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Application of variational method of homogeneous solutions for determination of stress concentration on the boundary of dissimilar isotropic elastic materials. Plane problem

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The stresses arising in plane strain conditions on the interfacial boundary dividing two dissimilar bodies of rectangle cross-sections under homogeneous heating/cooling and uniaxial tension/compression in normal to the boundary direction have been studied in the paper with the use of variational method of homogeneous solutions. Singularity parameters for the stresses in the corner points of the interfacial boundary have been studied on this basis with the application of known asymptotic solutions of plain elasticity problem for domains with corner points.

Key words: piece-wise homogeneous body, stress singularities in corner points, variational method of homogeneous solutions.

Introduction. Piece-wise homogeneous elements are widely used as components of various structures and devices. The operational stresses, arising in the interfaces between dissimilar materials, can be substantially higher then the applied external loading. As a result the strength of the elements can be decreased. It is known that the linear elasticity theory can brig to infinite values for stress components in corner points of the boundary that separates the materials with different elastic modules. The linear strength theories, that use the parameters of corner stress singularity in strength criterial relationships, are widely used during the last decades, and they have approved their effectiveness for some materials [1, 2].

Therefore the problem of development of effective mathematical methods for contact stresses determination, including behavior of the stresses in neighborhoods of corner points, is of the great importance. Such methods are necessary, in particular, for development of theoretical-experimental methods for determination of strength of adhesive or welded joints [3].

The asymptotic representations for stress components in a vicinity of corner points on the boundary between dissimilar materials were obtained, in particular, in [4]. In [5] the finite element method was used to solve the plane strain problem for piece-wise material rectangle. That enabled, in aggregate with asymptotic solutions for corner points, obtained in [4], to determine the stress singularity parameters in these points.

In [8] a variational method of homogeneous solutions was developed to solve 2-D elasticity problem in piece-wise rectangular material domain. Here we use this method to determine the stress concentration and singular behavior of the stress components near the corner points.

1. Formulation of the problems and the method for their solving

We consider a piece-wise body, consisting of two dissimilar parts with different values of their Young's moduli E^1 and E^2, Poisson's ratios ν^1 and ν^2 and thermal expansion coefficients α^1 and α^2. The parts are jointed at some fixed temperature T_0 along their common plane surface. The body is in plane strain state, caused by external loading or heating. Its cross-section is a rectangle $S = S_2 \cup S_1$, where $S_2 = \{(-a_2 < x < 0) \otimes (-1 < y < 1)\}$, $S_1 = \{(0 < x < a_1) \otimes (-1 < y < 1)\}$. Here x, y are the Cartesian coordinates in the cross-section, in which the line segment $x = 0,\ -1 \le y \le 1$ defines the interface between the dissimilar materials.

The stress components σ_{ij}^{λ}, $i, j \in \{x, y\}$, in the domains S_λ, $\lambda = 1,2$ satisfy the equations of equilibrium

$$\frac{\partial \sigma_{xx}^{\lambda}}{\partial x} + \frac{\partial \sigma_{xy}^{\lambda}}{\partial y} = 0, \qquad \frac{\partial \sigma_{xy}^{\lambda}}{\partial x} + \frac{\partial \sigma_{yy}^{\lambda}}{\partial y} = 0 . \tag{1}$$

The strain components $\varepsilon_{xx}^{\lambda}, \varepsilon_{yy}^{\lambda}, \varepsilon_{xy}^{\lambda}$, which are connected with the stresses $\sigma_{xx}^{\lambda}, \sigma_{yy}^{\lambda}, \sigma_{xy}^{\lambda}$ by the relationships

$$\varepsilon_{xx}^{\lambda} = \frac{1+\nu^{\lambda}}{E^{\lambda}}\left[\left(1-\nu^{\lambda}\right)\sigma_{xx}^{\lambda} - \nu^{\lambda}\sigma_{yy}^{\lambda}\right] + \alpha^{\lambda}\Delta T ,$$
$$\varepsilon_{yy}^{\lambda} = \frac{1+\nu^{\lambda}}{E^{\lambda}}\left[\left(1-\nu^{\lambda}\right)\sigma_{yy}^{\lambda} - \nu^{\lambda}\sigma_{xx}^{\lambda}\right] + \alpha^{\lambda}\Delta T, \quad \varepsilon_{xy}^{\lambda} = \frac{1+\nu^{\lambda}}{E^{\lambda}}\sigma_{xy}^{\lambda} , \tag{2}$$

