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Boundary element method for thermal identification of near-surface cylindrical cavity

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The problem for identification of the geometrical parameters of a tunnel cavity in a heat conductive half-space has been considered in the paper. Temperature field of body's external surface, caused by concentrated stationary heat fluxes, is used as input data for the identification problem. A stationary 2-d mathematical model for thermal sounding of the object has been built with the use of the boundary integral equations. The direct and inverse problems for cavity identification have been formulated within the model. The direct problem was numerically studied with the use of boundary-element method. On this basis informative parameters of the surface temperature field have been identified and quantitatively studied. Using these parameters, the inverse problem was reduced to an implicit nonlinear system of equations. An iterative boundary-element algorithm based on Newton method has been developed for solving this system. With the use of numerical experiment the efficiency of developed algorithm has been studied. The suggested method can be used for development nondestructive contactless methods for identification of cavities in solids with the use of IR-thermography.

Keywords: cavity identification, thermal sounding, inverse problems, boundary-element methods, iterative methods.

Introduction. The technique of IR-thermography can be used for identification of discontinuities in solids (cavities, inclusions, structural defects etc.) [1]. To do that a thermal process is exited in the object by heating it with external heat flow. The heterogeneity of body's thermal properties impacts on the thermal process. Hence the surface temperature of the object contains information about body's internal structure. Measuring this temperature with the use of IR-thermography one can obtain data applicable for identification of the object's structure.

To formulate inverse problems these data should be used simultaneously with appropriate mathematical model describing thermal sounding of the object. Approaches based on using finite Fourier transform [2], finite differences [1] and finite element [3, 4] methods are known. Realization of these approaches brings to non-linear ill-posed problems. To solve them variation-iterative methods [3] and neural networks [4] were used in particular.

Most of the known approaches are applied to slab-like objects. The thermal sounding process in such object is excited by a homogeneous heat flow, completely covering a one of object's plane surface. In the issue a contrast thermal image is formed. It reflects with some precision the object's internal heterogeneity. That enables to identify visually the inclusion or cavity. These approaches can be effectively used for thin-walled objects. The contrast range of the thermal image is depended on the thickness of the object, defect size, its occurrence depth etc. All that can decrease the identification precision in case of thick objects.

The problems of nondestructive identification of cavities in continuous media with the use of the data obtained by thermal sounding the object with external heat flows bring to inverse problems for the heat equation. The boundary integral method can be effectively used to do that [6-8]. This approach enables one to formulate the inverse problem as the nonlinear operator equation [7]. On this basis a regularized Newton iteration scheme for approximate reconstruction of the shape of the closed cavity in bounded 2d domain has been developed [6, 7].

In publications [9-11] we considered the approach to the identification of the geometrical parameters of the cylindrical cavity in a long cylindrical body. According to this approach, the body is sounding by its scanning with a concentrated heat flow and the surface temperature field in step is measured. In the frame of boundary-integral model direct and inverse problems for cavity identification were formulated. Analyzing the direct problem's solutions, obtained with use of the boundary-element method, informative parameters of the surface temperature field have been revealed. With the use the informative parameters and variational formulation of the inverse problem the problem was reduced to an overspecified system of nonlinear equations. That enables us to develop a regularized boundary-element iterative algorithm based on the Newton method.

In this paper we continue this approach for identification of subsurface cavities. In this view a heat conductive half-space with a cylindrical tunnel cavity is considered.

1. Mathematical model for thermal sounding

A heat conductive body in the form of half space is considered. The body contains a tunnel cylindrical cavity that is parallel to the plane, bounding the body. The cross-section of the surface bounded the cavity is a sufficiently smooth plane convex contour Γ. Let $\{x_1, x_2, x_3\}$ be the Cartesian coordinate system with axis x_3 being normal to the plane containing the contour Γ, origin O laying on the plane bounding the half-space and axis x_2 being normal to this plane. Then geometry of contour Γ can be defined by a function $\mathbf{x} = \mathbf{x}(s)$, where $\mathbf{x} = (x_1, x_2)^{\mathrm{T}}$, $s \in \Delta \subset \mathbb{R}$. The function $\mathbf{x} = \mathbf{x}(s)$ depends on a finite number $n \in \mathbb{N}$ of real parameters $\beta_1, \beta_2, \ldots, \beta_n$ defining geometry of the contour: $\mathbf{x} = \mathbf{x}(s; \beta_1, \beta_2, \ldots, \beta_n)$. Hence the problem of cavity identification is reduced to determination values of the parameters $\beta_k \in B_k \subset \mathbb{R}$, $k = \overline{1, n}$, where B_k is the admitted region for parameter β_k that form a 1-connected closed domain B in $\mathbb{R}^n$.

The body is heated by stationary heat flow that falls on half-space's boundary $x_2 = 0$. The normal component J of the flow intensity do not depend on x_3: $J = J(x_1)$. We consider function $J(x_1)$ as a member of some functional class **J**.

The body is cooled by convection with the ambient medium with temperature $T_m = const$. The cylindrical surface bounding the cavity is thermally insulated one.

