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Effect of spring fixation on dynamics of reservoir with liquid

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Mathematical model of combined motion of rigid cylindrical reservoir, filled by liquid with a free surface and fixed by spring to immovable point, is constructed. Nonlinear oscillations of the system under impulse force are investigated for springs of different stiffness. Potential of usage of spring with the purpose of liquid vibroprotection is studied.

Keywords: rigid cylindrical reservoir, liquid with a free surface, elastic fixation, nonlinear oscillations, variational algorithm.

Introduction. Problems of dynamics of liquid with a free surface in reservoirs for different variants of fixations are caused by demands of modern engineering. Reservoirs with liquid are used in engineering structures in machine building, aircraft and rocket construction, means of transport and storage of liquid cargo [1, 2, 3]. Special attention is paid to behavior of these structures under impulse loading, as well as to means of vibroprotection of such systems. Until now analytical solutions of these non-stationary boundary values problems are not obtained, therefore, for investigation of these problems approximate algorithms, based mostly on variational techniques are used. We state the problem of investigation of effect of spring fixation of reservoir on restriction of liquid oscillations for impulse disturbance of system motion, which models, for example, abnormal situations of operation.

1. Object of investigation

We consider translational motion in the horizontal plane of absolutely rigid cylindrical reservoir, partially filled by liquid and attached by spring to immovable point. At the initial time instant the system reservoir — liquid with a free surface is at rest state. Motion of the system is generated by force, applied to reservoir walls, in the form of rectangular impulse of duration Δ and height F.

The general scheme of such mechanical system is shown in Fig. 1. Here τ is the domain, occupied by liquid at arbitrary time, S and S_0 are perturbed and unperturbed free surface of liquid, Σ and Σ_0 are surfaces of contact of liquid with reservoir walls in perturbed and unperturbed states. We assume that liquid is ideal, homogeneous and at initial time vortex motions are absent, since we consider the problem for ground-based conditions we neglect contribution of capillary forces on liquid oscillations.

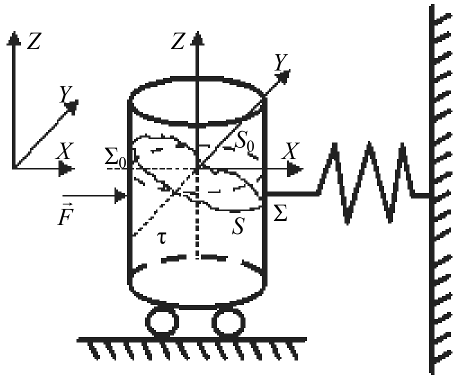

Fig. 1. General scheme of the mechanical system

2. Mathematical model

The problem is solved on the basis of variational algorithms, which are based on mathematical statement of the problem in the form of the Hamilton-Ostrogradskiy variational principle. Elevation of a liquid free surface is described by the equation $z = \xi(x, y, t)$. Mathematical problem represents a system of kinematic and dynamic conditions. We have the following kinematic restrictions: continuity equation for liquid

$$\Delta\phi = 0 \quad \text{in} \quad \tau, \tag{1}$$

here $\phi = \phi_0 + \dot{\vec{\varepsilon}} \cdot \vec{r}$, boundary conditions of non-flowing through surface of contact reservoir — liquid and a free surface

$$\left.\frac{\partial\phi}{\partial n}\right|_{\Sigma} = \dot{\vec{\varepsilon}} \cdot \vec{n}, \qquad \left.\frac{\partial\phi}{\partial n}\right|_{S} = \dot{\vec{\varepsilon}} \cdot \vec{n} + \frac{\dfrac{\partial\xi}{\partial t}}{\sqrt{1 + (\vec{\nabla}\xi)^2}}, \tag{2}$$

where $\vec{r}$ is radius-vector of points of the domain τ, $\vec{\varepsilon}$ is vector of translational motion of the reservoir, ϕ is the potential of velocities of liquid. At initial time instant displacements and velocities for all system components are accepted as zero. Dynamic boundary conditions are obtained from the Hamilton-Ostrogradskiy principle as natural.

