УДК 539.3

Mechanical and thermal effect of a filler of intercontact gaps on contact between two semi-infinite solids with microtextured surfaces

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The paper discusses the plane contact problem for two solids having microtextured surfaces that are forced together by a nominal pressure and carry the remotely established nominal heat flow. The interface between the contacting solids consists of a periodic array of gaps and a periodic array of contacts. The intercontact gaps are assumed to be filled with a heat-conducting ideal gas. Mechanical influence of the interstitial gas is taken into account by its pressure, while its thermal influence is simulated by thermal resistance. The problem considered is reduced to a Cauchy singular integral equation for a height of the gaps and a Prandtl singular integro-differential equation for a temperature jump between surfaces of the gaps. For calculating a gas pressure and a length of the gaps, the ideal gas law and the consistency condition for the Cauchy singular integral equation are used. Results are presented illustrating the effect of the applied pressure, the mass and the thermal conductivity of the gas on the average normal displacement and the average temperature jump.

Keywords: contact interaction, surface microtexture, intercontact gap, heat-conducting ideal gas, gas pressure, thermal resistance.

Introduction. The surface microtexturing, which consists in forming regularly (periodically) arranged grooves of the same shape on a surface of a solid, is one way of improving performance of joints [1, 2]. When microtextured surfaces are placed in contact, periodically arranged intercontact gaps occur at the interface. Under actual operating conditions, these gaps can be filled with a certain substance (grease, coolant medium, liquid or gas) that conducts heat and produces additional pressure on the contacting surfaces. Based on the solutions for contact of surfaces having periodic arrays of grooves or cavities, which account for mechanical and thermal influence of a filler of intercontact gaps, we can determine contact parameters and predict contact strength, stiffness and thermal contact resistance for microtextured solids operating in different gas-liquid mediums.

The investigations of elastic, thermal and thermoelastic contact between solids having regular surface texture concern mainly the case when there is no filler in intercontact gaps. Comprehensive reviews of studies of elastic contact between solids with regularly arranged surface microasperities (indenters) or wavy surfaces were provided, for instance, in [3, 4]. The elastic contact between a flat surface and a wavy surface when gaps between them are filled with a compressible liquid was considered in [5]. Assuming thermal insulation of intercontact gaps, the studies of thermal and thermoelastic contact between wavy surfaces and thermoelastic contact between two half-planes with a periodic array of thermally induced delamination zones were conducted in [6-8]. Based on the solution to the contact problem of heat conduction for a periodic array of intercontact gaps filled with a heat-conducting medium, the thermal effect of the interstitial medium on thermal constriction resistance was analyzed [9].

This article is devoted to investigation of contact between two semi-infinite solids with microtextured surfaces when both mechanical and thermal effects of a filler of intercontact gaps are taken into account. For solving the problem under consideration, the method of functions of intercontact gaps [10] is employed. This method was previously used for investigation of elastic contact between two semi-infinite solids with a single intercontact gap filled with a gas or a liquid [11-14], as well as for thermoelastic contact between such solids when only the thermal influence of a filler of intercontact gaps is considered [15-18].

1. Formulation of Problem

The model for the present analysis and the orientation of the coordinate axes with respect to the solids are shown in Fig. 1. Before the solids are placed in contact with each other, the surface of the lower solid is perfectly smooth, while the surface of the upper one has local grooves of length $2b$ arranged in a regular way at intervals d. The shape of each groove is described by the continuously differentiable function $r(x)$. The materials of the solids are assumed to be elastic, isotropic and identical. The thermal conductivity of the materials is denoted by K, Poisson's ratio by ν, and Young's modulus by E. The problem is posed in the framework of linear thermoelasticity, assuming plane strain conditions.

The solids are compressed against each other by a nominal pressure p, and a nominal heat flow q is imposed at infinity. Due to the initial surface microtexture of the upper solid, the interface between the solids consists of a periodic array of gaps and a periodic array of contacts. An unknown height and an unknown length of each gap is denoted by $h(x)$ and $2b$ respectively.

The gaps are assumed to be filled with an equal amount of a heat-conducting ideal gas. Mechanical influence of the interstitial gas is taken into account by its pressure P_1, which, according to the Pascal's law, is the same in each point of the gas. Thermal influence of the interstitial gas is simulated by thermal resistance $R(x) = h(x)/K_g$, where K_g denotes the thermal conductivity of the gas.

We also assume that the interface offers no thermal resistance outside the gaps, and the solids are in frictionless contact throughout the interface.

