K$ is the density of vortices, $\tilde{\mu}_0$ is the permeability of vacuum, H_0 is the applied magnetic field, h^K_z is the component of perturbed magnetic field coupled with displacements, $\tilde{\lambda}_{0K}$ is the London penetration depth and (x, y, z) are dimensionless coordinates (Fig. 1).

The boundary and jump conditions for the dimensionless variables in (5) across the characteristic planes of the heterostructure are

$$x = -1: \begin{cases} h^1_z = 0 & \text{(continuity of the tangent component of the magnetic field intensity)}, \\ u^1_{z,x} = 0 & \text{(the plane is stress free)}, \end{cases}$$

and

$$x = 0: \begin{cases} h^1_z - h^2_z = 0 & \text{(continuity of the tangent component of the magnetic field intensity)}, \\ u^1_z - u^2_z = 0 & \text{(continuity of the displacements)}, \\ u^1_{z,x} - u^2_{z,x} = 0 & \text{(the plane is stress free)}. \end{cases}$$

The characteristic equation of (6) for both layer and substrate reads

$$\begin{aligned}
&\tilde{\lambda}^2_{0K} \tilde{\mu}_K p^4 + \left[\tilde{\lambda}^2_{0K} B_K(\Omega, V) - F_K(\Omega, V)\tilde{\mu}_K - \tilde{\mu}_0 H^2_0\right] p^2 - \\
&- F_K(\Omega, V) B_K(\Omega, V) = 0,
\end{aligned} \tag{7}$$

where the solutions of (6) were assumed to be in the form

$$\left\{u^K_z, h^K_z\right\} = \left\{{}^0u^K_z, {}^0h^K_z\right\} e^{px} \tag{8}$$

and

$$B_K(\Omega,V)=\frac{\Omega^2}{V^2}\left(V^2\tilde{\rho}_K-\tilde{\mu}_K\right),\qquad F_K(\Omega,V)=\tilde{\lambda}_{0K}^2\frac{\Omega^2}{V^2}+1>0\,.$$

The waves under consideration propagate if the solutions of (8), u_z^1 and h_z^1, are convergent, that is, the squares of the roots p_1 and p_2 of the characteristic equation (7) in the layer are both real and p_3 and p_4 in the substrate are of opposite signs. To avoid divergence of solutions (8) in the substrate, it is additionally assumed that u_z^2 and h_z^2 vanish if $x \to \infty$. The above requirements for $p_1 - p_4$ are satisfied, if

$$p_1,p_2: B_1(\Omega,V)<0\to V^2<\tilde{\mu}_1/\tilde{\rho}_1\,,$$
$$p_3,p_4: B_2(\Omega,V)>0\to V^2>\tilde{\mu}_2/\tilde{\rho}_2\,.$$

Hence, the very important propagation condition for Love's wave is obtained. That wave runs if

$$\frac{\tilde{\mu}_2}{\tilde{\rho}_2}<V^2<\frac{\tilde{\mu}_1}{\tilde{\rho}_1}\ \text{(dimensionless form)}$$

or

$$v_{T2}<v<v_{T1}\ \text{(dimensional form)}\,. \tag{9}$$

That is a new result and it differs from the classical one for the elastic Love's wave propagating along the interface between two elastic materials (layer and substrate) [12-14]. For the latter case inequality (9) is reciprocal.

As a result, the solutions (8) for the layer are:

$$u_z^1=S_1e^{p_1x}+S_2e^{-p_1x}+S_3e^{p_2x}+S_4e^{-p_2x} \tag{10}$$

and

$$h_z^1=-M(p_1,\Omega,V)S_1e^{p_1x}+M(p_1,\Omega,V)S_2e^{-p_1x}-$$
$$-M(p_2,\Omega,V)S_3e^{p_2x}+M(p_2,\Omega,V)S_4e^{-p_2x}\,, \tag{11}$$

where

$$M(p_i,\Omega,V)=\frac{p_i}{\tilde{\mu}_0H_0}+\frac{\Omega^2\left(V^2-1\right)}{V^2H_0p_i},\qquad i=1,2\,. \tag{12}$$