Let us write kinetic and potential energy for every component of the system

$$T_l = \frac{1}{2}\rho\int_{\tau}\left(\vec{\nabla}\phi + \dot{\vec{\varepsilon}}\right)^2 d\tau, \quad T_r = \frac{1}{2}M_r\dot{\vec{\varepsilon}}^2, \tag{3}$$

$$\Pi_l = \frac{1}{2}\rho g\int_{S_0}\xi^2 dS, \quad \Pi_r = 0, \quad \Pi_f = F\varepsilon_x, \quad \Pi_s = \frac{1}{2}c\varepsilon_x^2. \tag{4}$$

Here T_l is kinetic energy of liquid, T_r kinetic energy of the reservoir, Π_l is potential energy of liquid, Π_r is potential energy of the reservoir, Π_f is potential energy of the applied force (conventional representation), Π_s is potential energy of elastic forces of the spring, ρ is liquid density, M_r is mass of the reservoir, c is stiffness factor of spring.

Taking into account the expressions (3), (4) let us write the Lagrange function for the mentioned system

$$L=\frac{1}{2}\rho\int_{\tau}\left(\vec{\nabla}\phi+\dot{\vec{\varepsilon}}\right)^2 d\tau+\frac{1}{2}M_r\dot{\vec{\varepsilon}}^2-\frac{1}{2}\rho g\int_{S_0}\xi^2 dS-F\varepsilon_x-\frac{1}{2}c\varepsilon_x^2. \tag{5}$$

Further we consider kinematic boundary conditions as mechanical constraints, which superimpose restriction on variations in the Hamilton-Ostrogradskiy variational principle

$$\delta\int_{t_1}^{t_2} L dt=0. \tag{6}$$

Dynamical boundary condition on a free surface of liquid follows from the Hamilton-Ostrogradskiy variational principle, and in the case of movable reservoir it takes the following form

$$\frac{\partial\phi}{\partial t}+\frac{1}{2}\left(\vec{\nabla}\phi\right)^2-\dot{\vec{\varepsilon}}\cdot\vec{\nabla}\phi+g\xi=0 \text{ on } S. \tag{7}$$

3. Construction of discrete model of the system

Investigation of nonlinear dynamics of combined motion of the system reservoir — liquid with a free surface was done according to the method of publication [1]. Nonlinear discrete model for this system is constructed on the basis of the Kantorovich method applied to kinematic restrictions of the problem and to variation formulation of the problem on the basis of the Hamilton-Ostrogradskiy variational principle. For efficient usage of the variational principle it is necessary to satisfy kinematic constraints before solving the variational problem. Since irrotational motion of ideal, homogeneous incompressible liquid is completely defined by motion of its boundaries, then elevation of a free surface ξ and displacements of tank walls $\vec{\varepsilon}$ completely characterize motion of liquid with a free surface, which is defined by liquid velocity potential ϕ.

For solutions of the nonlinear problem we shall use for values ϕ and ξ decompositions by normal modes of the linear problem ψ_n about motion of liquid with a free surface in movable reservoir, which holds kinematic boundary conditions on reservoir walls and linearized boundary conditions on a free surface

$$\phi=\sum_n b_n(t)\,\psi_n(r,\theta)\frac{\operatorname{ch}\chi_n\left(z+H\right)}{\chi_n\operatorname{sh}\chi_n H},\quad \xi=\sum_i a_i(t)\psi_i(r,\theta), \tag{8}$$

here a_i is amplitude parameter of excitation of normal modes of oscillations of liquid. Nonlinear kinematic boundary conditions on a free surface are satisfied by means of the Galerkin method [1]. According to this technique the coefficients $b_n(t)$ are expressed by the parameters a_n and their time derivatives. Decompositions (8) make it possible to transit from continuum system to its discrete constrained model. It is possible to accept amplitude parameters a_n and parameters of translational motion of the reservoir $\vec{\varepsilon}$ as independent variables. If we eliminate kinematic boundary conditions, we obtain the