Under such conditions the heat transfer process, exciting in the body by the flow, will be stationary and dependent just on x_1 and x_2 coordinates.

Let X be the line coinciding with the x_1-axis. Then caused by cavity disturbance $\tilde{T}(\boldsymbol{\eta}) = T(\boldsymbol{\eta}) - \bar{T}(\boldsymbol{\eta})$ of the body's surface temperature $T(\boldsymbol{\eta})$ satisfies the boundary integral equation [11]

$$\frac{1}{2}\tilde{T}(\boldsymbol{\eta}) + \int_{\mathbf{X}} \left(\Phi(\boldsymbol{\eta},\xi) + \frac{h}{\kappa}\Theta(\boldsymbol{\eta},\xi) \right) \tilde{T}(\xi)\, d\xi + \int_{\Gamma} \Phi(\boldsymbol{\eta},\xi)\tilde{T}(\xi)\, dl(\xi) =$$
$$-\int_{\Gamma} \Phi(\boldsymbol{\eta},\xi)\bar{T}(\xi)\, dl(\xi), \qquad (1)$$

where $\boldsymbol{\eta} = \boldsymbol{\eta}(x_1, x_2), \xi = \xi(x_1, x_2) \in X \cup \Gamma$ stand for the radius-vectors of two arbitrary points on the curve $X \cup \Gamma$, $\bar{T}(\boldsymbol{\eta})$ stands for the temperature field on the curve $X \cup \Gamma$ for the body without cavity, h and κ stand for the coefficients of convection heat exchange and thermal conductivity; $\Theta(\boldsymbol{\eta},\xi)$, $r(\boldsymbol{\eta},\xi)$ and $\Phi(\boldsymbol{\eta},\xi)$ denote the functions $\Theta(\boldsymbol{\eta},\xi) \equiv \frac{1}{2\pi}\ln\left(\frac{1}{r(\boldsymbol{\eta},\xi)}\right)$, $r(\boldsymbol{\eta},\xi) \equiv |\boldsymbol{\eta} - \xi|$ and $\Phi(\boldsymbol{\eta},\xi) \equiv \frac{\Theta(\boldsymbol{\eta},\xi)}{\partial \mathbf{n}(\xi)}$, where $\mathbf{n}$ — outer normal to $X \cup \Gamma$.

Temperature field $\bar{T}$ on the contour Γ is determined as

$$\bar{T}(\boldsymbol{\eta}) = -\int_X \Phi(\boldsymbol{\eta}, x_1)\bar{T}(x_1)\, dx_1 + \int_X \Theta(\boldsymbol{\eta}, x_1)\left(\frac{J(x_1)}{\kappa} + \frac{h}{\kappa}T_m\right) dx_1, \ \boldsymbol{\eta} \in \Gamma, \qquad (2)$$

where $\bar{T}(x_1)$ is the solution of the boundary integral equation

$$\frac{1}{2}\bar{T}(x_1) + \int_X \left(\Phi(x_1,\xi) + \frac{h}{\kappa}\Theta(x_1,\xi) \right) \bar{T}(\xi)\, d\xi =$$
$$= \int_X \Theta(x_1,\xi)\left(\frac{J(\xi)}{\kappa} + \frac{h}{\kappa}T_m\right) d\xi, \qquad x_1 \in X. \qquad (3)$$

Equations (1)-(3) specify the mathematical model for thermal sounding of the body with stationary heat flows.

When the sounding heat flow $J(x_1)$ is given, we can solve sequentially the equations (3) and (1) for any given values of the parameters $\beta_1, \beta_2, \ldots, \beta_n$ and determine in such way the temperature perturbation of $\tilde{T}(x_1)$ on the body's external surface. The calculated field $\tilde{T}(x_1)$ is be dependent on the sounding flow intensity $J(x_1)$ and geometrical parameters $\beta_1, \beta_2, \ldots, \beta_n$. We indicate that as $\tilde{T}(x_1) = \tilde{T}(x_1; J; \beta_1, \beta_2, \ldots, \beta_n)$.

From the other hand, we can impact on the real-life body, containing a near-surface cavity, by heat flow of intensity $J^e(x_1)$ and measure the body's stationary surface temperature distribution $T^e(x_1)$, caused by the flow. Then, calculating the disturbance $\tilde{T}^e(x_1) = T^e(x_1) - \bar{T}(x_1)$, we can compare it to the field $\tilde{T}(x_1; J; \beta_1, \beta_2, \ldots, \beta_n)$, being the solution of integral equation (1), obtaining for any chosen values of geometrical parameters $\beta_1, \beta_2, \ldots, \beta_n$. So, juxtaposing the measured data $T^e(x_1)$ to the mathematical model (1)-(3) gives some information about parameters $\beta_1, \beta_2, \ldots, \beta_n$.