For the substrate the solutions are as follows

$$u_z^2=S_5e^{-p_3x}\,, \tag{13}$$
$$h_z^2=N(p_3,\Omega,V)S_5e^{-p_3x}\,, \tag{14}$$

where

$$N(p_3,\Omega,V)=\frac{\tilde{\mu}_2p_2}{\tilde{\mu}_0H_0}+\frac{\Omega^2\left(V^2\tilde{\rho}_2-\tilde{\mu}_2\right)}{\tilde{\mu}_0V^2H_0p_3}\,. \tag{15}$$

Now using solutions (10)-(13) for the boundary and jumps conditions, we arrive at the homogeneous algebraic equations

$$W_{mn}\left(\Omega, V\right) S_n = 0, \qquad m,n = \overline{1,5}\,. \tag{16}$$

Equation (16) has nontrivial solutions only if its determinant satisfies the relation below

$$\det W_{mn}\left(\Omega, V\right) = 0\,. \tag{17}$$

We have thus proved that Love's waves can propagate in a superconducting heterostructure and that their dispersion relation is given by (17) [9] (Fig.2).

2. Numerical results

The numerical analysis of the problem considered in the paper has been done for the superconducting heterostructure consisting of two ceramics: $YBa_2Cu_3O_{6+x}$ (YBCO) as the layer and $La_{2-x}Sr_xCuO_4$ as the half-space. All the necessary data are collected in Table 1. The results of the use of these data in the solutions (10), (11), (13) and (14) are presented in Fig. 3-10.

Table 1

Data for the superconducting heterostructure

Quantity	$YBa_2Cu_3O_{6+X}$	$La_{2-x}Sr_xCuO_4$	Unit
λ_0	$4\cdot10^{-7}$	$2{,}5\cdot10^{-7}$	m
ρ	10^{-6}	$5\cdot10^{-6}$	kg/m^3
H_{c1}	$0{,}01/\mu_0$	$0{,}01/\mu_0$	A/m
H_{c2}	$120/\mu_0$	$120/\mu_0$	A/m
ξ	10^{-9}	$1{,}5\cdot10^{-9}$	m
H_c	$H_{c2}\xi\big/\left(\lambda_0\sqrt{2}\right)$	$H_{c2}\xi\big/\left(\lambda_0\sqrt{2}\right)$	A/m
c_{11}	$\mu_0 H_1^{02}/4\pi$	$\mu_0 H_1^{02}/4\pi$	N/m^2
c_{66}	$(1-0{,}29b)(1-b)^2\, bH_c^2/16\pi$	$(1-0{,}29b)(1-b)^2\, bH_c^2/16\pi$	N/m^2
b	$\mu_0 H_1^0/H_{c2}$	$\mu_0 H_1^0/H_{c2}$	Vs/Am
μ	c_{66}	c_{66}	N/m^2
λ	$c_{11}-2c_{66}$	$c_{11}-2c_{66}$	N/m^2
μ_0	$4\pi\cdot10^{-7}$	$4\pi\cdot10^{-7}$	Vs/Am

Very important result obtained from (17) is that the waves considered here propagate only if the thickness of the layer satisfies inequality

$$10^{-7}\,m < h < 10^{-5}\,m\,. \tag{18}$$

Then from Fig. 2 it is seen that there are two frequency regions where waves are practically nondispersive. That means that they can be stably modulated in order to transmit signals carrying information. Between those regions there is a forbidden interval (for frequencies)

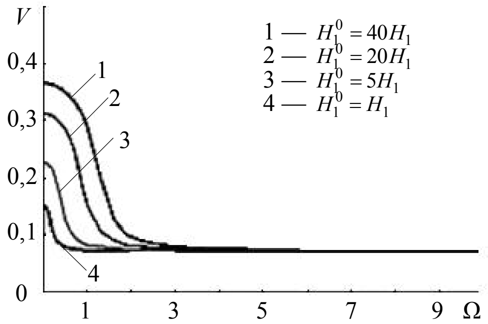

Fig. 2. Dispersion for various applied magnetic field intensities and fixed layer thickness $h = 10^{-7}\,m$

$$0,2 < \Omega < 2 \quad \left(10^8\, rad/s < \omega < 10^{12}\, rad/s\right), \qquad H_1^0 = 40H_1, \tag{19}$$

because of the very strong Love's wave dispersion is observed there. For the detailed discussion see [9]. In Figs. 3-8 the u_Z and h_Z wave amplitudes along the heterostructure have been shown then the dispersive region $\Omega \in (0{,}5;\ 1{,}0)$ is presented in Fig. 2.