satisfy in S_λ the compatibility equation

$$\frac{\partial^2 \varepsilon_{xx}^{\lambda}}{\partial x^2} + \frac{\partial^2 \varepsilon_{yy}^{\lambda}}{\partial y^2} + 2\frac{\partial^2 \varepsilon_{xy}^{\lambda}}{\partial x \partial y} = 0 . \tag{3}$$

In equations (2) ΔT stands for actual temperature increment with respect to its initial value T_0.

On the interface $x = 0$ the conditions of perfect elastic contact are valid:

$$\left.\left(\sigma_{xx}^{1} - \sigma_{xx}^{2}\right)\right|_{x=0} = 0, \ \left.\left(\sigma_{xy}^{1} - \sigma_{xy}^{2}\right)\right|_{x=0} = 0, \ \left.\left(u_{x}^{1} - u_{x}^{2}\right)\right|_{x=0} = 0, \ \left.\left(u_{y}^{1} - u_{y}^{2}\right)\right|_{x=0} = 0 . \tag{4}$$

Here $u_x^{\lambda}, u_y^{\lambda}$ ($\lambda = 1,2$) are Cartesian components of the translation vector in the domain S_λ.

We will study the stress-strained state of the body for two cases: a) homogeneous heating/cooling from the temperature T_0, at which stresses in the body are absent, to the temperature $T_0 + \Delta T$, where ΔT is a given constant, b) homogeneous stretch/compression by the normal forces p_0 applied to the surfaces $x = a_1$ and $x = -a_2$ at fixed temperature T_0.

The first case brings to a plane problem for the system (1)-(3) with given homogeneous temperature increment ΔT, contact conditions (4) and the conditions, prescribed on the free boundary:

$$\left.\sigma_{xx}^1\right|_{x=a_1} = 0, \quad \left.\sigma_{xy}^1\right|_{x=a_1} = 0, \quad \left.\sigma_{xx}^2\right|_{x=-a_2} = 0,$$
$$\left.\sigma_{xy}^2\right|_{x=-a_2} = 0, \quad \left.\sigma_{yy}^\lambda\right|_{y=\pm 1} = 0, \quad \left.\sigma_{xy}^\lambda\right|_{y=\pm 1} = 0. \tag{5}$$

We will present the solution of this problem as the superposition of a basic stress-strained state and a disturbance with stress components $\bar{\sigma}_{ij}^\lambda$ and $\tilde{\sigma}_{ij}^\lambda$, strain components $\bar{\varepsilon}_{ij}^\lambda$ and $\tilde{\varepsilon}_{ij}^\lambda$, and displacement components $\bar{u}_i^\lambda$ and $\tilde{u}_i^\lambda$ correspondingly:

$$\sigma_{ij}^\lambda = \bar{\sigma}_{ij}^\lambda + \tilde{\sigma}_{ij}^\lambda, \quad \varepsilon_{ij}^\lambda = \bar{\varepsilon}_{ij}^\lambda + \tilde{\varepsilon}_{ij}^\lambda, \quad u_i^\lambda = \bar{u}_i^\lambda + \tilde{u}_i^\lambda, \quad \lambda = 1,2, \quad i, j \in \{x, y\} \tag{6}$$

The basic state will be chosen as

$$\bar{\sigma}_{ij}^\lambda = 0, \quad \bar{\varepsilon}_{xx}^1 = \bar{\varepsilon}_{yy}^1 = \alpha^1 \Delta T, \quad \bar{\varepsilon}_{xx}^2 = \bar{\varepsilon}_{yy}^2 = \alpha^2 \Delta T, \quad \bar{\varepsilon}_{xy}^\lambda = 0. \tag{7}$$