2. Direct and inverse problems for cavity identification

Direct problem: a) determine perturbation of temperature field $\tilde{T}(x_1) = \tilde{T}(x_1; J; \beta_1, \beta_2, \ldots, \beta_n)$ on external body's surface for any given heat flow density $J(x_1) \in \mathcal{J}$ and any given values $\beta_k \in B_k$, $k = \overline{1, n}$ of all cavity's geometric parameters; b) determine on this basis characteristic parameters $F_1, F_2, \ldots F_m$ $(m \geq n)$ of temperature field $\tilde{T}(x_1)$, $x_1 \in X$, which can be used as informative parameters for cavity identification.

Points of function's $\tilde{T}(x_1)$, $x_1 \in X$ extremums $\arg\left(\max_{x_1}\left(\tilde{T}(x_1)\right)\right)$, $\arg\left(\min_{x_1}\left(\tilde{T}(x_1)\right)\right)$ and their values $\max_{x_1}\left(\tilde{T}(x_1)\right)$, $\min_{x_1}\left(\tilde{T}(x_1)\right)$ can be used as the characteristic parameters F_1, F_2, ... F_m in particular.

To solve the direct problem it is necessary to develop an algorithm for solving the integral equations (1), (3) and study numerically with its using the influence of values of the parameters $\beta_1, \beta_2, \ldots, \beta_n$ on temperature disturbance $\tilde{T}(x_1)$, $x_1 \in X$.

Inverse problem: functions for sounding heat flow $J^e(x_1)$, $x_1 \in X$ and temperature disturbance $\tilde{T}^e(x_1)$, $x_1 \in X$ on external body surface are given; the values of cavity's geometrical parameters $\beta_1, \beta_2, \ldots, \beta_n$ should to be determined.

We will use a variational formulation for the inverse problem.

Let $F_1^e, F_2^e, \ldots F_m^e$ be the values for characteristic parameters $F_1, F_2, \ldots F_m$ of the temperature disturbance $\tilde{T}^e(x_1)$, $x_1 \in X$. Function $\tilde{T}^e(x_1)$ is determined from the measured data obtained by thermal sounding of the real-world body, containing the cavity, with the heat flow of intensity $J^e(x_1)$, $x_1 \in X$. So, vector $\mathbf{F}^e \equiv \left(F_1^e, F_2^e, \ldots F_m^e\right)^{\mathrm{T}}$ carries posteriori information about cavity's geometry. Let $\tilde{T}(x_1; J; \beta_1, \beta_2, \ldots, \beta_n)$, $x_1 \in X$ be a solution of the direct problems, obtained at $J(x_1) = J^e(x_1)$, $x_1 \in X$ and some given values of geometric parameters $\beta_1, \beta_2, \ldots, \beta_n$, $\mathbf{F} \equiv \left(F_1, F_2, \ldots F_m\right)^{\mathrm{T}}$ is the vector of values of characteristic parameters $F_1, F_2, \ldots F_m$ for this solution. Obviously under fixed $J^e(x_1)$, $x_1 \in X$ vector $\mathbf{F}$ is dependent on just the geometrical parameters: $\mathbf{F} = \mathbf{F}(\beta_1, \beta_2, \ldots, \beta_n)$. Due to that we will consider the set $\{\beta_1, \beta_2, \ldots, \beta_n\}$ as the solution of the inverse problem if vector $\mathbf{F}(\beta_1,\beta_2,\ldots,\beta_n)$ is closest in some sense to vector $\mathbf{F}^e$, i.e. $\left\|\mathbf{F}(\beta_1,\beta_2,\ldots,\beta_n)-\mathbf{F}^e\right\|\to\min$, where $\|\ldots\|$ is a vector norm, for instance l_2.

Variational formulation jointly with iterative procedure will enable us to reduce the inverse problem to solving of some sequence of direct problems.

3. Boundary-element model for thermal sounding

In the direct problem the geometry of the cavity is known. So, we can solve boundary integral equations (2) and (1) for any given function $J(x_1), x_1\in X$ of sounding heat flow and determine the temperature disturbance $\tilde{T}(x_1), x_1\in X$. To do that we use the boundary-element method [11].

Let $\mathcal{A}\subset X$ be the projection of the contour Γ on x_1 axis. We choose the coordinate system x_1Ox_2 with the origin O in the center of the segment $\mathcal{A}$ and take a segment $\mathcal{L}(x_1)\subset X$: $x_1\in[-L/2,L/2]$ containing $\mathcal{A}$. We choose $L\gg a$, where $a=|\mathcal{A}|$ is the length of the segment $\mathcal{A}$. Then we choose $N\in\mathbb{Z}$, satisfying relation $L/N\ll a$, and select on $N+1$ regularly spaced nodes with Cartesian coordinates $(\xi_i,0)$, where $\xi_i=-L/2+i\,L/N$, $i=\overline{0,N}$. These nodes form N linear finite elements $E_i^X=[\xi_i,\xi_{i+1}]$, $i=\overline{0,N-1}$. Now we can consider line X as a union of $N+2$ segments, namely: N finite elements E_i^X and two semi-infinite elements $E_-^X\equiv(-\infty,-L/2]$ and $E_+^X\equiv[L/2,\infty)$: $X=E_-^X\cup\left(\cup_{i=0}^{N-1}E_i^X\right)\cup E_+^X$.

