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Existence and propagation character of spatial spin surface waves

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The problems of the existence and propagation character of spatial spin surface waves in ferromagnic media are considered. The condition of a surface wave existence is obtained depending on the physical constants of the medium and the angle between the wave vector direction and the direction of the axis of a ferromagnet easy magnetization. The regions of the wave numbers change, where the surface wave propagation becomes impossible (zone of silence) are determined. Formulas for determining the phase velocity and penetration depth of the surface wave are found. It is show that with a certain choice of the wave vector direction one can achieve the necessary localization of the spin wave at the body surface.

Key words: ferromagnetic, spatial spin surface wave, phase velocity, penetration depth.

Introduction. Based on the equation of motion of the magnetic moment and quasistatic Maxwell's equations, in work [1] the problem of existence of spin surface wave in a ferromagnetic semispace has been studied in a two-dimensional statement. It was taken that in the Cartesian rectangular system of the coordinates x_1, x_2, x_3 the semispace occupies the region $x_2 > 0$ and the Ox_3 axis coincides with the axis of easy magnetization of the ferromagnet. The two-dimensional problem (all quantities characterizing the wave motion in a medium do not depend on the coordinate x_3) was considered: in this case the wave propagates parallel to the Ox_1 axis (perpendicularly to the axis of easy magnetization) and damps with removal from the semispace surface. The dispersion equation was derived, the analysis of which reveals that: i) in a semispace two-dimensional surface spin waves (Damon-Eshbach waves) can propagate with a frequency independent of the wave vector modulus; ii) these waves depending on the wave vector direction, propagate either only along the positive direction of the Ox_1 axis or only along the opposite direction. More recently this problem was considered by many authors [2-6] which studied the influence of various factors (homogeneity of medium, exchange interaction, external magnetic fields, etc.) on the existence and propagation character of two-dimensional surface spin waves. Information on these investigations can be found in monographs [3, 5, 7] containing a sufficiently complete review of works related to spin waves. In all mentioned works the problem of surface spin waves was studied in a two-dimensional statement.

In this paper, the problem of existence and propagation of a spin surface wave is studied in a three-dimensional statement. The functions, which are the solutions of the considered three-dimensional problem, are represented in the form $f_k(x_2)\exp[i(\omega t - -k_1 x_1 - k_3 x_3)]$, where ω is the wave frequency, k_1 and k_3 are the components of the wave vector $\mathbf{k} = k_1\mathbf{e}_1 + k_3\mathbf{e}_3$, $\mathbf{e}_i$ are the unit vectors of coordinate axes, and $f_k(x_2)$ are unknown functions to be determined. With allowance for surface conditions of the problem, the functions $f_k(x_2)$ are derived and the dispersion equation with respect to ω is obtained, which at $k_3 = 0$ coincides with the dispersion equation of the Damon-Eshbach wave. By analysis of the dispersion equation, the following condition of existence of a spin surface wave is derived: $(\beta + H^0 M_0^{-1})(k_3 k_1^{-1})^2 < 4\pi$ where β is the anisotropy constant, M_0 is the magnitude of the saturation magnetic moment $\mathbf{M}_0 = M_0\mathbf{n}$, H_0 is the strength of magnetic field $\mathbf{H}_0 = H^0\mathbf{n}$, and $\mathbf{n}$ is the unit vector along the axis of easy magnetization of the ferromagnet. This means that in the considered ferromagnetic medium i) one can excite a surface wave if the angle θ made by the vectors $\mathbf{n}$ and k belongs to the following regions (zone of existence of the surface wave): $\theta_0 < \theta < \pi - \theta_0$ and $\pi + \theta_0 < \theta < 2\pi - \theta_0$ where $\mathrm{tg}^2\,\theta_0 = (\beta + H_0 M_0^{-1})/4\pi$ and $\beta + H_0 M_0^{-1} > 0$; ii) the surface wave cannot propagate, if θ belongs to the regions $\pi - \theta_0 < \theta < \pi + \theta_0$ and $2\pi - \theta_0 < \theta < 2\pi + \theta_0$ (zone of silence). From the solution of the dispersion equation if follows also that:

a) the surface wave can propagate either only along the positive direction of the wave vector or only along the opposite direction of this vector;
b) if one changes the direction of the wave vector to the opposite one, the direction of the surface wave propagation remains unchanged;
c) with a proper choice of the wave vector direction one can achieve a necessary localization of the spin waves at the surface of the body;
d) spatial surface waves, as well as the Damon-Eshbach waves, propagate with a dispersion.