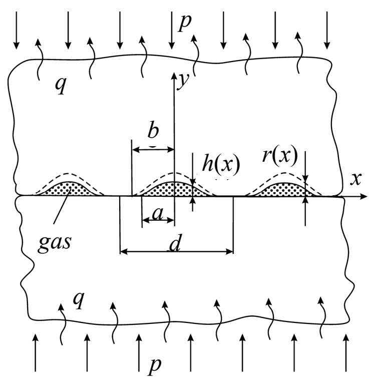

Fig. 1. The model of the problem

While the displacement perturbations arising out of a single interface gap vanish away from the interface ($y \to \pm\infty$), the cumulative effect of such perturbations arising out of an array of interface gaps will be noticeable far away from the interface. In the far field, this cumulative effect manifests itself in an average normal displacement $v_{av} = \frac{1}{d}\int_{-d/2}^{d/2}\left(r(x) - h(x)\right)dx$ and an average temperature jump $\gamma_{av} = \frac{1}{d}\int_{-d/2}^{d/2}\gamma(x)dx$.

2. Solution to Problem

Following the method of functions of intercontact gaps [10], we reduce the contact problem considered to a Cauchy singular integral equation (SIE) for the derivative of the gaps height $h(\xi)$

$$\int_{-\alpha}^{\alpha}\frac{h'(\eta)}{\eta-\xi}d\eta = \int_{-\beta}^{\beta}\frac{r'(\eta)}{\eta-\xi}d\eta + \frac{dE'(p-P_1)}{1+\xi^2}, \quad |\xi| < \alpha \tag{1}$$

and a Prandtl singular integro-differential equation for the temperature jump $\gamma(\xi)$ between surfaces of the gaps

$$\frac{K_g}{1+\xi^2}\frac{\gamma(\xi)}{h(\xi)} - \frac{K}{2d}\int_{-a}^{a}\frac{\gamma'(\eta)}{\eta-\xi}d\eta = \frac{q}{1+\xi^2}, \quad |\xi| < \alpha . \tag{2}$$

Here $E' = 4\left(1-\nu^2\right)/E$, $\xi = \tan(\pi x/d)$, $\eta = \tan(\pi t/d)$, $\alpha = \tan(\pi a/d)$, $\beta = \tan(\pi b/d)$.

The functions $h(\xi)$ and $\gamma(\xi)$ satisfy the following conditions [17, 18]:

$$h(\pm\alpha) = h'(\pm\alpha) = 0, \quad \gamma(\pm\alpha) = \gamma'(\pm\alpha) = 0 . \tag{3}$$

The right-hand side of the SIE (1) contains an unknown gas pressure P_1. For its calculating, we use the ideal gas law which in terms of the function $h(\xi)$ can be written as

$$P_1\int_{-\alpha}^{\alpha}\frac{h(\xi)}{1+\xi^2}d\xi = \frac{m}{\mu}RT , \tag{4}$$

there m and μ denote the mass and the molar mass of the gas respectively, T is the gas temperature, and R denotes the universal gas constant.

Let the shape of the grooves be described by the function $r(\xi) = r_0\left(1-\xi^2/\beta^2\right)^{3/2}$, $r_0 \ll \beta$.

Since $h'(\pm\alpha) = 0$, we will find the solution of the SIE (1) that is bounded at the points $\xi = \pm\alpha$. It has the form

$$h'(\xi) = \frac{dE'(p-P_1)}{\pi\sqrt{1+\alpha^2}}\frac{\xi\sqrt{\alpha^2-\xi^2}}{1+\xi^2} - \frac{3r_0}{\beta^2}\xi\sqrt{\alpha^2-\xi^2}, \quad |\xi| \le \alpha . \tag{5}$$

Integration of (5) from $-\alpha$ to ξ, in view of $h(\pm\alpha)=0$, gives

$$h(\xi)=\frac{dE'(p-P_1)}{\pi}\left[\frac{\sqrt{\alpha^2-\xi^2}}{\sqrt{1+\alpha^2}}-\operatorname{arctanh}\left(\frac{\sqrt{\alpha^2-\xi^2}}{\sqrt{1+\alpha^2}}\right)\right]+$$

$$+\frac{r_0}{\beta^3}\left(\alpha^2-\xi^2\right)^{3/2}, \quad |\xi|\le\alpha. \tag{6}$$