To satisfy conditions (4) the components of displacement vectors and stress tensor of the disturbance have to be subordinated on the interface $x = 0$ to the next conditions:

$$\left.\left(\tilde{u}_x^1 - \tilde{u}_x^2\right)\right|_{x=0} = 0, \quad \left.\left(\tilde{u}_y^1 - \tilde{u}_y^2\right)\right|_{x=0} = -\left(\alpha^1 - \alpha^2\right)\Delta T y,$$
$$\left.\left(\tilde{\sigma}_{xx}^1 - \tilde{\sigma}_{xx}^2\right)\right|_{x=0} = 0, \quad \left.\left(\tilde{\sigma}_{xy}^1 - \tilde{\sigma}_{xy}^2\right)\right|_{x=0} = 0. \tag{8}$$

The case b) brings to the plane elasticity problem for the system (1)-(3) at zero temperature increment $\Delta T = 0$, contact conditions (4) and the loading conditions:

$$\left.\sigma_{xx}^1\right|_{x=a_1} = p_0, \quad \left.\sigma_{xx}^2\right|_{x=-a_2} = -p_0, \quad \left.\sigma_{xy}^1\right|_{x=a_1} = 0,$$
$$\left.\sigma_{xy}^2\right|_{x=-a_2} = 0, \quad \left.\sigma_{yy}^\lambda\right|_{y=\pm 1} = \left.\sigma_{xy}^\lambda\right|_{y=\pm 1} = 0. \tag{9}$$

We also represent this problem's solution in the form (6), taking as the basic state parameters

$$\bar{\sigma}_{xx}^\lambda = p_0, \quad \bar{\sigma}_{yy}^\lambda = 0, \quad \bar{\sigma}_{xy}^\lambda = 0, \quad \bar{\varepsilon}_{xx}^\lambda = \frac{p_0}{E^\lambda}, \quad \bar{\varepsilon}_{yy}^\lambda = -\nu^\lambda \frac{p_0}{E^\lambda},$$
$$\bar{\varepsilon}_{xy}^\lambda = 0, \quad \bar{\varepsilon}_{zz}^\lambda = 0, \quad \bar{\varepsilon}_{xz}^\lambda = 0, \quad \bar{\varepsilon}_{yz}^\lambda = 0, \quad \bar{\varepsilon}_{yx}^\lambda = 0, \quad \bar{\varepsilon}_{zx}^\lambda = 0, \quad \bar{\varepsilon}_{zy}^\lambda = 0,$$
$$\bar{u}_x^\lambda = 1 - \left(\nu^\lambda\right)^2 p_0 x \Big/ E^\lambda, \quad \bar{u}_y^\lambda = -\nu^\lambda \left(1 + \nu^\lambda\right) p_0 y \Big/ E^\lambda. \tag{10}$$

Then the problem is reduced to the plane elasticity problem (1)-(3) for rectangle S, the external surface of which is unloaded:

$$\tilde{\sigma}^1_{xx}\Big|_{x=a_1}=0,\qquad \tilde{\sigma}^1_{xy}\Big|_{x=a_1}=0,\qquad \tilde{\sigma}^2_{xx}\Big|_{x=-a_2}=0,$$
$$\tilde{\sigma}^2_{xy}\Big|_{x=-a_2}=0,\qquad \tilde{\sigma}^\lambda_{yy}\Big|_{y=\pm 1}=0,\qquad \tilde{\sigma}^\lambda_{xy}\Big|_{y=\pm 1}=0, \tag{11}$$

with the prescribed on the interface $x = 0$ conditions:

$$\left(\tilde{u}^1_x-\tilde{u}^2_x\right)\Big|_{x=0}=0,\qquad \left(\tilde{u}^1_y-\tilde{u}^2_y\right)\Big|_{x=0}=\left(\nu^1/E^1-\nu^2/E^2\right)p_0 y,$$
$$\left(\tilde{\sigma}^1_{xx}-\tilde{\sigma}^2_{xx}\right)\Big|_{x=0}=0,\qquad \left(\tilde{\sigma}^1_{xy}-\tilde{\sigma}^2_{xy}\right)\Big|_{x=0}=0. \tag{12}$$