Lagrange function, which corresponds to free mechanical system with the number of independent parameters equal to the number of degrees of freedom of the system. Therefore, the obtained model is o minimal dimensionality. Further from the Hamilton-Ostrogradskiy variational principle we obtain the motion equations in parameters a_i and $\vec{\varepsilon}$

$$\sum_i \ddot{a}_i \left(\delta_{ir} + \sum_j a_j A^3_{rij} + \sum_{j,k} a_j a_k A^4_{rijk} \right) = -k_r \dot{a}_r + \sum_{i,j} \dot{a}_i \dot{a}_j C^3_{ijr} + \sum_{i,j,k} \dot{a}_i \dot{a}_j a_k C^4_{ijkr} +$$

$$+\dot{\vec{\varepsilon}} \cdot \left(\sum_i \dot{a}_i \vec{D}^2_{ir} + \sum_{i,j} \dot{a}_i a_j \vec{D}^3_{ijr} + \sum_{i,j,k} \dot{a}_i a_j a_k \vec{D}^4_{ijkr} \right), \tag{9}$$

$$\frac{\rho}{M_r + M_1} \sum_i \ddot{a}_i \left(\vec{B}^1_i + \sum_j a_j \vec{B}^2_{ij} + \sum_{j,k} a_j a_k \vec{B}^3_{ijk} \right) + \ddot{\vec{\varepsilon}} = \frac{\vec{F}}{M_r + M_1} + \vec{g} -$$

$$-\frac{\rho}{M_r + M_1} \left(\sum_{i,j} \dot{a}_i \dot{a}_j \vec{B}^2_{ij} + 2 \sum_{i,j,k} \dot{a}_i \dot{a}_j a_k \vec{B}^3_{ijk} \right). \tag{10}$$

The system (9), (10) is a system of ordinary differential equations, which is linear relative to the second derivatives of a_i and $\vec{\varepsilon}$. This enables its reduction to the Cauchy normal form with further usage of standard methods of numerical integration by time.

4. Numerical analysis of the system behavior

For numerical implementation we accept the model, which includes 12 normal modes of oscillations [4]. We analyzed numerically the problems about motion of the system cylindrical reservoir — liquid with a free surface in elastic fixation in the form of spring, attached to immovable point, under action of forces in the form of rectangular impulses with heights $F = 1000$ N, 4500 N, 8000 N and for different stiffness of spring $c = 0$, 50 N/m, 100 N/m, 200 N/m, 500 N/m, 1000 N/m, 2000 N/m with duration in time $\Delta = 1$ s. In the case of the absence of elastic fixation amplitudes of all harmonics perform oscillations near constant mean value and reduce in time. Under the presence of elastic fixation considerable modulation of these oscillations takes place with evident changes of mean value. Liquid oscillation essentially effects the motion of the reservoir. It is seen from Fig. 2 that on the increase of spring stiffness frequency of reservoir

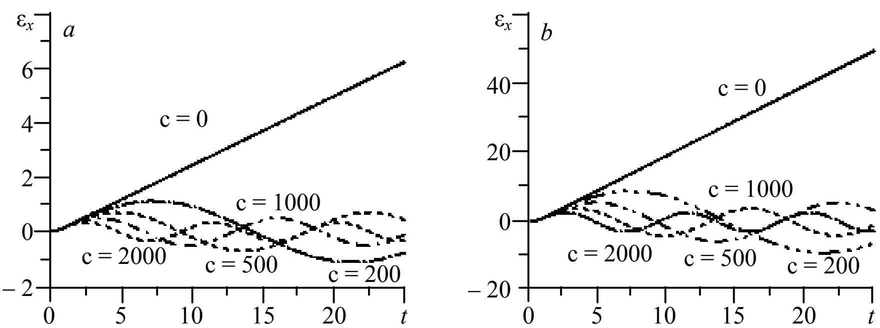

Fig. 2. Displacements ε_x for forces $F = 4500$ N (a) and $F = 8000$ N (b)

oscillations increases, and amplitude of reservoir displacements decreases. Variation of reservoir velocity in time shows that influence of liquid oscillations on reservoir motion decreases with growth of time (Fig. 3).