A bounded solution of a Cauchy singular integral equation is possible only if the right hand-side of the equation satisfies the consistency condition [19]. The consistency condition for the SIE (1) yields the relation between the gas pressure P_1, the applied load p and the gaps length α:

$$p-P_1=\frac{6r_0\pi}{dE'\beta}\sqrt{1+\alpha^2}\left(1-\frac{\alpha^2}{\beta^2}\right) \tag{7}$$

One more expression that links P_1, p and α is obtained from the ideal gas law (4) by substituting (6) into it and performing integration:

$$P_1\left\{dE'(p-P_1)\left[1-\frac{1}{\sqrt{\alpha^2+1}}-\frac{\ln\left(\alpha^2+1\right)}{2}\right]+\frac{r_0}{\beta^3}\left[\left(\alpha^2+1\right)\left(\sqrt{\alpha^2+1}-1\right)-\frac{\alpha^2}{2}\right]\right\}=$$

$$=\frac{m}{\mu}RT. \tag{8}$$

Substituting (7) in (8) gives an equation that determines P_1 for a given α. The applied pressure p required to produce a specified α is then calculated from (7).

For solving a Prandtl equation (2), the analytical-numerical approach proposed in [17] is applied.

3. Results

For illustration of the results, we introduce the following dimensionless quantities: $\tilde{r}_0=r_0/d$, $\tilde{b}=b/d$, $\tilde{m}=mRT/(\mu d)$, $\tilde{p}=E'p$, $\tilde{v}_{av}=v_{av}/d$, $\tilde{\gamma}_{av}=K_g\gamma_{av}/(qd)$. The results are shown for $\tilde{r}_0=0{,}001$, $\tilde{b}=0{,}233$. In Fig. 2, 3, the curves 1-3 correspond to $\tilde{m}=10^{-7}$; $5\cdot10^{-7}$; and 10^{-6} respectively. In Fig. 4, the curves 1-3 correspond to $\tilde{K}_g=0{,}01$; $0{,}02$; and $0{,}03$ respectively.

Fig. 2 shows the dependences of the average normal displacement $\tilde{v}_{av}$ on the applied pressure $\tilde{p}$ for different values of the gas mass $\tilde{m}$. The general trend seen from the curves is that $\tilde{v}_{av}$ increases with increasing $\tilde{p}$. As the amount of the gas $\tilde{m}$ in the gaps is increased, while the external load $\tilde{p}$ is held constant, the average normal displacement $\tilde{v}_{av}$ decreases.

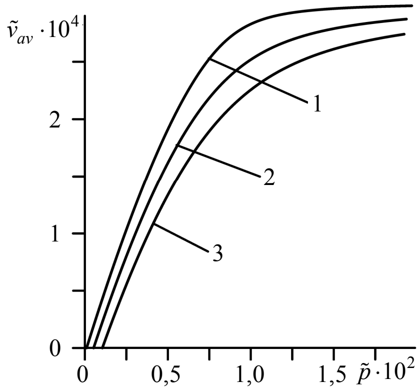

Fig. 2. Dependence of the average normal displacement on the applied pressure and the gas mass

Fig. 3 shows the average temperature jump $\tilde{\gamma}_{av}$ as a function of the applied pressure $\tilde{p}$ for the fixed value of the gas thermal conductivity $\tilde{K}_g = 0{,}01$ and different values of the gas mass $\tilde{m}$. As seen in the figure, $\tilde{\gamma}_{av}$ decreases with increasing $\tilde{p}$. For a fixed value of $\tilde{p}$, an increase of $\tilde{m}$ leads to an increase of $\tilde{\gamma}_{av}$.

Fig. 4 shows how the average temperature jump $\tilde{\gamma}_{av}$ depends on the gas thermal conductivity $\tilde{K}_g$ when $\tilde{m} = 10^{-7}$. As can be seen, increasing $\tilde{K}_g$ results in a decrease of $\tilde{\gamma}_{av}$.

Conclusions. The contact interaction between two semi-infinite solids separated by a periodic array of interface gaps that are filled with an equal amount of a heat-conducting ideal gas has been considered. The solids are subjected to a far filed nominal pressure and a far field nominal heat flow. The mechanical and thermal influence of the filler of the gaps has been taken into account through a pressure of the interstitial gas and a thermal resistance of the gaps.

For determining a height of the gaps and a temperature jump across the interface, a Cauchy singular integral equation and a Prandtl singular integro-differential equation have been obtained. The first equation has been solved exactly, and, for finding the solution to the second equation, the analytical-numerical approach developed previously has been applied. A gas pressure and a length of the gaps have been found from the ideal gas law and the auxiliary condition of existence of the bounded solution of the Cauchy singular integral equation.

