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Effect of viscous damping on forced oscillations of the system «reservoir – free surfaced liquid»

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The problem of transition of the mechanical system «cylindrical reservoir – liquid with a free surface» to the steady mode is under consideration. Behavior of the system is considered within the framework of the nonlinear model on durably time interval for system disturbance in the form of periodical force, applied to the reservoir. Character of the development of transient processes in the system is analyzed for different ratio of masses of reservoir and liquid, under the presence or absence of viscous damping. It was shown that only under the condition of artificial increase (in 7-30 times) of viscous damping, which was determined by Mikishev technique, transition to the steady mode of nonlinear oscillations of a free surface became possible.

Keywords: nonlinear dynamics, viscous damping, free surface of liquid, transient process, transition to steady mode.

Introduction. We consider the problem of dynamics of liquid with a free surface in the reservoir of cylindrical shape. The reservoir performs translational motion in the horizontal plane under action of active periodic force. Objective of the investigation is process of transition of oscillations of s free surface of liquid to steady mode of motion with considering viscous damping on the basis of nonlinear multimodal model (12 modes). In publications on steady mode of liquid motion [1-4] low-dimensional models (5 or less modes) were mostly used. Moreover, in these investigations reservoir moves according to the prescribed periodic law, and the hypothesis that liquid oscillations take place only with frequencies, multiple to the given law of motion of the reservoir (so, normal frequencies of liquid oscillations were ignored). The fact of preliminary specification of reservoir motion means that oscillations of a liquid free surface do not affect the reservoir motion, or, in other words, that reservoir has infinitely great mass. However, main practical applications are connected with the cases, when mass of liquid considerably exceeds the mass of the reservoir, so, considering the combined motion of the reservoir and liquid is required. In publication [5] mathematical model and the method of investigation of problems of combined motion of the reservoir and liquid with a free surface under action of active external forces and moments, applied to reservoir walls, were developed. Here the model [5] takes into account mutual influence of oscillations of the reservoir and liquid, and masses of the reservoir and liquid are parameters of the mathematical model. Using this model it was shown in the theoretical publication [6] that for reservoir of revolution shape transition of oscillations of a free surface of liquid on steady mode of motion does not occur for conservative system (without damping). This result correlates with conclusions of the experimental publications [7, 8] for the reservoir in the form of rectangular parallelepiped. It was shown in [6] that difference of results is caused by refuse from the hypothesis about potential of neglect of oscillations of a free surface of liquid on normal frequencies, investigation of system dynamics on the basis of nonlinear multimodal model (10 normal modes), considering combined motion of the reservoir and liquid. The approach, described in [6], was used in the present article for modeling of potential of transition of the system on steady mode of motion with considering damping effects.

1. Discrete model of the system «reservoir – liquid with a free surface»

We construct discrete model of the mechanical system «reservoir – liquid with a free surface» on the basis of the Hamilton-Ostrogradskiy variational principle, method of modal decomposition and method [5] of analytical elimination of all kinematic boundary conditions. Coefficients a_i of decomposition into series of elevations of a free surface ξ with respect to normal modes of oscillations of a free surface ψ_i and parameters of translational motion of the reservoir are used as independent system of variables

$$\sum_i \ddot{a}_i \left(\beta^q_{ri} + \sum_j a_j \gamma^q_{rij} + \sum_{i,j} a_i a_j \delta^q_{rijk} \right) + \\ + \ddot{\vec{\varepsilon}} \cdot \left(\vec{B}^1_r + \sum_i a_i \vec{B}^2_{ri} + \sum_{i,j} a_i a_j \vec{B}^3_{rij} + \sum_{i,j,k} a_i a_j a_k \vec{B}^4_{rijk} \right) = \\ = \frac{1}{2} \sum_{i,j} \dot{a}_i \dot{a}_j \left(\gamma^q_{ijr} - 2\gamma^q_{rij} \right) + \sum_{i,j,k} \dot{a}_i \dot{a}_j a_k \left(\delta^q_{ijkr} - 2\delta^q_{rijk} \right) - \frac{1}{2} g \alpha^s_r - g N_r a_r - \alpha^p_r a_r + \\ + \dot{\vec{\varepsilon}} \cdot \left[\vec{B}^1_r + \sum_i a_i \left(\vec{B}^2_{ir} - \vec{B}^2_{ri} \right) + \sum_{i,j} \dot{a}_i a_j 2 \left(\vec{B}^3_{ijr} - \vec{B}^3_{rij} \right) + \sum_{i,j,k} \dot{a}_i a_j a_k 3 \left(\vec{B}^4_{ijkr} - \vec{B}^4_{rijk} \right) \right], \quad (1)$$