It follows that both cases can be reduced to the plane elasticity problem for piece-wise material rectangle S with homogeneous in stresses boundary conditions on its sides:

$$\sigma^\lambda_{yy}\Big|_{y=\pm 1}=\sigma^\lambda_{xy}\Big|_{y=\pm 1}=0, \tag{13}$$

$$\sigma^\lambda_{xx}\Big|_{x=\begin{cases}a_1\\-a_2\end{cases}}=\sigma^\lambda_{xy}\Big|_{x=\begin{cases}a_1\\-a_2\end{cases}}=0 \tag{14}$$

and the next contact conditions on the common boundary of dissimilar materials:

$$\left(\sigma^1_{xx}-\sigma^2_{xx}\right)\Big|_{x=0}=0,\qquad \left(\sigma^1_{xy}-\sigma^2_{xy}\right)\Big|_{x=0}=0,$$
$$\left(u^1_x-u^2_x\right)\Big|_{x=0}=0,\qquad \left(u^1_y-u^2_y\right)\Big|_{x=0}=ay. \tag{15}$$

Here $a=-\left(\alpha^1-\alpha^2\right)\Delta T$ (in the case of homogeneous heating/cooling) and $a=\left(\nu^1/E^1-\nu^2/E^2\right)p_0$ (in the case of one-axial stretch/compression). From here we do not mark by tilde ($\sim$) the parameters of disturbed state.

The variational method of homogeneous solutions for solving the plane elasticity problem in a rectangle with conditions (13)-(15) was suggested in paper [6]. By this method the solution of the problem is presented as a series expansion by a functional basis. The complex-valued functions of the basis are eigenfunctions of some 2-D homogeneous elasticity problem (so called homogeneous solutions [7]). So each basic function satisfies the elasticity equations (1)-(3) within the open domains S_1 and S_2, and homogeneous in stresses conditions (13) on their sides $y=\pm 1$. The real and imaginary parts of these functions form independent functional basis. That enables to construct a general solution, which satisfies the equations (1)-(3), boundary conditions (13) and possesses an indeterminateness, that is sufficient to subordinate the solution to the boundary conditions (14) and contact conditions (15). Due to the method a weak subordination of the solution to the conditions (14), (15) in L_2 functional norm is used. That brings to the problem of minimization of a quadratic functional. The functional determines the mean square variation of the solution from boundary conditions (14) on the segments $x=-a_2$ and $x=a_1$, and contact conditions (15) on the segment $x = 0$.

Necessary criterion of minimum of the functional brings to an infinite system of algebraic equations in regard to the expansion coefficients. The system can be solved with the use of reduction method by retention in the searching solution a finite number N of members.

Consider, for an example, the uniaxial tension, which brings to the problem with contact conditions (12).

On Fig. 1 the distributions of the normal and tangential components of disturbed stress tensor on the segment $x = 0$ are shown. The problem was solved for particular case of materials with substantially different elasticity moduli $E_2 \gg E_1 = E$. The calculations are made for $p_0/E = 0,005$, $a_1 = 1$, $N = 20$.

The mean square errors of the solution on the segment $x = 0,\ y \in [-1,1]$ for the normal and tangential displacements are $\varepsilon_u = 1,31 \cdot 10^{-3}$ and $\varepsilon_v = 1,44 \cdot 10^{-3}$ correspondingly, whereas the errors on the segment $x = a_1,\ y \in [-1,1]$ for the normal and tangential tractions are $\varepsilon_\sigma = 8,63 \cdot 10^{-7}$ and $\varepsilon_\tau = 5,64 \cdot 10^{-7}$ correspondingly.

As it follows from the fulfilled quantitative analysis, the variational method of homogeneous solutions enables us to solve such problems with sufficient accuracy. But, in contrast to the known asymptotic method [5], this method gives finite values of stress components σ_{xx} and σ_{xy} in the corner points $x = 0,\ y = \pm 1$. This is an effect of reduction of infinite system of algebraic equations.