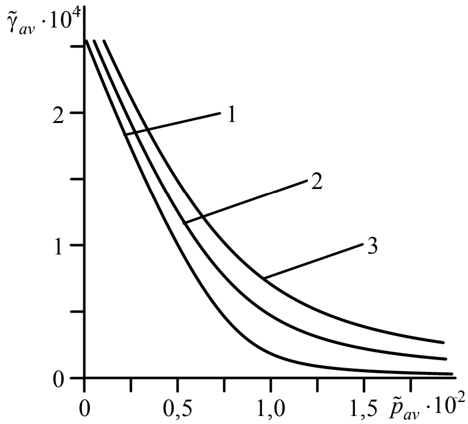

Fig. 3. Dependence of the average temperature jump on the applied pressure and the gas mass

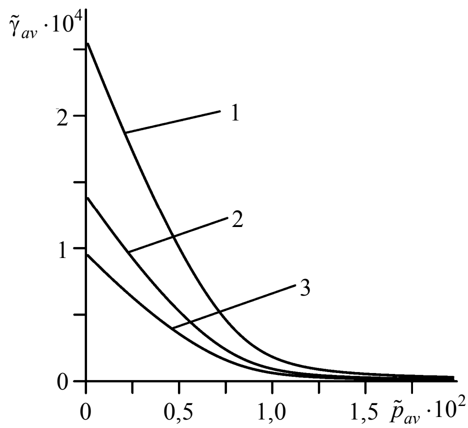

Fig. 4. Dependence of the average temperature jump on the applied pressure and the gas thermal conductivity

On this ground, the mechanical and thermal effect of the filler of the gaps has been analyzed. It has been observed that an increase of the amount of the gas in the gaps leads to a decrease in the average normal displacement and an increase in the average temperature jump. It has been also revealed that increasing the thermal conductivity of the filler results in a decrease in the average temperature jump.

This work was supported by National Academy of Sciences of Ukraine (Grant No. 23-08-12).

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Механічний і термічний вплив заповнювача міжконтактних зазорів на контакт двох півбезмежних тіл з мікротекстурованими поверхнями

Богдан Слободян, Костянтин Чумак, Ростислав Мартиняк

Розглядається плоска контактна задача для двох тіл з мікротекстурованими поверхнями, які притискаються одне до одного номінальним тиском і піддані дії теплового потоку, прикладеного на великій відстані від інтерфейсу. Інтерфейс між тілами, що контактують, утворений періодичною системою зазорів і періодичною системою ділянок контакту. Вважається, що міжконтактні зазори заповнені теплопровідним ідеальним газом. Механічний вплив газу в зазорах враховується його тиском, а його термічний вплив моделюється термоопором. Розглядувана задача зводиться до сингулярного інтегрального рівняння з ядром Коші стосовно висоти зазорів і сингулярного інтегро-диференціального рівняння типу Прандтля щодо стрибка температури між поверхнями зазорів. Для знаходження тиску газу та довжини зазорів використовуються рівняння стану ідеального газу й умова існування обмеженого розв'язку сингулярного інтегрального рівняння з ядром Коші. Проілюстровано вплив прикладеного тиску, маси та коефіцієнта теплопровідності газу на нормальне контактне зближення тіл та усереднений стрибок температури.

Механическое и термическое влияние заполнителя межконтактных зазоров на контакт двух полубесконечных тел с микротекстурированными поверхностями

Богдан Слободян, Константин Чумак, Ростислав Мартыняк

Рассматривается плоская контактная задача для двух тел с микротекстурированными поверхностями, которые прижимаются друг к другу номинальным давлением и подвергнуты действию теплового потока, приложенного вдали от интерфейса. Интерфейс между контактирующими телами образован периодической системой зазоров и периодической системой участков контакта. Полагается, что межконтактные зазоры заполнены теплопроводным идеальным газом. Механическое влияние газа в зазорах учитывается его давлением, а его термическое влияние моделируется термосопротивлением. Рассматриваемая задача сводится к сингулярному интегральному уравнению с ядром Коши относительно высоты зазоров и сингулярному интегро-дифференциальному уравнению типа Прандтля относительно скачка температуры между поверхностями зазоров. Для нахождения давлений газа и длины зазоров используются уравнение состояния идеального газа и условие существования ограниченного решения сингулярного интегрального уравнения с ядром Коши. Проиллюстрировано влияние приложенного давления, массы и коэффициента теплопроводности газа на нормальное контактное сближение тел и усредненный скачок температуры.

Отримано 02.03.13