$$\frac{\rho}{(M_F + M_T)} \left[\sum_i \ddot{a}_i \left(\vec{B}^1_i + \sum_j a_j \vec{B}^2_{ij} + \sum_{j,k} a_j a_k \vec{B}^3_{ijk} \right) \right] + \ddot{\vec{\varepsilon}} = \\ = \frac{\vec{F}}{(M_F + M_T)} - g\vec{k} - \frac{\rho}{(M_F + M_T)} \left(\sum_{i,j} \dot{a}_i \dot{a}_j \vec{B}^2_{ij} + \sum_{i,j,k} \dot{a}_i \dot{a}_j a_k 2\vec{B}^3_{ijk} \right), \quad (2)$$

here ρ is liquid density, α^p_r is the coefficient of the generalized dissipation [2], g is free falling acceleration, M_F and M_T are masses of liquid and reservoir.

The system (1), (2) contains $N + 3$ equations (N is the number of considered normal modes of oscillations of a free surface of liquid) and describes dynamics of combined motion of the system «reservoir – liquid with a free surface» under active external forces $\vec{F}$. The equations (1) describe dynamics of amplitudes of normal modes of oscillations of a free surface of liquid, and the equations (2) describe dynamics of translational motion of the cylindrical reservoir. However, these equations are inter-dependent and contain forces of interaction between the reservoir and liquid.

The system of variables, which enter the system (1), (2), within the framework of the accepted model defines properties of the considered mechanical system and peculiarities of manifestation of internal linear and nonlinear mechanisms in it. The coefficients of the system (1), (2) are determined as quadratures from the solutions of the boundary value problem for determination of normal modes of oscillations of a free surface of liquid. Here the coefficients β_{ir}^{q}, γ_{ijr}^{q}, δ_{rijk}^{q}, α_r^s, α_r^p, N_r, α_r^k, β_{ir}^k, γ_{ijr}^k, δ_{rijk}^k, λ_r, correspond to oscillations of liquid in immovable reservoir, and the coefficients $\vec{B}_r^1$, $\vec{B}_{ri}^2$, $\vec{B}_{rij}^3$, $\vec{B}_{rijk}^4$ reflect interaction of liquid oscillation with translational motion of liquid.

According to technique of the publication [5] we represent the equation of a liquid free surface ξ as

$$\xi = \sum_i a_i(t)\psi_i(r,\theta)\,,$$

The following system is accepted for numerical implementation

$$\psi_{1,2}(r,\theta) = J_1\left(\frac{\kappa_1^{(1)}}{R}r\right)\begin{matrix}\sin(\theta)\\ \cos(\theta)\end{matrix}, \quad \psi_{7,8}(r,\theta) = J_3\left(\frac{\kappa_3^{(1)}}{R}r\right)\begin{matrix}\sin(3\theta)\\ \cos(3\theta)\end{matrix},$$

$$\psi_3(r,\theta) = J_0\left(\frac{\kappa_0^{(1)}}{R}r\right), \quad \psi_{9,10}(r,\theta) = J_1\left(\frac{\kappa_1^{(2)}}{R}r\right)\begin{matrix}\sin(\theta)\\ \cos(\theta)\end{matrix},$$