This means, that in some neighborhood $r < \rho_0,\ \rho_0 > 0$ (where $r \in S$, $r =$, $= \sqrt{x^2 + (1-|y|)^2}$) of the corner points $x = 0,\ y = \pm 1$ the error of the solution is higher than its mean square error on the segment $x = 0,\ y \in [-1,1]$.

But numerical experiments have shown that the radius ρ_0 of the neighborhood, in which the solution error is exceeded the mean square error on the segment $x = 0$, $y \in [-1,1]$, diminishes with growing N.

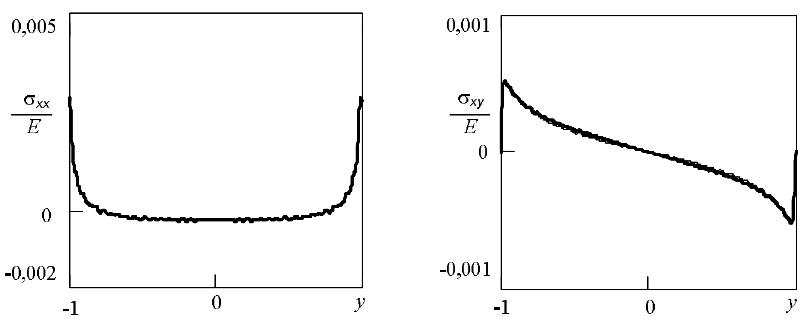

Fig. 1

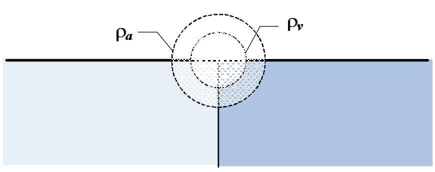

Fig. 2

Due to Bogy's theory [4] asymptotic behavior of stress component σ_{xx} in vicinity of the corner point for the problem (13)-(15) is described by the formula [5]:

$$\sigma_{xx}^{\text{asymp}}(0,y) = \frac{K}{(1-|y|)^{\omega}}, \qquad (16)$$

where K and ω are constants; stress intensity factor K is dependent on the materials elasticity constants, body's geometry and applied loading, and index ω is dependent on the materials elasticity constants. Formula (15) describes the stress component σ_{xx} sufficiently good in some small vicinity $r < \rho_a$, where $0 < \rho_a \ll 1$.

Choosing the number N big enough, we can reach the situation, when the radius ρ_v, determining the validity domain $r > \rho_v$ for the variational solution, becomes lesser than the radius ρ_a, determining the validity domain $r < \rho_a$ for asymptotic solution: $\rho_v < \rho_a$ (Fig. 2). As the validity domains of the solutions are overlapped within the range $y \in (1-\rho_a, 1-\rho_v)$, $x = 0$, we can approximate the variational solution by the function (16) and determine in such manner the parameters K and ω.

We use this approach, based of approximation of the variational solution in a vicinity of corner point by the asymptotic solution, to study in 2-D formulation the influence of the materials' elastic properties, geometrical parameters and applied loading on stress singularity in the corner points.

2. The case of homogeneous heating/cooling

We start from the case when the elastic modulus of the two materials are substantially different: $E^2 \gg E^1 = E$. In this case we come to the plane elasticity problem in the rectangle $(0 < x < a_1) \otimes (-1 < y < 1)$ with boundary conditions

$$u_x\big|_{x=0} = 0, \qquad u_y\big|_{x=0} = -\left(\alpha^1 - \alpha^2\right)(T - T_0)\,y,$$
$$\sigma_{xx}\big|_{x=a_1} = 0, \quad \sigma_{xy}\big|_{x=a_1} = 0, \quad \sigma_{yy}\big|_{y=\pm 1} = 0, \quad \sigma_{xy}\big|_{y=\pm 1} = 0 \qquad (17)$$

This problem is solved by variational method of homogeneous solutions with the use the procedure described in paper [7]. On Fig. 3 the distribution of stress component σ_{xx} in vicinity of the corner point $x = 0, y = 1$ is depicted in the logarithmical scale. The calculations were made at $N = 40$ for $\alpha^1 = 8{,}1 \cdot 10^{-6} K^{-1}$, $\alpha^2 = 9{,}18 \cdot 10^{-6} K^{-1}$, $\nu^1 = 0{,}2$. The straight line, depicted on this figure, is calculated through approximation of the dependence of $\sigma_{xx}(0,y)$ by the function (16) in the range $-2{,}5 < \lg(1-y) < -1$ with the use of least-squares procedure. The free term and angular coefficient of this linear function determine stress singularity parameters K and ω.