The influence of the wave vector direction on the phase velocity and penetration depth of the spatial spin surface wave is also studied.

2. Problem of propagation of spin (magnetic) waves in ferromagnets

Let us consider a dielectrical ferromagnetic crystal occupied a region Ω (inner region) of a three-dimensional Euclidean space. It is assumed that properties of a medium outside the crystal (in outer region) coincide with the properties of vacuum. A Cartesian rectangular system of coordinates x_1, x_2, x_3 is chosen in such a way that the Ox_3 axis coincides with the axis of easy magnetization of the ferromagnet.

Our investigation of the wave process is carried out on the basis of the equation of motion for the magnetic moment of the ferromagnet. In the absence of dissipative processes, this equation has the following form [7]

$$\frac{d\boldsymbol{\mu}}{dt} = g\left(\boldsymbol{\mu}\times\mathbf{H}^{(ef)}\right). \tag{1}$$

Here g is the gyromagnetic ratio ($g \approx 1{,}76\cdot 10^{7}\text{c}^{-1}\text{e}^{-1}$), $\boldsymbol{\mu}(x_1, x_2, x_3, t)$ is the magnetic moment of a unit mass of the ferromagnet, and $\mathbf{H}^{(ef)}$ is the effective magnetic field, the components of which are defined by formulas [7]

$$H_i^{(ef)} = H_i - \frac{\partial F}{\partial \mu_i} + \frac{1}{\rho}\frac{\partial}{\partial x_k}\left(\rho\frac{\partial F}{\partial \gamma_{ik}}\right),$$

$$\gamma_{ik} = \frac{\partial \mu_i}{\partial x_k}, \qquad \frac{d\boldsymbol{\mu}}{dt} = \frac{\partial \boldsymbol{\mu}}{\partial t} + v_i\frac{\partial \boldsymbol{\mu}}{\partial x_i}, \qquad i = \overline{1,3}. \tag{2}$$

In formulas (2) ρ is the density, $\mathbf{v}$ is the velocity, $F(\mu_i, \gamma_{ik})$ is the potential energy of the ferromagnet per unit mass, and $\mathbf{H}$ is the magnetic filed in the ferromagnet. Hereinafter we assume a summation over repetitive indices.

It is necessary to add to equation (1) the equations of magnetostatics in the region occupied by the ferromagnet (region Ω)

$$\text{rot}\,\mathbf{H} = 0, \qquad \text{div}\,\mathbf{B} = 0,$$

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M} = \mathbf{H} + 4\pi\rho\boldsymbol{\mu}, \tag{3}$$

and also the boundary conditions

$$\left[\mathbf{B} - \mathbf{B}^{(e)}\right]\cdot\mathbf{N} = 0, \qquad \left[\mathbf{H} - \mathbf{H}^{(e)}\right]\times\mathbf{N} = 0 \tag{4}$$

on the surface S of the ferromagnet. In formulas (4) $\mathbf{N}$ is the unit vector of the outer normal to the body surface, $\mathbf{M}$ is the magnetic induction, and index «e» implies that a considered quantity belongs to the external medium. Quantities with this index satisfy the equations of magnetostatics for vacuum

$$\text{rot}\,\mathbf{H}^{(e)} = 0, \qquad \text{div}\,\mathbf{B}^{(e)} = 0, \qquad \mathbf{B}^{(e)} = \mathbf{H}^{(e)} \tag{5}$$

and conditions at infinity.

Consider two states of magnetization of the ferromagnet. The first one will be named the equilibrium state and all quantities related to this state will be denoted by the index «0». It is assumed that in this state the medium is homogeneously magnetized up to saturation in the direction of the axis of easy magnetization: $\mathbf{M}_0 = M_0\mathbf{n} = M_S\mathbf{n}$ (M_S is the saturation magnetic moment). The second state will be named the excited state. All quantities related to this state will be denoted by the tilde and they will be represented as a sum of quantities related to the equilibrium state and to the perturbation of corresponding quantities: $\tilde{Q} = Q_0 + q$. Perturbations are considered as small quantities as compared to the corresponding quantities of the equilibrium state and are not marked by any additional indices.