$$\psi_{4,5}(r,\theta) = J_2\left(\frac{\kappa_2^{(1)}}{R}r\right)\begin{matrix}\sin(2\theta)\\ \cos(2\theta)\end{matrix}, \quad \psi_{11,12}(r,\theta) = J_1\left(\frac{\kappa_1^{(3)}}{R}r\right)\begin{matrix}\sin(\theta)\\ \cos(\theta)\end{matrix},$$

$$\psi_6(r,\theta) = J_0\left(\frac{\kappa_0^{(2)}}{R}r\right).$$

The coefficients of generalized dissipation α_r^p are computed according to well-known technique of G. M. Mikishev [2] (with factor of correction $\sqrt{2}$). In numerical experiments we increased this coefficient 30 times for providing the required effect on development of oscillatory processes.

2. Results of numerical experiments

Let us consider circular cylindrical reservoir with vertical longitudinal axis Oz, which performs translational motion in the plane xOy. The reservoir of radius $R = 1$ m and mass M_T is partially filled by water with mass M_F with depth $H = R$.

Since the system of equations (1), (2) is linear relative to second derivatives, this makes it possible to transit to the Cauchy normal form on every step of numerical integration by means of numerical methods with further integration of this systems on the basis of the standard Runge-Kutta method. Here on the stage of transition to the Cauchy normal form order of derivatives was reduced by introduction of the generalized velocities $\dot{a}_i$ as equivalent independent variables (together with a_i). On investigation of dynamics of the system reservoir – liquid we considered $n_1 = n_2 = 12$ normal modes in decompositions for linear and quadratic terms and $n_3 = 6$ normal modes for cubic nonlinearities. Normal modes are arranged in ascending order of their frequencies except the mode second normal mode ψ_6, which inclusion is required for fitness of some experimental results. Step of numerical integration was specified as $\Delta t = 0{,}02$ s. On analysis of the results and construction of graphs amplitudes were reduced to dimensionless form relative to characteristic dimension of the system, namely radius of the reservoir R, and time was normalized by period of oscillations of the first antisymmetric mode ψ_1.

Motion is initiated by horizontal force, applied to reservoir walls, which is changed according to the harmonic law $F = F_y \cos(pt)$, initial excitation of a free surface of liquid is absent. In all numerical examples value of amplitude of external horizontal force, applied to the reservoir, was accepted in such a way, that oscillations of a free surface of liquid hit into nonlinear range of variation of waves amplitudes, i. e., elevation of a free surface of liquid were about $(0{,}2 \div 0{,}25)R$.

Let us consider nonlinear oscillations of a free surface of liquid, when frequency of external force is in below resonance range (Fig. 1, 2), i. e., $p = \omega_1$, where ω_1 is partial frequency of the first antisymmetric normal mode. Normal (resonant) frequency of the mechanical system for the specified mass ratio $M_T = 0{,}1 M_F$ is equal to $\omega_e = 1{,}2834\omega_1$. As it is seen from Fig. 1*a*, oscillations of a free surface under the absence of dissipation occur with noticeable amplitude modulation with the presence of the varied in time mean value. The presence of amplitude modulation is explained