Selecting on contour Γ $N_\Gamma\in\mathbb{Z}$ regularly spaced nodes with Cartesian coordinates $\xi_i\equiv(\xi_{1i},\xi_{2i})$, where $\xi_{1i}=x_1(s_i)$, $\xi_{2i}=x_2(s_i)$, $\{s_i, i=\overline{0,N_\Gamma}\}$, we introduce N_Γ curvilinear finite elements $E_i^\Gamma=\{\mathbf{x}(s), s\in[s_i,s_{i+1})\}$, each of which is a segment of the contour: $\Gamma=\bigcup_{i=0}^{N^\Gamma-1}E_i^\Gamma$.

We introduce in equations (1), (2) new unknown functions $u\equiv\tilde{T}-T_m$ and $\overline{u}\equiv\overline{T}-T_m$ which decay when $x_1\to\pm\infty$, and apply to them linear approximations on each finite elements $E_i^X\left(i=\overline{0,N-1}\right)$ and $E_i^\Gamma\left(i=\overline{0,N_\Gamma-1}\right)$ using the functions $\varphi_1(t)=(1-t)/2$ and $\varphi_2(t)=(1+t)/2$ [9].

To reflect the semi-infinite elements E_-^X and E_+^X on segment $[-1,1]$ we use for them nonlinear transformations of coordinates $x_1=\xi_1(t-3)/(1+t)$ and $x_1=\xi_N(t+3)/(1-t)$ ($t\in[-1,1]$) correspondingly. Then, we approximate the unknown functions on these elements by the functions $\varphi_-(t)=(1+t)^2/4$ and $\varphi_+(t)=(1-t)^2/4$ respectively.

Finally, of the unknown function we can replace the integrals in the equations (1), (2) by sums of boundary element integrals. For instance we have

$$\int_{-\infty}^{\infty} H(\boldsymbol{\eta},x)u(x)dx = u_1\int_{-1}^{1} H(\boldsymbol{\eta},t)\varphi_-(t)J_-(t)dt +$$
$$+\sum_{i=2}^{N-2}\frac{L}{2N}\int_{-1}^{1} H(\boldsymbol{\eta},t)\left(u_i\varphi_1(t)+u_{i+1}\varphi_2(t)\right)dt + u_N\int_{-1}^{1} H(\boldsymbol{\eta},t)\varphi_+(t)J_+(t)dt \qquad (4)$$

where $H(\boldsymbol{\eta},x)=\Phi(\boldsymbol{\eta},x)+\frac{h}{\kappa}\Theta(\boldsymbol{\eta},x)$, $J_+(t)=4\xi_N/(1-t)^2$, $J_-(t)=4\xi_1/(1+t)^2$.

In the issue the boundary integral equations (1) and (2) will be replaced by the systems of linear algebraic equations (5) and (6) respectively:

$$\mathbf{M}_{(11)}\tilde{\mathbf{U}}_{(1)}+\mathbf{M}_{(12)}\tilde{\mathbf{U}}_{(2)}=\mathbf{0}, \quad \mathbf{M}_{(21)}\tilde{\mathbf{U}}_{(1)}+\mathbf{M}_{(22)}\tilde{\mathbf{U}}_{(2)}=\mathbf{B}, \qquad (5)$$
$$\mathbf{M}_{(11)}\overline{\mathbf{U}}_{(1)}=\overline{\mathbf{B}}. \qquad (6)$$

Here $\tilde{\mathbf{U}}_{(1)}$ and $\tilde{\mathbf{U}}_{(2)}$ are vectors of function's u node values on the line X and contour Γ correspondingly, $\overline{\mathbf{U}}_{(1)}$ is vector of function's $\overline{u}$ node values on the line X; $\mathbf{M}_{(11)}$, $\mathbf{M}_{(12)}$, $\mathbf{M}_{(21)}$ and $\mathbf{M}_{(22)}$ are matrixes of dimensions $N\times N_\Gamma$, $N_\Gamma\times N$ and $N_\Gamma\times N_\Gamma$; $\mathbf{B}=\mathbf{\Phi}\cdot\left(\mathbf{M}^{-1}\cdot\overline{\mathbf{B}}\right)$, where $\mathbf{M}$ and $\mathbf{\Phi}$ are $N\times N$-matrixes; $\overline{\mathbf{B}}=\mathbf{\Theta}\cdot\left(\mathbf{J}+\left(T_m h/\kappa\right)\mathbf{I}\right)$, where $\mathbf{\Theta}$ is $N\times N$-matrix, $\mathbf{J}$ is vector of node values of sounding flux intensity function on the line X, $\mathbf{I}$ is unit N-vector.