According to formulas (3)-(5), characteristics of the magnetic field of the equilibrium state should satisfy the equations

$$\text{rot}\,\mathbf{H}_0 = 0, \qquad \text{div}\,\mathbf{B}_0 = 0, \qquad \mathbf{B}_0 = \mathbf{H}_0 + 4\pi\mathbf{M}_0 \tag{6}$$

in the region Ω, the equations

$$\operatorname{rot}\mathbf{H}_0^{(e)}=0,\qquad \operatorname{div}\mathbf{B}_0^{(e)}=0,\qquad \mathbf{B}_0^{(e)}=\mathbf{H}_0^{(e)} \tag{7}$$

in the external region, and the condition of interfacing on the ferromagnet surface

$$\left[\mathbf{B}_0-\mathbf{B}_0^{(e)}\right]\cdot\mathbf{N}=0,\qquad \left[\mathbf{H}_0-\mathbf{H}_0^{(e)}\right]\times\mathbf{N}=0\,. \tag{8}$$

In addition, the conditions at infinity should be fulfilled

$$\lim_{r\to\infty}\mathbf{H}_0^{(e)}=\mathbf{H}^0 \tag{9}$$

(here $r^2=x_1^2+x_2^2+x_3^2$, and $\mathbf{H}^0$ is a given external magnetic field directed along the axis of easy magnetization: $\mathbf{H}^0=H^0\mathbf{n}$), as well as the condition following from equation (1)

$$\mathbf{n}\times\mathbf{H}_0^{(ef)}=0\,. \tag{10}$$

Further, for simplicity, we consider such regions Ω (occupied by the body) where the problem (6)-(9) has a solution $\mathbf{H}_0$ parallel to the vector $\mathbf{n}$: $\mathbf{H}_0=H_0\mathbf{n}$. Such regions are, for instance, the regions occupied by an infinite cylindrical body, the axis of which is parallel to the axis of easy magnetization, and the region occupied by a body in the form of an infinite layer or a semispace the boundary planes of which are parallel to the vector $\mathbf{n}$.

According to the above, the quantities characterizing a perturbed state of the medium should be represented as

$$\tilde{\mu}_i=\mu_{0i}+\mu_i,\qquad \tilde{\rho}=\rho_0+\rho,\qquad \tilde{H}_i=H_{0i}+h_i,$$
$$\tilde{H}_i^{(ef)}=H_{oi}^{(ef)}+h_i^{(ef)}=H_{0i}+h_i^{(ef)},\qquad \tilde{v}_i=v_{0i}+v_i=v_i\,. \tag{11}$$

Here $\rho_0=const$ is the equilibrium density of the ferromagnet, $\boldsymbol{\mu}_0=\rho_0^{-1}\mathbf{M}_0$ is the magnetic moment density of the equilibrium state, μ_i,ρ,h_i and $h_i^{(ef)}$ are perturbations of the corresponding quantities of the equilibrium state.

As seen from formulas (2), in order write the equations with respect to the perturbations μ_i,h_i, and $h_i^{(ef)}$ of the equilibrium state, it is necessary to give an expression for the density of the potential energy F of the ferromagnet. Here we consider the expression for F in the case of small perturbations and a low gradient of the magnetic moment density, restricting ourselves to uniaxial ferromagnets. Then, expanding the function $F\left(\mu_i,\partial\mu_i/\partial x_k\right)$ into a Taylor series in the vicinity of equilibrium state and limiting ourselves up to the terms of the second-order smallness, we obtain for F the representation [7]

$$2\rho_0^{-1}F=\beta\left(\boldsymbol{\mu}\cdot\boldsymbol{\mu}\right)+b\left(\boldsymbol{\mu}\cdot\mathbf{n}\right)^2+\lambda\frac{\partial\mu_i}{\partial x_k}\frac{\partial\mu_i}{\partial x_k}, \tag{12}$$

where β is the magnetic anisotropy constant of the medium and λ is the exchange constant (modulus of the exchange interaction).

Taking into account expression (12) and basic assumption of the theory of small perturbations (for instance, such as $|q| << |Q_0|$, $|q|^2 << |q|$, etc.) from formulas (2) we get the following linearized expression for the perturbation of the effective magnetic field

$$\mathrm{h}^{(ef)} = \mathrm{h} - \rho_0 \beta \mu + \rho_0 \lambda \Delta \mu - \rho_0 b (\mathrm{n} \cdot \mu) \mathrm{n}, \tag{13}$$

where Δ is the three-dimensional Laplace operator.