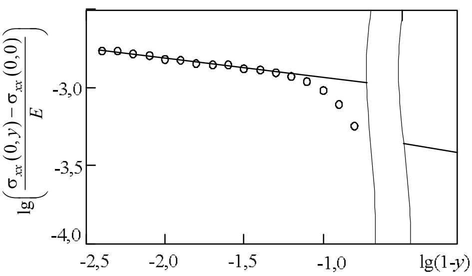

Fig. 3

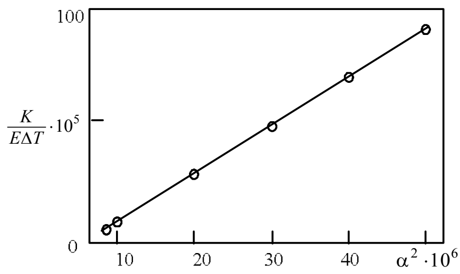

Fig. 4

Solving the problem (17) for various values of parameter α^2, the influence of α^2 on stress intensity factor *K* for component σ_{xx} at fixed parameters α^1, *E*, and ν^1 were studied. Dependence of the normalized intensity factor $K/(E\Delta T)$ on α^2, calculated for six values of α^2, is shown on the Fig. 4. The obtained linear dependence *K* on α^2 is expected: it results directly from the problem formulation. Though, the obtained result has corroborated indirectly, that the range for approximation of variational solution by the function (16) was chosen well.

Next we consider the case, when the dimensions a_1 and a_2 of the rectangles S_1 and S_2 are sufficiently big. As under the loading conditions (17) the surface forces, erasing on the boundary $x = 0$ of dissimilar materials, are self-balanced, the stresses will quickly decay with the distance from this boundary. This enables to substitute the initial problem (17) by the plane elasticity problem for piece-wise homogeneous strip $\{(-\infty < x < \infty) \otimes (-1 < y < 1)\}$, on the sides $y = \pm 1$ of which the homogeneous in stresses boundary conditions (13) are given and on the boundary of dissimilar materials the conditions of type (15) are valid. We should demand additionally, that the solution vanishes on infinity $x \to \pm\infty$.

The procedure for this problem solving with the use of variational method of homogeneous solutions is given in paper [8].

The previously carried out numerical experiments [7] show, that the stresses caused by self-balanced loading, applied on the boundary $x = 0$, decay practically to zero on the distances *x* of order 0,5. Therefore results obtained for infinite piece-wise homogeneous rectangle strip will be valid for finite piece-wise homogeneous rectangle, the dimensions a_1 and a_2 of which are equal halve its width or greater.

Studies of stress concentration in such formulation were carried out for the case of dissimilar materials joint with elasticity parameters $\nu^1 = 0{,}312$, $E^1 = 199{,}5 \cdot 10^9$ Pa and $\nu^2 = 0{,}293$, $E^2 = 211{,}4 \cdot 10^9$ Pa.

The dependence of stress intensity factor *K* for component σ_{xx} on the difference $\Delta\alpha = \alpha^2 - \alpha^1$ of thermal expansion coefficients is depictured on Fig. 5.

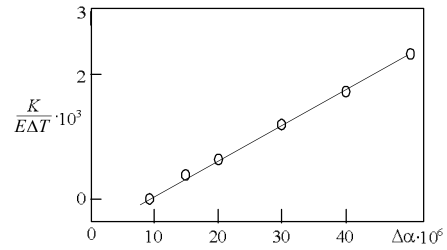

Fig. 5

The graph is built on the basis of the solutions obtained by the variational method of homogeneous solutions for six values of the difference $\Delta\alpha$. The solution was obtained at $N=0$. The relative mean-value errors of conditions (15) satisfaction for the normal ε_u and tangential ε_v displacements are equal $5{,}46\cdot 10^{-9}$ and $1{,}33\cdot 10^{-8}$ correspondently.