4. Solution of direct problem and informative parameters for cavity identification

Solving sequentially the systems (6) and (5), we can determine vector $\tilde{\mathbf{U}}_{(1)}$, representing temperature disturbance $\tilde{T}(x_1)$, $x_1\in X$ on external surface of the body, and study on this basis influence of the geometrical parameters $\beta_1,\beta_2,\ldots,\beta_n$ on the characteristic parameters $F_1,F_2,\ldots F_m$ of the temperature field $\tilde{T}(x_1)$.

Further we restrict ourselves by the case of circular cylindrical cavity. Its geometry is determined by three independent parameters i.e. — coordinates $x_1=x_0$ and $x_2=y_0$ of the circle's center, and its radius r_0.

Let's the function

$$J(x_1)\equiv\frac{j_0}{b\sqrt{2\pi}}\exp\left(-\frac{(x_1-\zeta)^2}{2b^2}\right), \quad \zeta\in\mathcal{L}, \qquad (7)$$

where $j_0=const>0$, $b=const>0$, determines intensity of the sounding heat flow.

In this case the direct problem's solution $\tilde{T}(x_1), x_1 \in X$ is dependent on three parameters of sounding heat flow j_0, b, ζ and three geometrical parameters: $\tilde{T}(x_1) = \tilde{T}(x_1; j_0, b, \zeta; x_0, y_0, r_0)$.

Applying the boundary-element model (5), (6) we can study influence of sounding heat flow's parameters and geometrical parameters of the cavity on temperature disturbance on body's external surface.

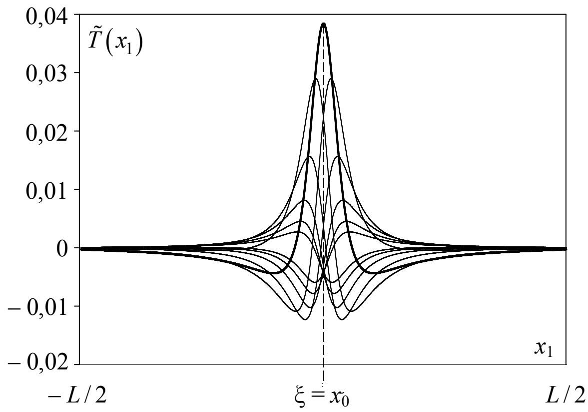

Fig. 1. Temperature disturbances for different heat flow's position $\zeta \in \{-L/8 + i\}$, $i = \overline{0,10}$

On fig. 1 the plots for temperature disturbance $\tilde{T}(x_1), x_1 \in X$, calculated for different positions of sounding heat flow, are presented. The calculations were made for the next values of the geometrical parameters: $x_0 = 0,\ y_0 = -2,\ r_0 = 0{,}5$; values for parameters of the sounding heat flow were chosen $j_0 = 100\,\mathrm{W/m^2}$ and $b = 1$; the coefficients of thermal conductivity and convective heat exchange were taken equal $\kappa = 1{,}15\ \mathrm{W/m \cdot K}$ and $h = 2\ \mathrm{W/m^2 \cdot K}$, the value $T_m = 300\,\mathrm{K}$ was used for ambient temperature. The boundary elements was defined by $L = 40,\ N = 300$ and $N_\Gamma = 320$.

At fixed parameters j_0, b, x_0, y_0, r_0 the function of temperature disturbance $\tilde{T}(x_1), x_1 \in X$ is dependent just on parameter ζ, determining position of sounding heat flow: $\tilde{T}(x_1) = \tilde{T}(x_1; \zeta)$. We considered characteristic parameters $F_1, F_2, \ldots, F_6$ for functions $\tilde{T}(x_1; \zeta), x_1 \in X$, namely: the maximum $F_1(\zeta) = \max_{x_1} \tilde{T}(x_1; \zeta)$ and two minimums $F_{2,3}(\zeta) = \min_{x_1} \tilde{T}(x_1; \zeta)$, as well as the coordinates of these extremums: $F_4(\zeta) = \arg\left(\max_{x_1} \tilde{T}(x_1, \zeta)\right)$, $F_{5,6}(\zeta) = \arg\left(\min_{x_1} \tilde{T}(x_1, \zeta)\right)$. The values of the parameters are dependent on ζ . Theses dependence, calculated for the curves, presented on fig. 1, are depicted on the fig. 2.

As we can see from the plots fig. 1 and 2, the coordinate $F_4(\zeta)$ of maximum of function $\tilde{T}(x_1; \zeta)$ in general (for arbitrary ζ) doesn't coincide with coordinate ζ of maximum of the sounding flow. Just when ζ coincides with abscissa x_0 of the cavity's center, $F_4(\zeta)$ coincides with coordinate ζ. That can be written as