Similarly, considering expressions (6)-(11) and conditions for the smallness of perturbations, one can derive from formulas (1) and (3)-(5) the linearized equations and boundary conditions describing the perturbations of corresponding quantities, characterizing the equilibrium state of the considered ferromagnetic medium:
equations in the region Ω

$$\frac{\partial \boldsymbol{\mu}}{\partial t} = g\mu_0 \mathbf{n} \times \left[\mathbf{h} - \rho_0 \left(\beta + \frac{H^0}{M_0} \right) \boldsymbol{\mu} + \rho_0 \lambda \Delta \boldsymbol{\mu} \right],$$

$$\mathrm{rot}\,\mathbf{h} = 0, \qquad \mathrm{div}\left(\mathbf{h} + 4\pi\rho_0 \boldsymbol{\mu} \right) = 0; \tag{14}$$

equations in the external region

$$\mathrm{rot}\,\mathbf{h}^{(e)} = 0, \qquad \mathrm{div}\,\mathbf{h}^{(e)} = 0, \tag{15}$$

boundary conditions on the surface S

$$\left(\mathbf{h} - \mathbf{h}^{(e)} + 4\pi\rho_0 \boldsymbol{\mu} \right) \cdot \mathbf{N} = 0, \qquad \left(\mathbf{h} - \mathbf{h}^{(e)} \right) \times \mathbf{N} = 0 \tag{16}$$

and conditions of perturbations damping at infinity

$$\lim_{r \to \infty} \mathbf{h}^{(e)} = 0. \tag{17}$$

Equations and boundary conditions of type (14)-(17) characterizing the propagation of the spin (magnetic) waves in ferromagnets, based on different approaches, have been obtained in many works (see [3, 5, 7]). The method of small perturbations used in this paper is similar to the approach applied in [7], and the final equations and surface conditions derived in [7] completely coincide with formulas (13)-(17) at $H^0 = 0$.

3. Dispersion equation of spatial spin surface waves

Let Ω be a semispace the boundary of which is parallel to the axis of easy magnetization of a ferromagnet occupying the region Ω. The system of coordinates x_1, x_2, x_3 is chosen so that Ω coincides with the region $x_2 > 0$ and the Ox_3 axis is directed along the vector $\mathbf{n}$, i. e. along the axis of easy magnetization. Then let a medium be in a permanent magnetic field $\mathbf{H}^0(0, 0, H^0)$. In this case the problem of determining the magnetic field $\mathbf{H}_0$ of the equilibrium state, i. e. the problem (6)-(9), has the solution

$$H_{01} = H_{02} = 0, \qquad H_{03} = H^0. \tag{18}$$

Based on formulas (18) and (14)-(17), the study of the wave process in a considered magnetic system, in the case of the long-wavelength approximation $\left(|\beta\boldsymbol{\mu}| >> |\lambda\Delta\boldsymbol{\mu}|\right)$, is reduced to solving the equations

$$\frac{\partial\mu_1}{\partial t} - \omega_M\hat{\beta}\mu_2 - g\mu_0\frac{\partial\varphi}{\partial x_2} = 0,$$
$$\frac{\partial\mu_2}{\partial t} + \omega_M\hat{\beta}\mu_1 + g\mu_0\frac{\partial\varphi}{\partial x_1} = 0,$$
$$\frac{\partial\mu_3}{\partial t} = 0,$$
$$\Delta\varphi - 4\pi\rho_0 \operatorname{div}\boldsymbol{\mu} = 0 \tag{19}$$

at $x_2 > 0$ and the equation

$$\Delta\varphi^{(e)} = 0 \tag{20}$$

at $x_2 < 0$ with the conditions at the surface $x_2 = 0$

$$\varphi = \varphi^{(e)}, \qquad \frac{\partial\varphi}{\partial x_2} - 4\pi\rho_0\mu_2 = \frac{\partial\varphi^{(e)}}{\partial x_2} \tag{21}$$

and the conditions of damping of perturbations at infinity

$$\lim_{x_2\to-\infty}\varphi^{(e)} = 0, \qquad \lim_{x_2\to+\infty}\varphi = 0, \qquad \lim_{x_2\to+\infty}\mu = 0. \tag{22}$$

In formulas (19)-(22) the functions $\phi\left(x_1,x_2,x_3,t\right)$ and $\phi^{(e)}\left(x_1,x_2,x_3,t\right)$ are the potentials of the perturbed magnetic filed in the medium and vacuum, respectively, Δ is the three-dimensional Laplace operator

$$\mathbf{h} = -\operatorname{grad}\varphi, \qquad \mathbf{h}^{(e)} = -\operatorname{grad}\varphi^{(e)},$$
$$\omega_M = g\rho_0\mu_0 = gM_0, \qquad \hat{\beta} = \beta + H^0 M_0^{-1}.$$

Note that in formula (22) the conditions at $x_2 \to +\infty$ are also the necessary conditions for existence of a surface wave.