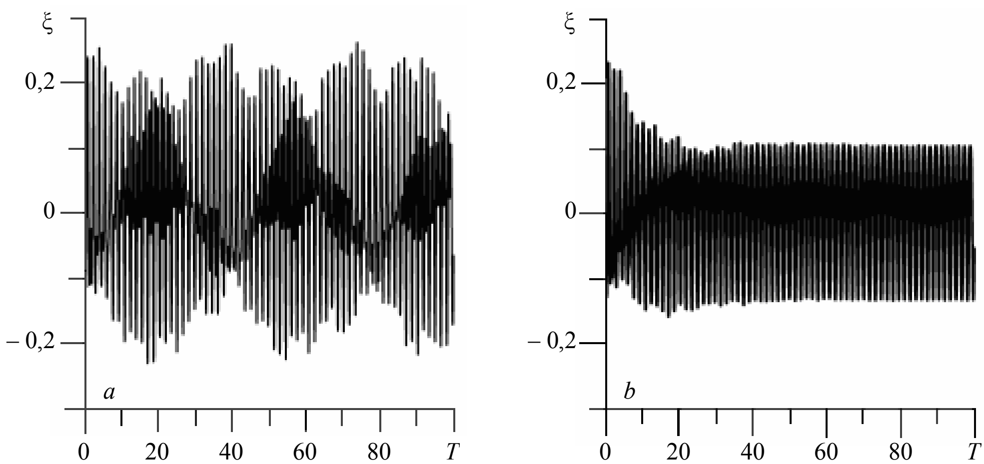

Fig. 1. Amplitude of oscillations of liquid on tank wall under the absence (*a*) and presence (*b*) of dissipative forces for disturbance of motion with below resonance frequency

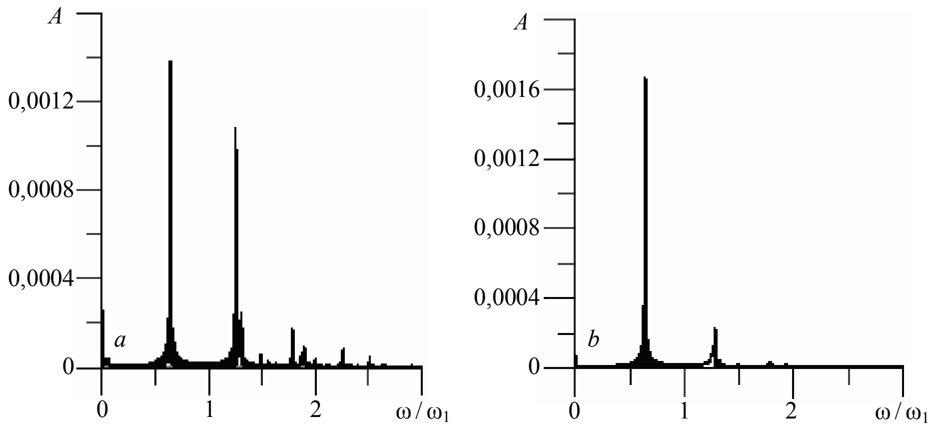

Fig. 2. Frequency spectrum of oscillations of liquid on tank wall under the absence (*a*) and presence (*b*) of dissipative forces for disturbance of motion with below resonance frequency

by the presence of two practically equal by amplitude harmonics on external and normal frequencies of the system (Fig. 2*a*), which are also close one to another. Variation of mean value in time can be explained by the presence of time harmonics with extremely low (close to zero) frequencies, caused by difference of two close values of frequencies. Moreover, in frequency spectrum high harmonics with frequencies, which are not multiple to the frequency of external disturbance present. The presence of amplitude modulation, time variable mean value and high harmonics made it possible to draw conclusion that under the absence of dissipation on disturbance of motion in below resonance range steady oscillations in the system are absent. Increase of mass of the reservoir under the absence of dissipation results only in redistribution of high harmonics frequencies, however, these harmonics affect dynamics of the process and manifest in graphs of oscillations in the form of abrupt changes of curves. Thus, under the absence of dissipation oscillations of a free surface in heavy reservoir on motion disturbance in below resonance range of frequencies do not transit to steady mode.

As it is seen from Fig. 2*b*, insertion into the system of the generalized dissipation acts similar to selective filter, namely, suppress oscillations on high and combination frequencies with conservation of dominate harmonic on the frequency of external disturbance. The harmonic on normal frequency of the system has amplitudes an order of magnitude lesser the dominant one. Therefore, we can draw conclusion that under the presence of dissipation disturbance of motion in below resonance range of frequencies after certain time of completion of transient processes results in transition of oscillations of a free surface to conventionally steady (close to one-frequency) mode (Fig. 2*b*). Increase of the reservoir mass promotes more rapid transition of oscillations to conventionally steady mode.