To compare the above results for the dispersive region in Fig. 9 and 10 the u_Z and h_Z amplitude distributions have been presented for the nondispersive region $\Omega \in (2,\ 4)$ (Fig. 2).

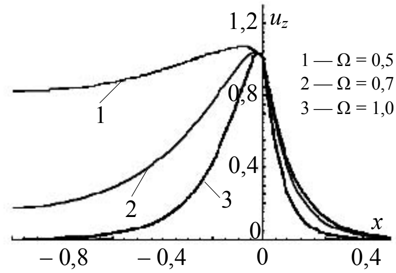

Fig. 3. u_Z wave amplitude distribution for fixed $h = 10^{-7}$m, fixed $H_0 = 5H_1$ and various frequencies Ω

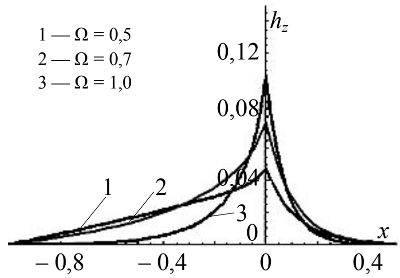

Fig. 4. h_Z wave amplitude distribution for fixed $h = 10^{-7}$m, fixed $H_0 = 5H_1$ and various frequencies Ω

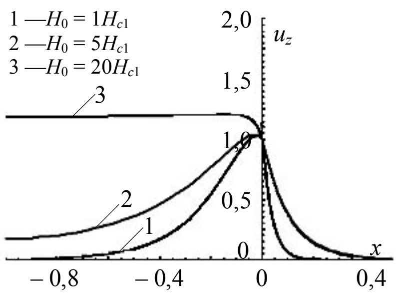

Fig. 5. u_Z wave amplitude distribution for fixed $h = 10^{-7}$m, Ω = 0,7 and various magnetic field intensities H_0

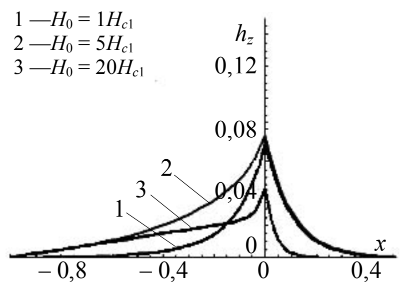

Fig. 6. h_Z wave amplitude distribution for fixed $h = 10^{-7}$m, Ω = 0,7 and various magnetic field intensities H_0

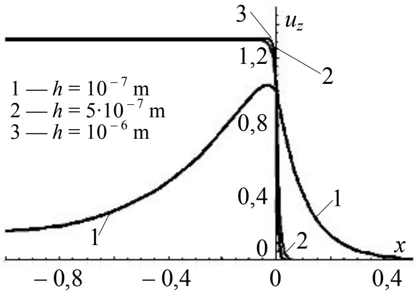

Fig. 7. u_Z wave amplitude distribution for fixed $H_0 = 5H_1$, $\Omega = 0{,}7$ and various thickness h

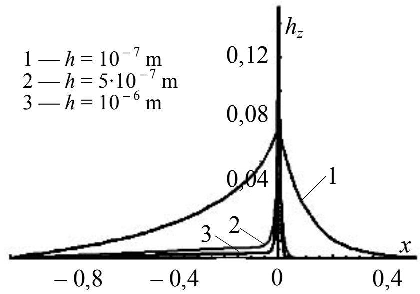

Fig. 8. h_Z wave amplitude distribution for fixed $H_0 = 5H_1$, $\Omega = 0{,}7$ and various thickness h