$$\zeta = x_0 \ \Rightarrow\ F_4(\zeta) = \zeta . \tag{8}$$

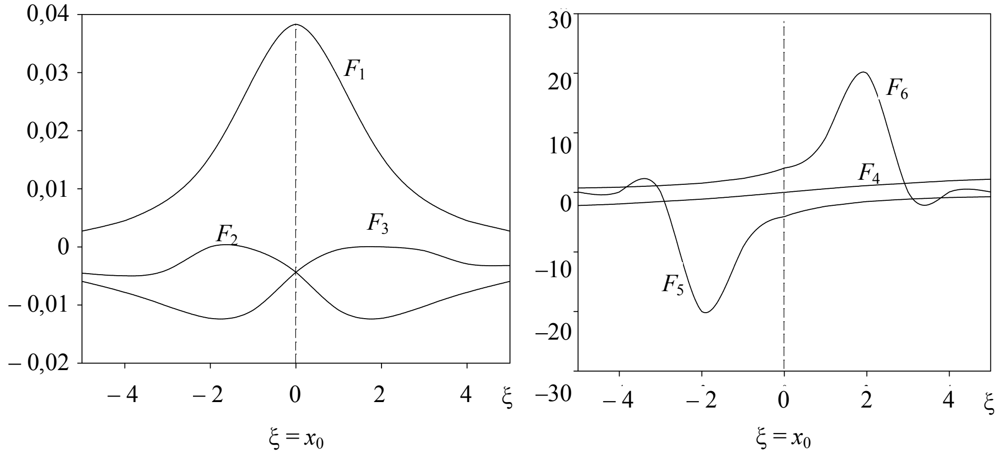

Fig. 2. Dependences the parameters F_1, F_2, ..., F_6 on the position of sounding flow

This property follows from symmetry of the temperature field $\tilde{T}(x_1,\zeta)$ at $\zeta = x_0$ with respect to variable x_1.

Taking into account the uniqueness of the solution $\tilde{T}(x_1;\zeta)$ of the direct problem, we can conclude that property (8) is biunique one, i.e.

$$F_4(\zeta) = \zeta \Rightarrow \zeta = x_0 . \tag{9}$$

Analyzing behavior of minimums of functions $\tilde{T}(x_1;\zeta)$ we can see, that

$$|x_0 - F_4| < |x_0 - F_5| \Rightarrow F_2(\zeta) > F_3(\zeta), \tag{10}$$

$$\zeta = x_0 \Rightarrow F_2(\zeta) = F_3(\zeta), \quad \zeta = x_0 \Rightarrow F_5(\zeta) + F_6(\zeta) = 2\zeta . \tag{11}$$

Taking into account the uniqueness of the solution $\tilde{T}(x_1;\zeta)$ of the direct problem, we can conclude that property (10), (11) are biunique, i.e.

$$F_2(\zeta) > F_3(\zeta) \Rightarrow |x_0 - F_4| < |x_0 - F_5|, \tag{12}$$

$$F_2(\zeta) = F_3(\zeta) \Rightarrow \zeta = x_0, \quad F_5(\zeta) + F_6(\zeta) = 2\zeta \Rightarrow \zeta = x_0 . \tag{13}$$

The property (8)-(13) reflect the influence of the relative position of cavity and sounding flow centers on temperature field of body's external surface. They can be used for identification of x_0 independently on two other parameters y_0 and r_0.

To estimate a possibility to use the characteristic parameters F_1, F_2, ..., F_6 as informative parameters for identification of the geometric parameters we studied numerically the influence of the parameters y_0 and r_0 on F_1, F_2, ..., F_6. We restricted ourselves by the case of central sounding, when the center ζ of the sounding flow distribution coincides with abscissa x_0 of cavity's center.

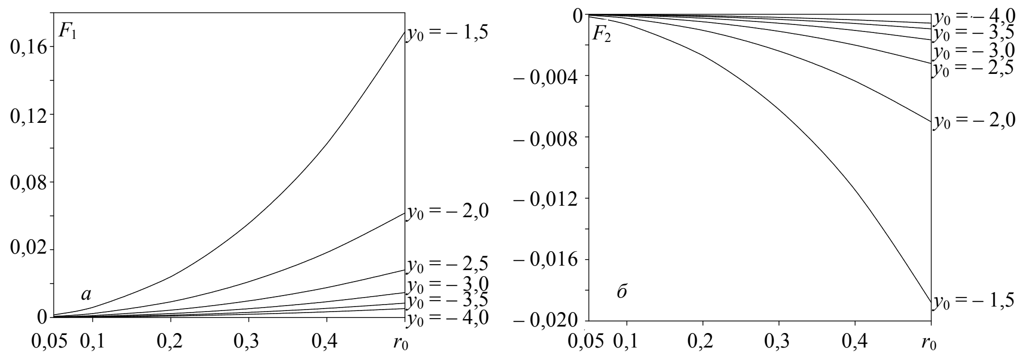

Fig. 3. Dependences of parameters F_1 (a) and F_2 (b) on geometrical parameters r_0 and y_0

On fig. 3 influence of geometric parameters y_0, r_0 on characteristic parameters F_1 and $F_2 = F_3$ are presented for example. The calculations were made by solving the direct problem for case of central sounding for discrete sets of values for parameters $y_0 \in \{-4;-3,5;-3;-2,5;-2;-1,5\}$ and $r_0 = \{0,05;0,1;0,2;0,3;0,4;0,5\}$.