Solutions of equations (19), corresponding to propagation of the wave with a frequency ω, wave numbers k_1, k_3 and an amplitude depending on the coordinate x_2 can be sought for as

$$\mu_j = f_j\left(x_2\right)\exp\left[i\left(\omega t - k_1x_1 - k_3x_3\right)\right],$$
$$\varphi = \Phi\left(x_2\right)\exp\left[i\left(\omega t - k_1x_1 - k_3x_3\right)\right] \qquad \left(j = \overline{1,3}\right). \tag{23}$$

Substitution of formulas (23) into the first three equations of system (19), with allowance for the last condition from the set (22), leads to the following expressions for unknown functions $f_j\left(x_2\right)$

$$f_1(x_2) = \frac{ig\mu_0}{\omega_M^2\hat{\beta}^2 - \omega^2}\left(\omega\frac{d\Phi}{dx_2} + \omega_M\hat{\beta}k_1\Phi\right),$$

$$f_2(x_2) = -\frac{g\mu_0}{\omega_M^2\hat{\beta}^2 - \omega^2}\left(\omega_M\hat{\beta}\frac{d\Phi}{dx_2} + \omega k_1\Phi\right),$$

$$f_3(x_2) \equiv 0, \qquad \omega^2 \neq \omega_M^2\hat{\beta}^2. \tag{24}$$

By substituting expression (23) and (24) into the last equation of system (19) we obtain the following ordinary differential equation with respect to the unknown function $\Phi(x_2)$

$$\alpha\frac{d^2\Phi}{dx_2^2} - \left(k_3^2 + \alpha k_1^2\right)\Phi = 0, \qquad \alpha = \frac{\hat{\beta}\left(\hat{\beta} + 4\pi\right)\omega_M^2 - \omega^2}{\omega_M^2\hat{\beta}^2 - \omega^2}. \tag{25}$$

Solutions of equation (20) (in the region $x_2 < 0$) corresponding to representation (23) and satisfying the first condition from the set (22), have the form

$$\varphi^{(e)} = A^{(e)}e^{kx_2}\exp i\left(\omega t - k_1x_1 - k_3x_3\right), \qquad k = \sqrt{k_1^2 + k_3^2} > 0, \tag{26}$$

where $A^{(e)}$ is an arbitrary constant.

As seen from equation (25), at $\alpha = 0$ one has either a trial solution $\left(\mu_i \equiv 0, h_i \equiv 0\right)$ or a transverse bulk wave. Therefore we assume that $\alpha \neq 0$, because further only the problems of existence and propagation of surface waves are considered. Then, by using formulas (23), (24), (26) and surface condition (21) one can show that, if $k_1 = 0$, then spin surface waves cannot propagate in the ferromagnetic medium under consideration. Indeed, if $k_1 = 0$, equation (25) can have a solution satisfying the condition $\lim_{x_2\to+\infty}\Phi(x_2) = 0$ (condition of perturbations damping at infinity) only at $\alpha > 0$. Then for $\Phi(x_2)$ we obtain

$$\Phi(x_2) = B\exp\left(-\frac{|k_3|}{\alpha}x_2\right). \tag{27}$$

Substituting expressions (26) and (27) into formula (21), we get the dispersion equation

$$1 + \sqrt{\alpha} = 0,$$

which does not have real roots. Hence, it is impossible to excite surface spin wave propagation along the axis of easy magnetization.

Now we proceed to consideration of the general case $\alpha \neq 0, k_1 \neq 0$. In this case equation (25) has a solution vanishing at infinity $\left(x_2 \to +\infty\right)$ only in the case when the following condition is fulfilled

$$\delta = k_1^2 + \frac{1}{\alpha} k_3^2 > 0\,, \tag{28}$$

which is a necessary condition for existence of a surface wave.

Under condition (28), the solution to equation (25), satisfying the conditions of damping of perturbations at infinity, has the form

$$\Phi\left(x_2\right) = A \exp\left(-\sqrt{\delta} x_2\right), \tag{29}$$

where A is an arbitrary constant.