Let us analyze behavior of amplitudes of perturbation of a liquid free surface. It is seen from Fig. 3, where variation of amplitude the first antisymmetric mode a_2 is shown, that under the absence of spring oscillations of liquid decrease with time. In the case of elastic fixation of the reservoir liquid performs oscillations with strong variation of mean value of graph. In this case frequency of oscillations of the mean value increases with increase of string stiffness. Since reservoir performs motion with the frequency, defined by stiffness of elastic fixation, free surface of liquid has systematic inclination with this frequency. Namely oscillations with this frequency manifest in variation of mean value of oscillations. At the same time maximum of aggregate oscillations of a liquid free surface remain practically the same. This testify, that elastic support cannot be considered as efficient technique of vibroprotection. The system shows similar behavior for other magnitudes of external force *F*.

Amplitudes of oscillations of normal modes a_i decrease with growth of their number i. Thus, if a_1 has magnitude about 0,3, amplitudes of the first and second axis-

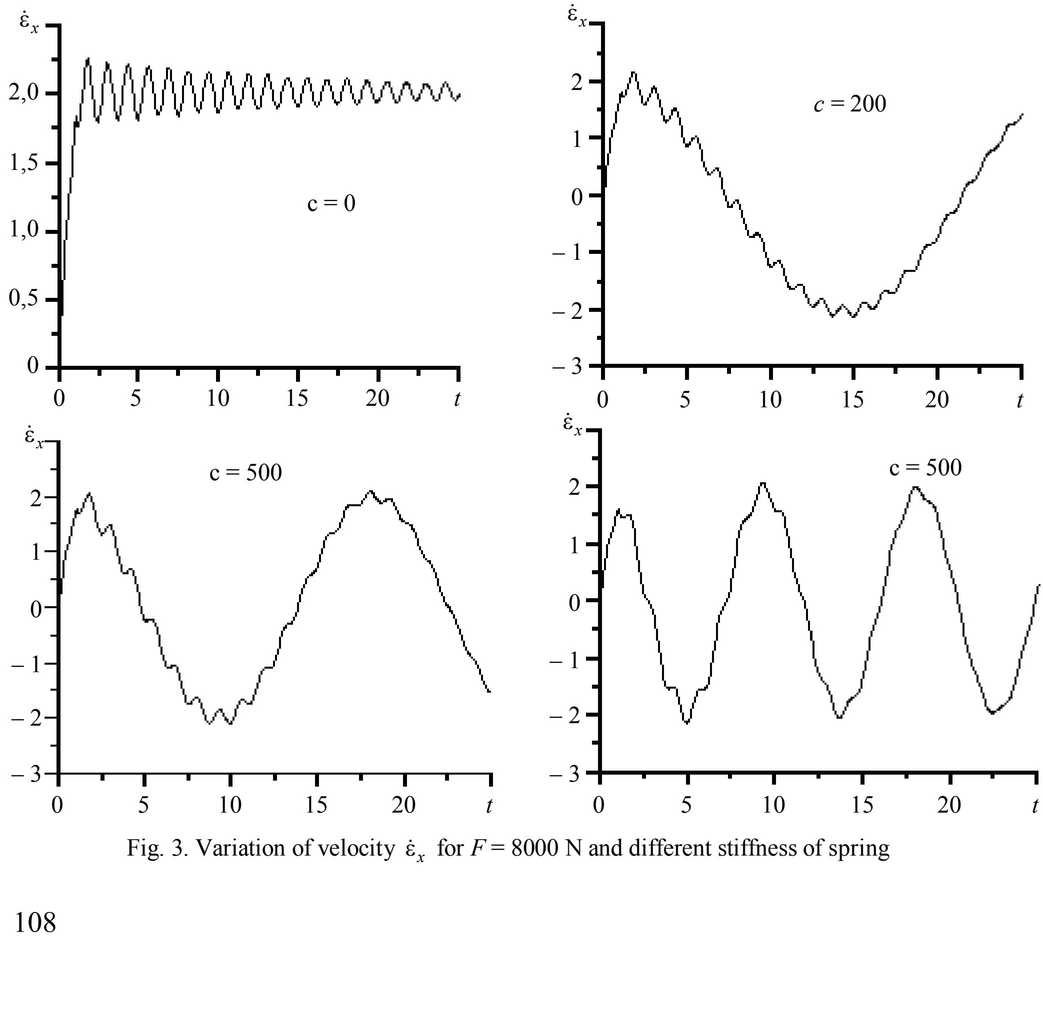

Fig. 3. Variation of velocity $\dot{\varepsilon}_x$ for F = 8000 N and different stiffness of spring