Let us consider nonlinear oscillations of a free surface of liquid, when frequency of the external force F is in a close vicinity of resonant frequency (Fig. 3, 4), namely, $p = 1{,}25\,\omega_1 \approx \omega_e$ for ratio of masses $M_T = 0{,}1 M_F$. As it is seen from Fig. 3*a*, under the absence of dissipation oscillations of a free surface have pronounced amplitude modulation and

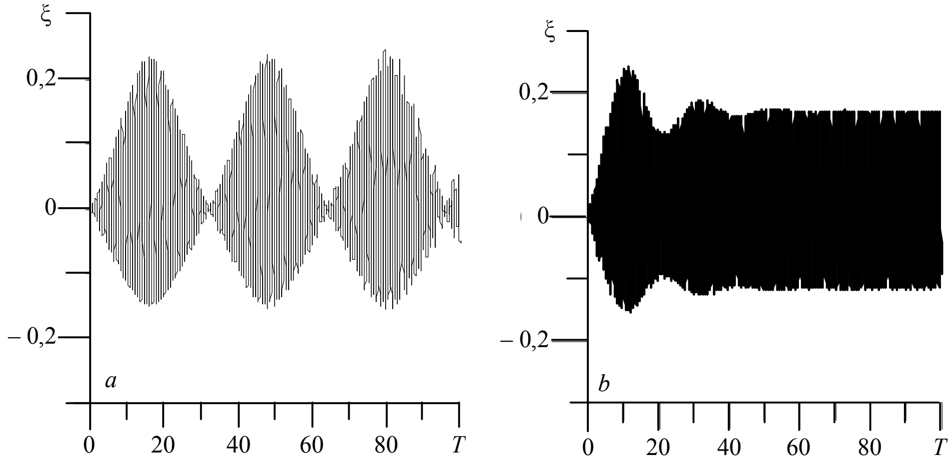

Fig. 3. Amplitude of oscillations of liquid on tank wall under the absence (*a*) and presence (*b*) of dissipative forces for disturbance of motion with near resonance frequency

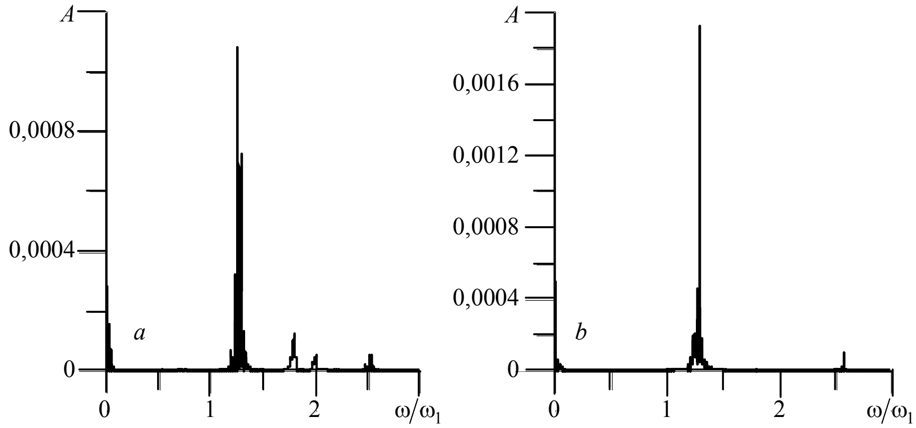

Fig. 4. Frequency spectrum of oscillations of liquid on tank wall under the absence (*a*) and presence (*b*) of dissipative forces for disturbance of motion with near resonance frequency

time variable mean value. This property is also confirmed by structure if frequency spectrum, namely, all dominating harmonics are focused near frequencies of disturbance, but harmonics with extremely low and high frequencies have amplitudes of the same order (Fig. 4*a*).

Increase of reservoir mass results in the property that frequencies of dominating harmonics differ considerably, therefore period of amplitude envelope decreases. In addition on increase of reservoir mass contribution of energy of harmonics on low frequencies decreases, but even for mass ration $M_T = 10M_F$ their energy amounts 15-30 % of the total energy of the system. Normal modes with high frequencies do not manifest in the case of absence of dissipation both foe light and heavy reservoir, there are no abrupt changes in curves and double-peaks in Fig. 4.