Satisfying the surface condition (21), one can derive the following dispersion equation with respect to the frequency ω

$$\frac{4\pi x}{\hat{\beta}^2 - x^2} - \sqrt{1 + r^2} = \alpha \sqrt{1 + \frac{r^2}{\alpha}}\,, \tag{30}$$

where

$$x = \frac{\omega}{\omega_M} \frac{k_1}{|k_1|}, \qquad r = k_3 k_1^{-1}\,, \qquad \alpha = \frac{\hat{\beta}\left(\hat{\beta} + 4\pi\right) - x^2}{\hat{\beta}^2 - x^2}\,. \tag{31}$$

Thus, the problem of existence of a spatial surface wave (and, hence, the character of its propagation) depends on whether equation (30) has real roots satisfying condition (28) or not.

4. Solution of dispersion equation. Condition of existence and character of propagation of surface waves

Taking into account that the quantity α is nonzero and in view of formula (31), it is convenient to represent the dispersion equation (30) in the form

$$f(x) = \sqrt{g(x)}\,, \tag{32}$$

where

$$f(x) = \frac{\sqrt{1 + r^2}\left(x^2 - \hat{\beta}^2\right) + 4\pi x}{\hat{\beta}\left(\hat{\beta} + 4\pi\right) - x^2}, \qquad g(x) = \frac{\left(1 + r^2\right)\left(\hat{\beta}^2 - x^2\right) + 4\pi\hat{\beta}}{\hat{\beta}\left(\hat{\beta} + 4\pi\right) - x^2}\,. \tag{33}$$

In consideration of equation (32) we restrict ourselves to the case $\hat{\beta} > 0$. This equation has real roots only in those cases when

$$f(x) > 0, \qquad g(x) > 0\,. \tag{34}$$

With allowance for expression (33), one can establish that the function $f(x)$ is positive in the regions

$$x^+ < x < \beta_1\,; \tag{35}$$

$$-\beta_1 < x < x^-\,, \quad \text{if} \quad \hat{\beta} r^2 > 4\pi\,; \tag{36}$$

$$x^- < x < -\beta_1\,, \quad \text{if} \quad \hat{\beta} r^2 < 4\pi\,, \tag{37}$$

and the function $g(x)$ in the regions

$$-\sqrt{\hat{\beta}^2 + \frac{4\pi\hat{\beta}}{1+r^2}} < x < \sqrt{\hat{\beta}^2 + \frac{4\pi\hat{\beta}}{1+r^2}}\ ; \qquad (38)$$

$$x > \beta_1\ ; \qquad (39)$$

$$x < -\beta_1\ . \qquad (40)$$

In expressions (35)-(40) the following notations were introduced

$$x^{\pm} = -\frac{2\pi}{\sqrt{1+r^2}} \pm \sqrt{\hat{\beta}^2 + \left(\frac{2\pi}{\sqrt{1+r^2}}\right)^2}\ , \qquad \beta_1 = \sqrt{\hat{\beta}^2 + 4\pi\hat{\beta}}\ ,$$

From these expressions one can conclude that conditions (34) (at which equation (32) can have real roots) are fulfilled in the following cases:

$$0 < x^+ < x < \sqrt{\hat{\beta}^2 + \frac{4\pi\hat{\beta}}{1+r^2}}\ ; \qquad (41)$$

$$x^- < x < -\beta_1 \text{ if } \hat{\beta}r^2 < 4\pi\ . \qquad (42)$$

Taking into account expressions (41) and (42), it is easy to show that equation (32) does not have positive roots. Hence, if

$$\hat{\beta}r^2 > 4\pi\ , \qquad (43)$$

equation (32) does not have real roots, i. e., under condition (43) the existence of a spin surface wave becomes impossible.

However, if

$$\hat{\beta}r^2 < 4\pi\ , \qquad (44)$$

equation (32) has a single root belonging to region (42) and determined by the formula

$$x = -\frac{\hat{\beta}\left(2+r^2\right)+4\pi}{2\sqrt{1+r^2}}\ . \qquad (45)$$

Thus, expression (44) is the condition of existence of a surface spin wave in the ferromagnetic medium under consideration.