As we can see, maximum $F_1(x_0)$ and minimum $F_2(x_0)$ of temperature field $\tilde{T}(x_1), x \in X$, excited by central thermal sounding, are strongly dependent on parameters y_0 and r_0. Therefore F_1 and F_2 can be used as informative parameters for identification the cavity's geometrical parameters y_0 and r_0.

5. Iterative method for solving of inverse problem

Since the parameter x_0 can be determined independently, we consider the inverse problem for identification parameters y_0 and r_0 with the use of the data, obtained by central sounding of the body. In this case only three characteristic parameters F_1, F_2 and F_5 are independent.

Let $\tilde{T}^e(x_1), x_1 \in X$ be a known function, describing the disturbance of temperature field on the external surface of the half-space that contains the circular tunnel cavity with geometrical parameters x_0^e, y_0^e and r_0^e. Parameter x_0^e is known. The function $\tilde{T}^e(x_1)$ was determined from the data, obtained by central sounding $\left(\zeta = x_0^e\right)$ the body with heat flow, intensity of that is described by formula (7) at $\zeta = x_0^e$ and given $j_0 = j_0^e$ and $b = b^e$. It is necessary to determine values of cavity's geometrical parameters y_0^e and r_0^e.

Since function $\tilde{T}^e(x_1), x_1 \in X$ is known we can determine its characteristic parameters $F_1^e = \max\limits_{x_1}\left(\tilde{T}^e(x_1)\right)$, $F_2^e = \min\limits_{x_1}\left(\tilde{T}^e\right)$ and $F_5^e = \arg\left(\min\limits_{x_1}\left(\tilde{T}^e\right)\right)$.

Let's take arbitrary values y_0 and r_0 for unknown parameters and solve the direct problem for case of central sounding by the flow (7) with the parameters $\zeta = x_0^e$,

$j_0 = j_0^e$ and $b = b^e$. Function $\tilde{T}(x_1)$, $x_1 \in X$ determined in such way will be dependent on the chosen values of the parameters y_0 and r_0: $\tilde{T}(x_1) = T(x_1; y_0, r_0)$. Now we can calculate the values of parameters F_1, F_2 and F_3 for the field $\tilde{T}(x_1)$, $x_1 \in X$: $F_1(y_0, r_0) =$ $= \max\limits_{x_1}\left(\tilde{T}(x_1; y_0, r_0)\right)$, $F_2(y_0, r_0) = \min\limits_{x_1}\left(\tilde{T}(x_1; y_0, r_0)\right)$, $F_5(y_0, r_0) = \arg\left(\min\limits_{x_1}\left(\tilde{T}(x_1; y_0, r_0)\right)\right)$.

If chosen values y_0 and r_0 are equal to the corresponding actual values y_0^e and r_0^e, the obtained value F_1, F_2 and F_5 will be equal to corresponding values F_1^e, F_2^e and F_5^e:

$$F_k\left(y_0^e, r_0^e\right) - F_k^e = 0, \quad k = 1,2,5, \tag{14}$$

So, we obtain the system of equations for determination parameters y_0^e and r_0^e.

Explicit analytical structures of the functions $F_i(y_0, r_0)$ are unknown. But we can calculate the values of each of them for any given y_0 and r_0.

Three equations (14) make up nonlinear overdetermined system. It can be solved by the iterative method developed in [8]. But since this system is dependent just on two unknown variables we can consider only two equations

$$F_k\left(y_0^e, r_0^e\right) - F_k^e = 0, \quad k = 1,2, \tag{15}$$

To solve the system (15) we developed an iterative algorithm based of Newton method [10].

Let $y_0 \in B_y = \left[y_0^{\min}, y_0^{\max}\right]$, $r_0 \in B_r = \left[r_0^{\min}, r_0^{\max}\right]$, where $y_0^{\min}, y_0^{\max}$ and $r_0^{\min}, r_0^{\max}$ stand for the lower and upper bounds for parameters y_0 and r_0. Using the denotation $\mathbf{f} = (f_1, f_2)^{\mathrm{T}}$, where $f_i = f_i(y_0, r_0) \equiv F_i(y_0, r_0) - F_i^e$, $i = 1,2$, $\mathbf{Y} = (y_0, r_0)^{\mathrm{T}}$, we can represent the iterative procedure for numerical solving the system (15):

$$\mathbf{Y}^{(k+1)} = \mathbf{Y}^{(k)} - \mathbf{G}^{-1}\left(\mathbf{Y}^{(k)}\right)\mathbf{f}\left(\mathbf{Y}^{(k)}\right), \tag{16}$$

where, $\mathbf{Y}^{(k)}$ and $\mathbf{Y}^{(k+1)}$ are vectors of k-th and $(k+1)$-th approaches, $\mathbf{G}$ stands for Jacobi matrix for functions $f_1(y_0, r_0)$ and $f_2(y_0, r_0)$.