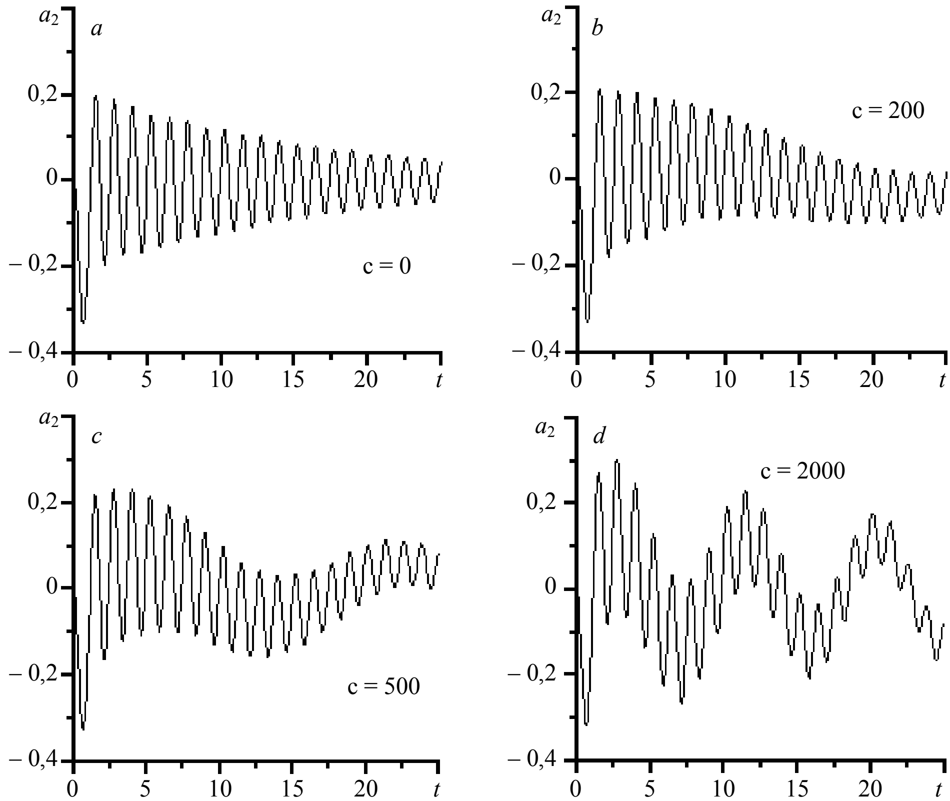

Fig. 4. Variation of amplitude a_2 in time for F = 8000 N

symmetric normal modes a_3, a_6 are about 0,05 and 0,006 correspondingly, and the second and the third antisymmetric normal modes a_{10}, a_{12} are about is about 0,05. So, the first normal mode significantly dominates. Nevertheless, let us analyze variation of the first axis-symmetric normal mode a_3, because it characterizes nonlinear mechanism of energy redistribution between normal modes and defines non-symmetry of waves on a free surface of liquid. It is seen from Fig. 5 that mean value of graph of oscillations of this amplitude is greater than zero. This just corresponds to the property of non-symmetry of wave profile, then height of wave crest is greater than depth of wave foot [1, 3]. It is also interest to note that if general amplitude of wave decrease, it hits into linear range of oscillations and, therefore, mean value of oscillations of the amplitude a_3 tends to zero (Fig. 5*c*).

Fig. 6 shows variation in time of high (third) antisymmetric mode a_{12}. It is seen that mean value of this amplitude also changes in time with frequency proportional to spring stiffness. This is caused by accelerated motion of the reservoir, then equilibrium position of liquid will be inclined. Similar results were obtained for other variants of external force F and spring stiffness c.