Let us introduce an angle θ between the direction of the axis of easy magnetization (direction of the vector **n**) and the direction of the wave vector $\mathbf{k}\left(k_1, 0, k_3\right)$ counted from the vector **n** clockwise. Then from equation (44) we conclude that in the considered ferromagnetic medium:

a) one can excite a surface wave if θ belongs to the following regions (zones of existence of a surface wave)

$$\theta_0 < \theta < \pi - \theta_0 \quad \text{and} \quad \pi + \theta_0 < \theta < 2\pi - \theta_0\ , \qquad \theta_0 = \sqrt{\operatorname{arctg}\left(\frac{\beta}{4\pi}\right)}\ ; \qquad (46)$$

b) a surface wave cannot propagate if θ belongs to the regions (zone of silence)

$$\pi-\theta_0<\theta<\pi+\theta_0, \qquad 2\pi-\theta_0<\theta<2\pi+\theta_0. \tag{47}$$

Let us return to expression (45) which, in view of formulas (31), allows one to obtain the following formulas for determining the frequency ω of a surface wave depending on the square of the ratio of wave numbers $\left(k_3^2 k_1^{-2}\right)$ and the sign of the wave number k_1

$$\omega=-\omega_M \frac{|k_1|}{k_1} \frac{\hat{\beta}\left(2+r^2\right)+4\pi}{2\left(1+r^2\right)^{1/2}}. \tag{48}$$

Substituting $k_3 = 0$ into formula (48), we get the known expression for the frequency of a two-dimensional surface wave (Damon-Eshbach waves [1])

$$\omega_{DE}=-\left(\hat{\beta}+2\pi\right)\omega_M \frac{|k_1|}{k_1}. \tag{49}$$

By comparing formulas (48) and (49) we arrive at the inequality $|\omega| \le |\omega_{DE}|$ (the equality takes place only at $k_3 = 0$).

It is easy to establish from expressions (48) and (23) that:

i) if $k_1 > 0$, the surface wave can propagate only in the opposite direction of the wave vector **k**;
ii) if $k_1 < 0$, the direction of the surface wave propagation coincides with the direction of the wave vector **k**;
iii) if one changes the wave vector direction to the opposite one, the direction of the surface wave propagation remains unchanged.

Formula (48) allows one also to derive the following expression for determining the modulus v of the phase velocity $\mathbf{v}\left(\mathbf{v}=\mathbf{N}\omega/k, \mathbf{N}=\mathbf{k}/k\right)$ is the wave normal, $k=\sqrt{k_1^2+k_3^2}$ is the wave numbers) of the surface waves

$$v=\frac{|\omega|}{\sqrt{k_1^2+k_3^2}}=\frac{\omega_M}{2|k_1|}\frac{2\left(\hat{\beta}+2\pi\right)k_1^2+\hat{\beta}k_3^2}{k_1^2+k_3^2}. \tag{50}$$

By taking $k_3 = 0$ in expression (50) we obtain the following formula to determine the phase velocity of the Damon-Eshbach wave $\left(k_1 \ne 0\right)$

$$v_{DE}=\left(\hat{\beta}+2\pi\right)\frac{\omega_M}{|k_1|}. \tag{51}$$

Formulas (50) and (51) show that spatial surface waves as well as the Damon-Eshbach waves propagate with dispersion.

Using expressions (28)-(31) and (45), it is easy to show that the penetration depth γ of a surface wave (i. e., the depth, at which the wave amplitude falls e times) is determined by

$$\gamma = \frac{1}{|k_1|\sqrt{1+r^2}} \frac{4\pi - \hat{\beta}r^2}{4\pi + \hat{\beta}r^2} . \tag{52}$$

Note that in this case $\gamma > 0$, according to condition (44) (condition of existence of a surface wave).

Let us consider the dependence of the phase velocity **v** and penetration depth γ of a surface wave on the direction of the wave vector **k**, taking $|\mathbf{k}| = R = const$. Then the mentioned quantities are represented in the following way $\left(k_1^2 = R^2 - k_3^2 > 0\right)$

$$v = \frac{\omega_M}{2R} \frac{2\left(\hat{\beta}+2\pi\right) - \left(\hat{\beta}+4\pi\right)\bar{k}_3^2}{\left(1-\bar{k}_3^2\right)^{\frac{1}{2}}}, \quad \gamma = \frac{1}{R} \frac{4\pi\left(1-\bar{k}_3^2\right) - \hat{\beta}\bar{k}_3^2}{4\pi\left(1-\bar{k}_3^2\right) + \hat{\beta}\bar{k}_3^2}, \quad \bar{k}_3^2 = \frac{k_3^2}{R^2} . \tag{53}$$