To calculate Jacobi matrix it is necessary to determine the derivatives $\partial f_i(y_0, r_0)/\partial y_0$, $\partial f_i(y_0, r_0)/\partial r_0$, $i = 1,2$. Since the analytical structure of functions $F_i^c(y_0, r_0)$ is unknown, these derivatives be replaced by the finite differences [8]

$$\frac{\partial f_i(y_0, r_0)}{\partial y_0} \approx \frac{f_i(y_0 + \Delta y_0, r_0) - f_i(y_0, r_0)}{\Delta y_0}, \tag{17}$$

$$\frac{\partial f_i(y_0, r_0)}{\partial r_0} \approx \frac{f_i(y_0, r_0 + \Delta r_0) - f_i(y_0, r_0)}{\Delta r_0}. \tag{18}$$

where Δy_0 and Δr_0 are positive constants small as compared to $\left|y_0^{\max} - y_0^{\min}\right|$ and $\left|r_0^{\max} - r_0^{\min}\right|$.

Calculations due to the formula (16) should be terminated, when the condition $\left\|\mathbf{Y}^{(k+1)} - \mathbf{Y}^{(k)}\right\| \le \varepsilon$ becomes true. Here is a small positive parameter.

6. Numerical results

To estimate the effectiveness of the developed algorithms for solving of the inverse problem we used a numerical experiment. To obtain the input data for the inverse problem the values for cavity's geometric parameters have been assigned: $x_0 = 0$, $y_0 = — 2$, $r_0 = 0{,}5$. Then the direct problem for assigned parameters was solved. Obtained in such way solution was treated as function $\tilde{T}^e(x_1)$, $x_1 \in X$. Next, the values of the characteristic parameters $F_1^e = 0{,}0617$ and $F_2^e = -0{,}007$ for field $\tilde{T}^e(x_1)$ were determined. These values were used in the system (15). The values $y_0 = -3$, $r_0 = 1$ were used as zero-approximation for the iterative process (16). They have been taken as an input data for solving system (14) by algorithm (15). The values $y_0 = -1{,}999984$ and $r_0 = 0{,}499976$ were obtained on the 6-th iteration.

Conclusions. Direct and inverse problems for identification of tunnel cavity in the half-space on the base of given surface temperature field, excited by concentrated heat flow, incident on the half-space's surface, have been considered. The boundary element method for solving of the direct problem has been developed. Analyzing numerical solutions of the direct problem, a set of characteristic parameters of surface's temperature field has been established. These parameters can be used as informative ones for identification of the cavity. A variational formulation for inverse problem of determination of cavity's geometric parameters has been done with the use of the characteristic parameters. An iterative boundary-element method for solving of the inverse problem has been developed. Effectiveness of the developed method has been corroborated with the use of numerical experiment. This method can be used under development of nondestructive contactless methods for cavities identification in solids with the use of technique of IR-thermography.

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Гранично-елементний метод теплової ідентифікації параметрів приповерхневої тунельної циліндричної порожнини у тілі з плоскою поверхнею

Василь Чекурін, Олег Сінькевич

Розглянуто задачу визначення геометричних параметрів циліндричної тунельної порожнини у теплопровідному півпросторі за заданим на зовнішній поверхні температурним полем, збудженим стаціонарним нагрівом тіла зосередженими тепловими потоками. Із застосуванням граничних інтегральних рівнянь побудовано двовимірну математичну модель теплового зондування тіла, в межах якої сформульовані пряма й обернена задачі ідентифікації параметрів порожнини. Гранично-елементним методом досліджено пряму задачу. На цій основі виявлено та досліджено інформативні параметри теплової ідентифікації. Із використанням цих параметрів сформульовано обернену задачу, яку зведено до нелінійної системи неявно заданих рівнянь. Розроблено ітераційний алгоритм розв’язування цієї системи, який ґрунтується на методі Ньютона. Проведено числове дослідження ефективності цього алгоритму. Запропонований метод можна застосувати для створення засобів ідентифікації порожнин у твердих тілах на основі даних ІЧ-термографії.

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Гранично-элементный метод идентификации приповерхностной туннельной цилиндрической полости в теле с плоской поверхностью

Василь Чекурин, Олег Синькевич

Рассмотрена задача определения геометрических параметров цилиндрической туннельной полости в теплопроводном полупространстве, исходя из заданного температурного поля внешней поверхности тела, возбуждаемого сосредоточенным стационарным тепловым потоком в условиях конвективного теплообмена с внешней средой. С использованием граничных интегральных уравнений построена двумерная математическая модель теплового зондирования тела. В рамках этой модели сформулированы прямая и обратная задачи идентификации параметров полости. Гранично-элементным методом проведено исследование прямой задачи. На этой основе выявлены и исследованы информативные параметры. С использованием этих параметров сформулировано обратную задачу, которая сведена к системе нелинейных неявно заданных уравнений. Разработан итерационный алгоритм решения этой системы, основанный на методе Ньютона. С использованием численного эксперимента проведено исследование эффективности разработанного метода. Предложенный метод можно использовать для создания методов идентификации полостей в твердых телах с использованием техники ИК-термографии.

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