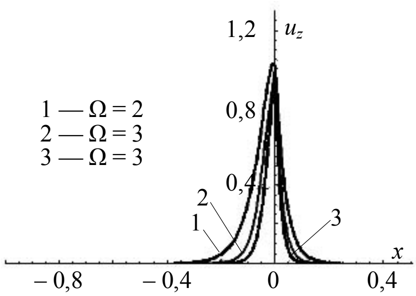

Fig. 9. u_Z wave amplitude distribution for fixed $h = 10^{-7}$m, fixed $H_0 = 5H_1$ and various frequencies Ω

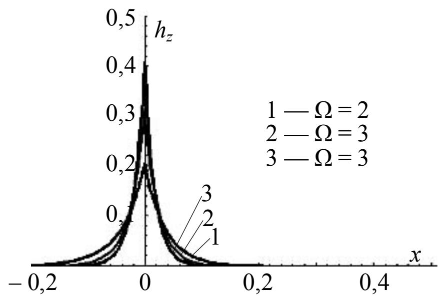

Fig. 10. h_Z wave amplitude distribution for fixed $h = 10^{-7}$m, fixed $H_0 = 5H_1$ and various frequencies Ω

Figs. 3, 5, 7 demonstrate that for a lower frequency and a higher thickness of the layer and stronger external magnetic field the distribution of the u_z waves in the layer remains practically constant within the dispersion region. So, in that region the bigger frequency of those waves, the thinner layer and the weaker magnetic field, the u_Z waves become more stable. That situation confirms Fig. 9 for u_Z (its dispersion disappears). Similar situation one can observe in Fig. 4, 6, 8 and 10 for h_Z wave amplitude distribution, respectively. Only the energy of h_Z wave is generally much more focused in the vicinity of the heterostructure interface.

Conclusions.

- It occurred that the Love-like wave can propagate in the heterostructure consisting of two vortex fields coming from two different superconductors.
- There are two dispersionless regions concerning Love's modes in the structure.
- There is a forbidden region where dispersion is very high.
- The waves under consideration propagate with acoustic phase velocity and optical wave frequency. It is a very peculiar, anomalous and interesting property.
- In the case of strong dispersion the u_z component amplitude indicates that the majority of the mechanical energy of vortices remains in the layer decreasing rapidly along the half-space depth.

- Similar situation has been observed for u_z amplitude distribution.
- However, the h_z component amplitude distribution behaves differently: the magnetic signals propagate almost solely along the interface of the heterostructure (like Love's modes in elastic structures).
- For the nondispersive region we observe both for u_z amplitudes and h_z amplitudes that the Love-like energy perturbations propagate almost solely along interface of the structure being very stable.

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Хвилі типу Лява в гетероструктурі, утвореній надпровідними $YBa_2Cu_3O_{6+x}$ та $La_{2-x}Sr_xCuO_4$

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Досліджено поширення хвиль типу Лява в полі магнітних вихорів, що виникають за наявності зовнішнього постійного магнітного поля в гетероструктурі, утвореній надпровідним півпростором, на поверхню якого нанесено надпровідне покриття скінченної товщини. Наведено розподіл амплітуд хвиль зміщення ґратки магнітних вихорів і збурення магнітного поля в околі межі поділу різнорідних надпровідних матеріалів.

Волны типа Лява в гетероструктуре, образованной сверхпроводящими $YBa_2Cu_3O_{6+x}$ и $La_{2-x}Sr_xCuO_4$

Богдан Марушевски, Анджей Джевецки, Роман Староста

Исследовано распространение волн типа Лява в поле магнитных вихрей, возникающих при наличии внешнего постоянного магнитного поля в гетероструктуре, образованной сверхпроводящим полупространством, на поверхность которого нанесено сверхпроводящие покрытия конечной толщины. Представлено распределение амплитуд волн смещения решетки магнитных вихрей и возмущения магнитного поля вблизи границы раздела разнородных сверхпроводящих материалов.

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