From the condition of existence of a surface wave (44) it follows that $\bar{k}_3 \in (-a, a)$, where $a = \left[4\pi/\left(\hat{\beta}+4\pi\right)\right]^{1/2}$. As seen from expression (53), $v(k_3)$ and $\gamma(k_3)$ are the even functions of k_3 and have a maximum at the point $k_3 = 0$. There are not any other points of extremum in the interval $(-a, a)$. The case of $k_3 = 0$ corresponds to the Damon-Eshbach wave, and the maximum values of these functions are the phase velocity $\left(v(0) = \left(\hat{\beta}+2\pi\right)\omega_M / R\right)$ and penetration depth $\left(\gamma(0) = R^{-1}\right)$ of this wave. Hence, one can conclude that:

i) the phase velocity of a spatial surface wave is lower than that of the Damon-Eshbach wave;

ii) the penetration depth $\gamma(k_3)$ of a spatial surface wave satisfies the condition $\gamma(k_3) \le \gamma(0)$ (where $\gamma(0) = R^{-1}$ is the penetration depth of the Damon-Eshbach wave) and is an infinitely small function in the vicinity of the points $k_3 = \pm aR$;

iii) in contrast to spatial surface waves, the penetration depth of the Damon-Eshbach wave does not depend on the magnetic properties of the medium.

Based on the above properties of a spatial surface wave (particularly, the property ii)) we conclude also that with a certain choice of the wave vector direction one can achieve a necessary localization of the spin wave at the surface of the body.

References

[1] *Damon, R. W.* Magnetostatic modes of a ferromagnetic slab / *R. W. Damon, I. R. Eshbach* // J. Phys. Chem. Solids. — 1961. — Vol. 19. — P. 308.
[2] *Benson, H.* Spin Waves in Thin Films; Dipolar Effects / *H. Benson, D. L. Mills* // Phis. Rev. — 1969. — Vol. 178. — P. 839.
[3] *Гуревич, А. Г.* Магнитный резонанс в ферритах и антиферромагнетиках / *А. Г. Гуревич*. — Москва: Наука, 1973. — 593 с.
[4] *Tucker, J. W.* Microwave ultrasonics in solid state physics / *J. W. Tucker, V. W. Rampton*. — Amsterdam: North-Holland, 1972.
[5] *Можен, Ж.* Механика электромагнитных сплошных сред / *Ж. Можен*. — Москва: Мир, 1991. — 560 с.
[6] *Baghdasaryan, G. Y.* Surface spin waves in laminated media / *G. Y. Baghdasaryan, D. J. Hasanayn, S. L. Sahakyan.* — Proc. Int. Conf. on Applied and Mathematical Aspects of Natural Sciences. — 1999. — Vol. 3. — P. 34.
[7] *Ахиезер, А. И.* Спиновые волны / *В. Г. Барьяхтар, С. В. Пелетминский.* — Москва: Наука, 1973. — 368 с.

Існування та характер поширення просторових спінових поверхневих хвиль у феромагнетиках

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Досліджено питання існування та характер поширення просторових спінових поверхневих хвиль у феромагнітних середовищах. Отримано умову існування поверхневої хвилі залежно від фізичних постійних матеріалу середовища та від кута, утвореного напрямком хвильового вектора з напрямком осі легкого намагнічування феромагнетику. Визначені області зміни хвильових чисел, за яких поширення поверхневої хвилі є неможливе (зони мовчання). Знайдені формули для визначення фазової швидкості та глибини проникнення поверхневої хвилі. Показано, що з вибором напрямку хвильового вектора можна досягти необхідної локалізації спінової хвилі біля поверхні тіла.

Существование и характер распространения пространственных спиновых поверхностных волн в ферромагнетиках

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Исследованы вопросы существования и характер распространения пространственных спиновых поверхностных волн в ферромагнитных средах. Получено условие существования поверхностной волны в зависимости от физических постоянных материала среды и от угла, составленного направлением волнового вектора с направлением оси легкого намагничивания ферромагнетика. Определены области изменения волновых чисел, при которых распространение поверхностной волны становится невозможным (зоны молчания). Найдены формулы для определения фазовой скорости и глубины проникновения поверхностной волны. Показано, что с выбором направления волнового вектора можно достичь необходимой локализации спиновой волны у поверхности тела.

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