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On taking into account the medium moving in investigation on thermomechanical processes in electroconductive ferromagnetic bodies

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A version of quantitative description is offered in interconnection of electromagnetothermomechanical processes in electroconducting ferromagnetic bodies in external time-periodic magnetic field. Using the methods of similarity and dimensional theory, a system of electrodynamics and thermoelasticity equations is reduced to the dimensionless form. The quantitative analysis of characteristic dimensionless criteria for technically pure iron was carried out. The effects for which characteristic dimensionless parameters are much less in comparison to unit are neglected. A simplified system of equations of the model to find the magnetic field, temperature, displacements and mechanical stresses in electroconducting ferromagnetic bodies is written. The parameters at which the medium mobility should be considered are found.

Keywords: electromagnetic field, magnetothermomechanics, conducting ferromagnetic bodies, methods of similarity and dimensional theory.

Introduction. Mathematical models that describe the interaction of electromagnetic field (EMF) with a substance are usually quite bulky and generally are of little use for practical calculations [1-4 and others]. Original equations of such models contain many terms which make negligible contribution compared to other analogical. Generally the contribution of many terms is neglected, not giving sufficient justification. It turns out that depending on the concrete external action and material, many terms can be neglected, and the model in this case is considerably simplified.

We assume that the considered body is exposed to an external time-periodic EMF, given by the vector of the magnetic field intensity on the surface of the body

$$\vec{H}^{(ext)}\left(\vec{r},t\right)=\vec{H}_1\left(\vec{r}_1\right)\cos(\omega t), \quad (1)$$

where $\vec{r}_1$ is the radius-vector of the point of the surface; $\omega = 2\pi\nu$, ν is frequency, t is time; $\vec{H}_1(\vec{r}_1)$ is amplitude of the time-harmonic magnetic field.

It is important to find conditions under which you need to consider the medium mobility [5-10] on investigating thermomechanical processes in ferromagnetic electro-conducting bodies in external magnetic field given by the law (1).

We proceed from the fact that the equations both of electrodynamics and thermo-mechanics are invariant with respect to Galilean transformations [11-16]. For this purpose we consider two coordinate reference systems K and K'. The system K' is moving relative to the system K with constant velocity $\vec{v}$, which direction is randomly chosen. Space-time points in these systems are denoted, respectively, by $(\vec{r}, t)$ and $(\vec{r}', t')$, and by all values of the system K' under consideration we place a prime. In continuum mechanics native reference frame K' is associated with Lagrange and the laboratory one K with Euler. We write the transformation laws of the coordinates $\vec{r}$, time t and operators of differentiation with respect to coordinates and time:

$$\vec{r}' = \vec{r} - \vec{v}\,t, \quad t' = t, \quad \vec{\nabla}' = \vec{\nabla}, \quad \Delta' = \Delta, \quad \partial/\partial t' = \partial/\partial t + \vec{v}\cdot\vec{\nabla} \equiv d/dt, \qquad (2)$$

where $\vec{\nabla}$ and Δ are, respectively, Hamiltonian and Laplace operators. The value d/dt in the literature is called also as a total time derivative [2, 8].

1. Mathematical model of electromagnetothermomechanics in its native reference frame

We write the basic formula of electrodynamics and thermomechanics of slowly moving ferromagnetic [8-17] (invariant with respect to Galilean transformation) in native (moving) reference frame.

Consider the ferromagnetic materials which are characterized by non-linear relationship between the induction vectors $\vec{B}'$ (magnetization $\vec{M}'$) and the intensity of the magnetic field $\vec{H}'$ and are connected by the relation:

$$\vec{B}' = \mu_0\left(\vec{H}' + \vec{M}'\left(\vec{H}'\right)\right). \qquad (3)$$

Confine ourselves to consideration isotropic ferromagnetic media, in which induction vector $\vec{B}'$ (or magnetization $\vec{M}'$) is parallel to the stress vector $\vec{H}'$ of magnetic field, i. e.:

$$\vec{B}'\left(\vec{H}'\right) = \mu_0\,\mu\left(\vec{H}'\right)\vec{H}', \quad \vec{M}'\left(\vec{H}'\right) = \chi\left(\vec{H}'\right)\vec{H}'. \qquad (4)$$

Here $\chi\left(\vec{H}'\right)$ and $\mu\left(\vec{H}'\right) \equiv 1 + \chi\left(\vec{H}'\right)$ are, respectively, the relative magnetic susceptibility and permeability of the medium; μ_0 is magnetic constant.

Note that a variety of relations $\mu\left(\vec{H}'\right)$ (or $\chi\left(\vec{H}'\right)$) for different ferromagnetic materials is sufficiently well described in [18-20 et al.].