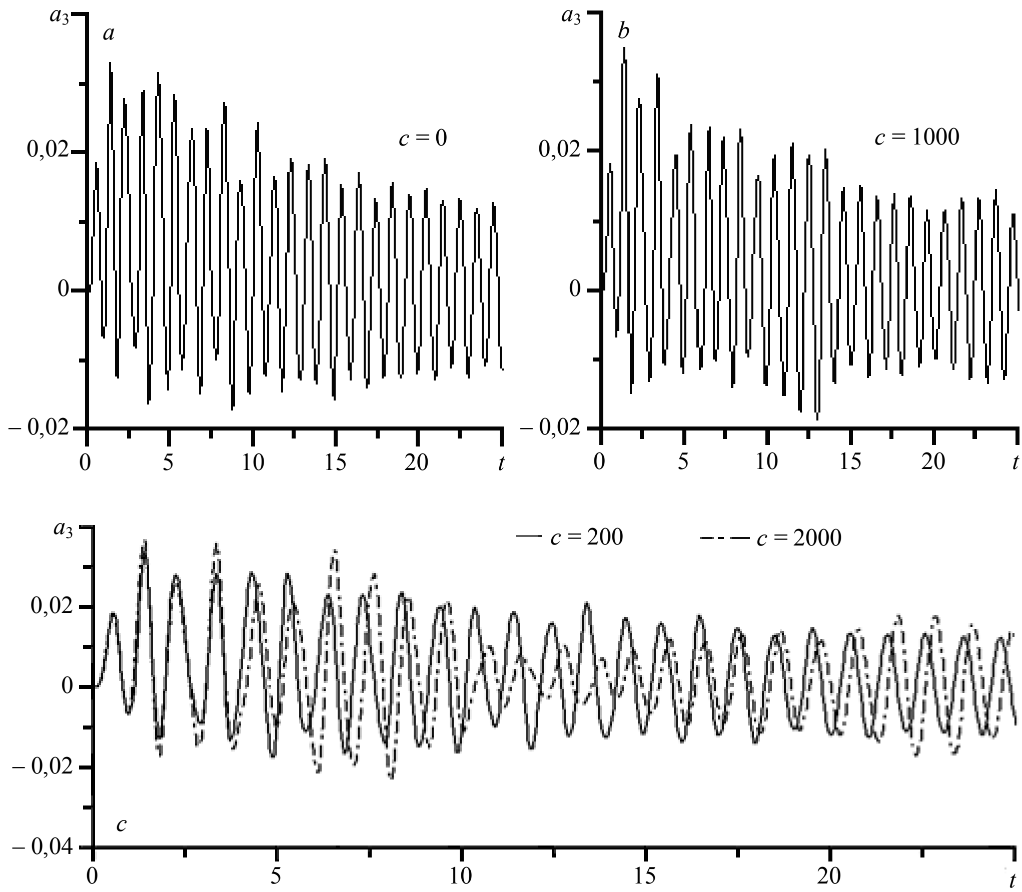

Fig. 5. Variation of the amplitude a_3 in time for F = 8000 N

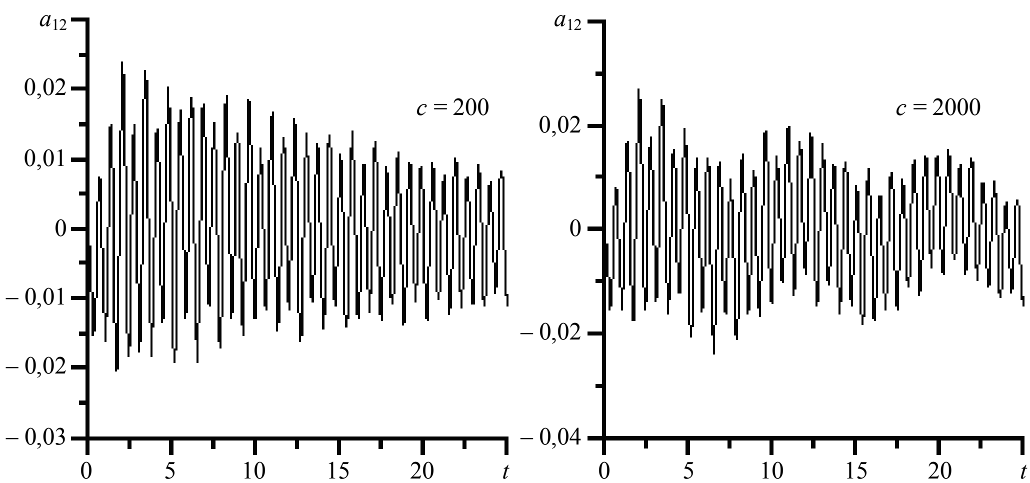

Fig. 6. Variation of amplitude a_{12} in time for F = 8000 N

Conclusion. Nonlinear mathematical model of dynamics of combined motion of rigid cylindrical reservoir, filled by liquid with a free surface and fixed by elastic spring, caused by impulse force applied to the reservoir, was constructed. It was shown that on increase of the value of external force F and spring stiffness c amplitudes of every normal mode increases; for the same value of the force F amplitudes a_i decrease with increase of the number of normal mode i; on increase of spring stiffness displacements of the reservoir ε_x decrease, and frequency of reservoir oscillations increases; for limiting case of zero stiffness of spring displacement of reservoir has growing character; mean value of antisymmetric normal modes performs oscillations with frequency, proportional to spring stiffness. Analysis of magnitudes of amplitudes shows that it is not expedient to use elastic fixation of the reservoir as technique for reduction of amplitudes of oscillations of a free surface, however, elastic fixation promotes lowering of displacements of the reservoir.

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Вплив пружинного закріплення на динаміку резервуара з рідиною

Олег Лимарченко, Роксолана Ткаченко

Розроблено математичну модель сумісного руху абсолютно твердого циліндричного резервуара, заповненого рідиною з вільною поверхнею з пружинним закріпленням до нерухомої точки. Досліджено нелінійні коливання системи під дією імпульсної сили для різних жорсткостей пружини. Вивчено можливість використання пружинного закріплення як засобу віброзахисту.

Влияние пружинного закрепления на динамику резервуара с жидкостью

Олег Лимарченко, Роксолана Ткаченко

Разработана математическая модель совместного движения абсолютно твердого цилиндрического резервуара, заполненного жидкостью со свободной поверхностью с пружинным закреплением с неподвижной точкой. Исследованы нелинейные колебания системы под действием импульсной силы для разных жесткостей пружины. Изучена возможность применения пружинного закрепления как средства виброзащиты.

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