3. The case of uniaxial tension/compression

We start from the case when the material of S_2 is more rigid then other one: $E^2 \gg E^1 \equiv E$. In this case the boundary-contact plane elasticity problem for piece-homogeneous material rectangle S with conditions (13)-(15) can be substituted by the problem for rectangle S_2 with boundary conditions

$$u_x\big|_{x=0}=0, \qquad u_y\big|_{x=0}=\frac{\nu}{E}p_0 y\,,$$
$$\sigma_{xx}\big|_{x=a_1}=0, \qquad \sigma_{xy}\big|_{x=a_1}=0, \qquad \sigma_{yy}\big|_{y=\pm 1}=\sigma_{xy}\big|_{y=\pm 1}=0\,. \tag{18}$$

The problem was solved by the procedure described in the paper [7] for $a_1=1$ at $N=40$.

Fig. 6 illustrates the influence of applied tension on the stress intensity factor K. Fig. 7 and 8 give dependences K and ω on Poisson's ratio. The dependences are linear. This agrees with problem's linearity and confirms indirectly that the range for approximation stress distribution $\sigma_{xx}(0,y)$ near-by the corner point $x=0,\ y=1$ by function (16) is chosen well.

On Fig. 9 the dependence of stress intensity factor K on the parameter ω is shown. The graph is calculated on the basis of the data presented on Fig. 7 and 8.

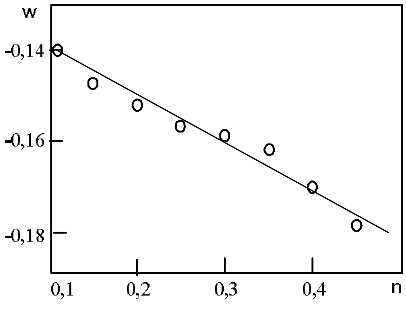

Fig. 6

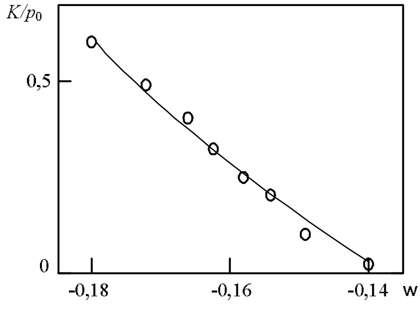

Fig. 7

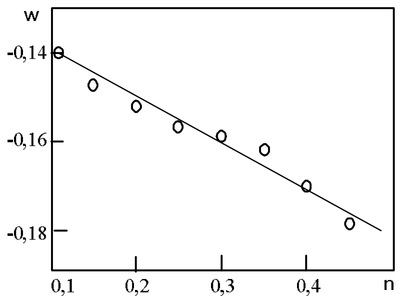

Fig. 8

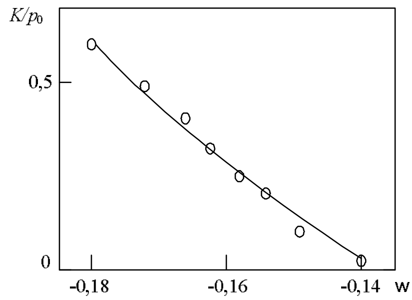

Fig. 9

Conclusions. The elasticity problems for determination of stress concentration on the boundary dividing two dissimilar materials have been considered in 2-D formulation. The problems were formulated for the piece-wise homogeneous material rectangle with the interfacial boundary that is normal to its two opposite sides. Two cases of loading were considered: homogeneous heating/cooling and uniaxial tension/compression by normal tractions applied to the opposite rectangle's sides which are parallel to the boundary.

It has been shown, that both problems can be reduced to the plane elasticity problem for piece-wise homogeneous material rectangle with strain incompatibility lumped on its interfacial boundary. All external sides of the rectangle are free of tractions. The strain incompatibility is specified in the problems by jumps of tangential displacement components, given on the boundary. The variational method of homogeneous solutions, previously developed by the authors, can be effectively used to solve this problem.