As regards the electrical properties, ferromagnetic materials are conventional dielectrics for which the relationship between the induction vector $\vec{D}'$ and stress vector $\vec{E}'$ of electric field has the form:

$$\vec{D}' = \varepsilon_0 \varepsilon \vec{E}', \tag{5}$$

where ε is relative dielectric permeability of the medium; ε_0 is electric constant.

The third phenomenological electrodynamics relation refers to electroconductive bodies, which are ferromagnets. This is Ohm's law, which relates the conduction current density $\vec{j}'$ with the intensity of magnetic field. Based on the work [21], we write it as:

$$\vec{j}' = \lambda\left(\vec{E}' + \vec{E}_s'\right). \tag{6}$$

Here $\vec{E}_s' = \varphi_0 \vec{\nabla}' \left(\alpha_T T' + \dfrac{1-2\nu_p}{3E_p}\sigma' - \dfrac{q'}{3q_n} \right)$ is the electric field intensity due to the fields of non-electromagnetic physical nature; λ is electric conductivity; T' is absolute temperature; σ' is the first invariant of the stress tensor $\hat{\sigma}'$; q' is specific electric charge per unit mass of the body; α_T is the temperature coefficient of linear expansion; E_p and ν_p are elasticity modulus and Poisson ratio; $\varphi_0 = 2\varphi_1$. The formulas for the values φ_1 and q_n are given in [21], which because of the awkwardness, will not be given here.

Taking into account the relation (5), we write the Maxwell equation and the law of conservation of electric charge for the body region:

$$\vec{\nabla}' \times \vec{H}' = \varepsilon_0 \varepsilon \frac{\partial \vec{E}'}{\partial t'} + \vec{j}', \quad \vec{\nabla}' \times \vec{E}' = -\frac{\partial \vec{B}'}{\partial t'}, \quad \frac{\partial (\rho' q')}{\partial t'} + \vec{\nabla}' \cdot \vec{j}' = 0,$$

$$\varepsilon_0 \varepsilon \vec{\nabla}' \cdot \vec{E}' = \rho' q', \quad \vec{\nabla}' \cdot \vec{B}' = 0. \tag{7}$$

Here ρ' is the density of the body and the symbols $\vec{\nabla}'\times$ and $\vec{\nabla}'\cdot$ denote the operations of the rotor and divergence in their native reference frame.

EMF as regards the material continuum we consider to be external effect, which manifests itself in the body through the energetic and power factors of interaction. Energy characteristics of the fields we introduce using the law of conservation of EMF as Poynting theorem condition [20]. Then for this case, the stored energy in the body W' of EMF (including the relations (3)-(6)) can be written as:

$$W' = 0{,}5\left[\varepsilon_0 \varepsilon \left(\vec{E}'\right)^2 + \vec{H}' \cdot \vec{B}' \right]. \tag{8}$$

The volume total thermal output Q' in the ferromagnet due to the flow of electric current and reversal of magnetization is [20]:

$$Q' = \vec{j}' \cdot \vec{E}' + \frac{\mu_0}{2}\left(\vec{H}' \cdot \frac{\partial \vec{M}'}{\partial t'} - \vec{M}' \cdot \frac{\partial \vec{H}'}{\partial t'} \right). \quad (9)$$

The ponderomotive force $\vec{F}$ and force moment $\vec{N}$ are the force factors of EMF interaction with the material medium. The expression for the ponderomotive force density using a statistical model of electromechanic interaction between the field and the medium is convenient to represent as [20, 22]:

$$\vec{F}' = \vec{F}_0' + \vec{\mathcal{F}}', \quad (10)$$

where

$$\vec{F}_0' = \rho' q' \vec{E}' + \vec{j}' \times \vec{B}', \quad (11)$$

$$\vec{\mathcal{F}}' = \varepsilon_0 (\varepsilon - 1)\left[\left(\vec{E}' \cdot \vec{\nabla}' \right) \vec{E}' + \vec{E}' \times \left(\vec{E}' \times \vec{\nabla}' \right) \right] + \left(\vec{M}' \cdot \vec{\nabla}' \right) \vec{B}' + \vec{M}' \times \left(\vec{\nabla}' \times \vec{B}' \right), \quad (12)$$

and the force $\vec{F}_0'$ is created by electric charges and currents. The first two terms of the force $\vec{\mathcal{F}}'$ are responsible for the polarization, and two others — for reversal of magnetization the medium.

The ponderomotive force acting on an arbitrary region of the medium can be reduced to tension force acting on the surface of this region [7, 14, 23, 24]:

$$\vec{F}' = \vec{\nabla}' \cdot \hat{P}' - \partial \vec{G}' / \partial t', \quad (13)$$

where

$$\vec{G}' = \varepsilon_0 \varepsilon \vec{E}' \times \vec{B}' \quad (14)$$

is EMF vector of momentum density in the medium.