Therefore, under the absence of dissipation in the case of system disturbance in a small vicinity of resonant frequency transition of the system on steady mode in its classical sense does not occur, however it is possible to say about conventionally steady mode, for which the presence of 2-3 dominating frequencies. Noticeable amplitude modulation and the absence of manifestation of high frequency modes are also peculiar.

Under the presence of dissipation (Fig. 3*b*, 4*b*) after completion of transient processes oscillations of a free surface of liquid transit to steady mode. As it is seen from frequency spectrum (Fig. 4*b*), harmonic on frequency of external disturbance dominates, thus we can suppose oscillations as practically one-frequency. Effect of harmonic with zero frequency is considerable, but it manifests only in the fact that mean value of oscillations is not zero, but has certain positive shift. Reduction of the reservoir mass provides more rapid transition of oscillations of a free surface of liquid on conventionally steady mode.

Let us consider nonlinear oscillations of a free surface of liquid, when frequency of the external force F is in above resonance range of frequencies (Fig. 5, 6), here $p = 1,66\omega_1$ for mass ration $M_T = 0,1M_F$ As it is seen from Fig. 5*a*, 6*a*, oscillations of a free surface is characterized by amplitude modulation and variable in time mean value of amplitudes. In contrast to below resonance range third dominating harmonic with combination frequency $\omega \approx 1,8\omega_1$ participates in formation of processes (Fig. 6*b*). All other features of character of oscillations of a free surface for above resonance range are similar to the case of below resonance range, i. e., contribution of high normal modes is considerable, increase of reservoir mass does not suppress high harmonics of spectrum, but provides only redistribution of their energy contribution. Thus, transition of oscillations of a free surface of liquid on steady mode does not occur for above resonance range.

Inclusion of the generalized dissipation into the considered system conserves the dominating harmonic on the frequency of external disturbance (Fig. 6*b*), however amplitudes of harmonics on the normal frequency $\omega \approx 1,28\omega_1$ and $\omega \approx 1,8\omega_1$ have such

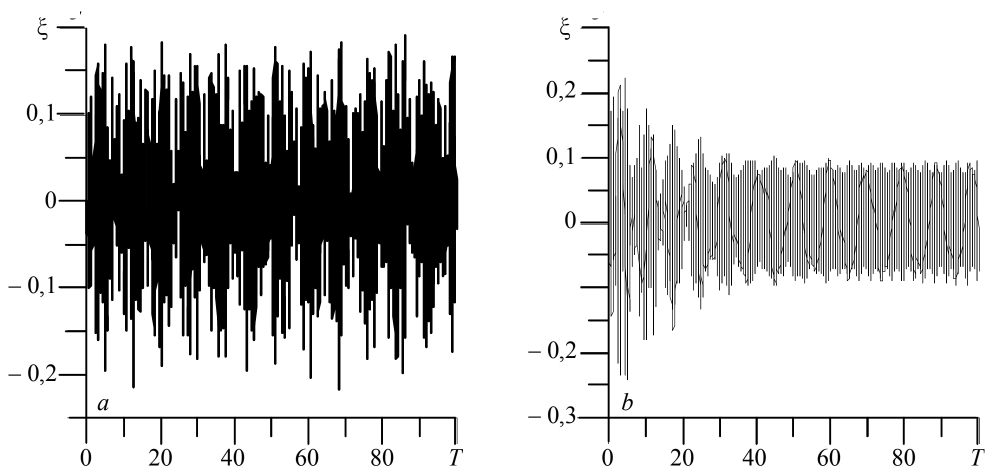

Fig. 5. Amplitude of oscillations of liquid on tank wall under the absence (*a*) and presence (*b*) of dissipative forces for disturbance of motion on above resonance frequency