It follows from the solutions, obtained by the variational method, that behavior of stresses near-by the corner points is in qualitative agreement with the known Bogy's asymptotic solutions. On this basis determination of stress singularity parameters in the corner points starting from solutions obtained by the variational method has been carried out.

The study of stress singularity in the corner points, that have been carried out in this paper with the use of variational method of homogeneous solutions, brings to the results which are consistent with the results obtained in [5] by finite element method.

So, stress distributions on the boundary dividing dissimilar materials can be effectively determined with the use of variational method of homogeneous solutions and the stress singularity parameters can also be calculated on this basis. The results of these calculations can be used, in particular, to evaluate the strength of joints of dissimilar materials within the bounds of strength theories, that apply the parameters of corner stress singularity in strength criterial relationships.

Література

[1] *Божидарник В. В., Сулим Г. Т.* Елементи теорії пластичності та міцності. Т. 2. — Львів: Світ, 1999. — 418 с.

[2] *Khai M. V., Stepanyuk O. I.* The effect of the interface between media on stress concentration in the vicinity of a crack // Journal of Mathematical Sciences. — 1998. — Vol. 90, Issue 2. — P. 1961-1966.

[3] *Chowdhuri M. A. K., Xia Z.* Application of a New Experimental Method to Determine Bi-Material Interface Bonding Strength // Conference Proceeding of the Society for Experimental Mechanics Series. — 2013. — Vol. 34. — P. 247-254.
[4] *Bogy D. B.* On the problem of edge-bonded elastic quarter-planes loaded at the boundary // Int. J. Solids Structures. — 1970. — Vol. 6, No 9. — P. 1287-1313.
[5] *Munz D., Yang Y.Y.* Stress singularities at the interface in bonded dissimilar materials under mechanical and thermal loading // J. Appl. Mech. — 1992. — 59. — P. 57-861.
[6] *Чекурін В. Ф., Постолакі Л. І.* Теоретично-експериментальне визначення залишкових напружень у плоских з'єднаннях // Фіз.-хім. механіка матеріалів. — 2009. — Т. 45, № 2. — С. 153-162.
[7] *Чекурін В. Ф., Постолакі Л. І.* Варіаційний метод розв'язування бігармонічних задач для прямокутної області // Мат. методи та фіз.-мех. поля. — 2008. — Т. 51, № 1. — С. 88-98.
[8] *Чекурін В. Ф., Постолакі Л. І.* Обернена задача визначення залишкових напружень в околі з'єднання різнорідних матеріалів // Машинознавство. — 2010. — № 6. — С. 3-7.

Застосування варіаційного методу однорідних розв'язків для визначення концентрації напружень на поверхні розділу різнорідних ізотропних пружних матеріалів. Плоска задача

Василь Чекурін, Леся Постолакі

Із використанням варіаційного методу однорідних розв'язків досліджено напруження, які виникають в умовах плоскої деформації на поверхні розділу двох різнорідних тіл прямокутного перерізу за однорідного нагріву й одновісного розтягу, нормального до цієї поверхні. На основі отриманих розв'язків, із використанням відомих асимптотичних представлень для компонент напружень у околах кутових точок, проведено дослідження параметрів сингулярності напружень у кутових точках залежно від характеристик матеріалу та параметрів прикладеного навантаження.

Применение вариационного метода однородных решений для определения концентрации напряжений на поверхности раздела разнородных изотропных упругих материалов. Плоская задача

Василь Чекурин, Леся Постолаки

С использованием вариационного метода однородных решений исследованы напряжения, возникающие в условиях плоской деформации на поверхности раздела двух разнородных тел прямоугольного сечения при нагружении одноосным растяжением перпендикулярно к этой поверхности и однородном нагреве. На основе полученных решений, с использованием известных асимптотических решений, проведено исследование параметров сингулярности напряжений в угловых точках в зависимости от характеристик материала и параметров приложенного нагружения.

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