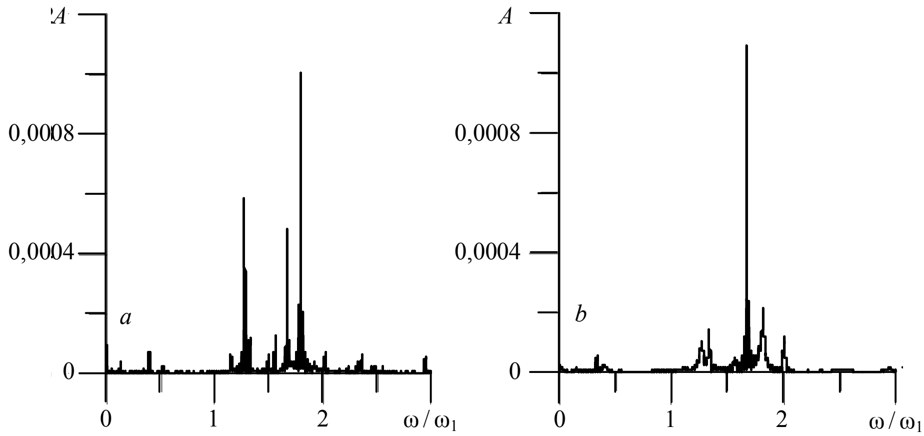

Fig. 6. Frequency spectrum of oscillations of liquid on tank wall under the absence (*a*) and presence (*b*) of dissipative forces for disturbance of motion with near resonance frequency

magnitudes, that due to them after completion of transient processes oscillations have character of amplitude modulation (Fig. 5*b*). Thus, in the above resonance range disturbance of motion for small mass of the reservoir even under the presence of the generalized dissipation increased 30 times does not lead to steady mode of motion in classical sense. Only considerable increase of the reservoir mass results in transition of oscillations to conventionally steady mode, the mode with weak modulation.

Conclusions. Transition to steady mode of motion for the problem of dynamics of liquid with a free surface in cylindrical reservoir with considering generalized viscous dissipation was considered. Behavior of the system is considered under horizontal periodic disturbance, when frequency of the external force in a vicinity of resonance, namely, below, near and above resonance values. It was shown that for all case transition of the system to the steady mode of oscillations in classical sense does not occur. Insertion of viscous dissipation, determined according to the boundary layer approach, can promote transition of the system to the mode, close to steady one, for which after completion of transient processes insignificant amplitude modulation and the presence of constant mean value are peculiar. However, this transition is attained by means of considerable (7-30 times) increase of the coefficients of the generalized dissipation, therefore, for transition of the mechanical system «reservoir – liquid with a free surface» under periodic disturbance it is necessary to use passive dampers in the form of structural components (barriers, baffles).

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Вплив в'язкого демпфування на вимушені коливання системи «резервуар – рідина з вільною поверхнею»

Олександр Константінов

Розглянуто задачу про вихід механічної системи «циліндричний резервуар – рідина з вільною поверхнею» на режим усталених коливань. Поведінка системи розглядається в рамках нелінійної моделі на тривалому проміжку часу у разі збудження руху системи періодичною силою, прикладеною до резервуара. Аналізується характер розвитку перехідних процесів у системі для різних співвідношень мас резервуара і рідини за умови наявності та відсутності в'язкого демпфування. Показано, що лише за штучного збільшення в'язкості в 7-30 разів, визначеною за методикою Г. Н. Мікішева, можливий вихід системи на усталений режим.

Влияние вязкого демпфирования на вынужденные колебания системы «резервуар – жидкость со свободной поверхностью»

Александр Константинов

Рассмотрена задача о выходе механической системы «цилиндрический резервуар – жидкость со свободной поверхностью» на режим установившихся колебаний. Поведение системы рассматривается в рамках нелинейной модели на продолжительном промежутке времени при возбуждении движения системы периодической силой, приложенной к резервуару. Анализируется характер развития переходных процессов в системе для разных соотношений масс резервуара и жидкости в условиях наличия или отсутствия вязкого демпфирования. Показано, что только при искусственном увеличении вязкости в 7-30 раз, определенной по методике Г. Н. Микишева, возможен выход системы на установившийся режим.

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