In the case of ponderomotive force (10) for the Maxwell tension tensor we have:

$$\hat{P}' = \varepsilon_0 \varepsilon \vec{E} \otimes \vec{E}' + \vec{H}' \otimes \vec{B}' + w' \hat{I}. \quad (15)$$

Here

$$w' = \frac{\mu_0}{2}\left[\left(\vec{M}' \right)^2 - \left(\vec{H}' \right)^2 \right] - \frac{\varepsilon_0}{2}\left(\vec{E}' \right)^2, \quad (16)$$

$\hat{I}$ is unit tensor, and the symbol «$\otimes$» denotes operation of dyadic product.

The ponderomotive moment $\vec{N}$ has the form [3, 5, 14, 25]:

$$\vec{N}' = \vec{D}' \times \vec{E}' + \vec{B}' \times \vec{H}'. \quad (17)$$

For isotropic materials the relations (3) and (5) hold (i.e., when the vectors $\vec{D}' \parallel \vec{E}'$ and $\vec{B}' \parallel \vec{H}'$) and as a result we have $\vec{N}' = 0$.

We write the balance relations [2, 26] for the density of mass ρ', momentum $\rho' \vec{v}'$:

$$\frac{\partial \rho'}{\partial t'} + \vec{\nabla}' \cdot (\rho' \vec{v}') = 0, \quad \frac{\partial (\rho' \vec{v}')}{\partial t'} = \vec{\nabla}' \cdot \left(\hat{\sigma}' + \hat{P}' \right) - \frac{\partial \vec{G}'}{\partial t'}, \tag{18}$$

and also the definition of the strain tensor $\hat{e}'$ according to Cauchy:

$$\hat{e}' = 0,5 \left(\vec{\nabla}' \otimes \vec{u}' + \vec{u}' \otimes \vec{\nabla}' + \vec{\nabla}' \otimes \vec{u}' \cdot \vec{u}' \otimes \vec{\nabla}' \right). \tag{19}$$

In mechanics of deformable solids the velocity $\vec{v}'$ of center of mass is associated with the movement of the center of mass by the relation $\vec{v}' = \partial \vec{u}' / \partial t'$.

We write also Duhamel-Neumann law:

$$\hat{\sigma}' = \frac{E_p}{(1+\nu_p)(1-2\nu_p)} \left\{ (1-2\nu_p) \hat{e}' + \left[\nu_p e' - (1+\nu_p) \alpha_T (T' - T_0) \right] \hat{I} \right\} \tag{20}$$

and the heat conduction equation [2, 27]

$$\kappa \Delta' T' + Q' = \rho' C_e \frac{\partial T'}{\partial t'} + \frac{\alpha_T E_p}{1-2\nu_p} T' \frac{\partial e'}{\partial t'} = \rho' C_\sigma \frac{\partial T'}{\partial t'} + \alpha_T T' \frac{\partial \sigma'}{\partial t'}. \tag{21}$$

Here $e' = \sum_{\alpha=1}^{3} e'_{\alpha\alpha} = \vec{\nabla}' \cdot \vec{u}'$ is the first invariant of the strain tensor; T_0 is the initial temperature; C_e and C_σ are the specific heat capacities at constant strains and stresses, which are coupled by relation [27]:

$$C_e(T') = C_\sigma - \frac{2\alpha_T E_p}{(1-2\nu_p)} \frac{T'}{\rho'}. \tag{22}$$

In the experimental determination of heat capacities C_σ, the heat capacity C_e is measured and is calculated theoretically using the formula (22).

Here are the laws of change of the values of electromagnetothermomechanics at transition from moving (primed) to fixed (unprimed) reference systems.

The laws of transformation of EMF vectors and material relations of electrodynamics:

$$\vec{H}' = \vec{H} - \varepsilon_0 \varepsilon \vec{v} \times \vec{E}, \quad \vec{B}' = B, \quad \vec{M}' = \vec{M} + \varepsilon_0 \varepsilon \vec{v} \times \vec{E},$$
$$\vec{E}' = \vec{E} + \vec{v} \times \vec{B}, \; \vec{D}' = \vec{D}, \; \vec{D}' = \vec{D}, \; \vec{E}'_s = \vec{E}_s,$$
$$\vec{j}' = \vec{j} - \rho q \vec{v}, \; \rho' q' = \rho q, \; \vec{j} = \rho q \vec{v} + \lambda \left(\vec{E} + \vec{E}_s + \vec{v} \times \vec{B} \right),$$
$$\vec{B} = \mu_0 \left[\vec{H} + \vec{M} \left(\vec{H} - \varepsilon_0 \varepsilon \vec{v} \times \vec{E} \right) \right], \; \vec{D} = \varepsilon_0 \varepsilon \left(\vec{E} + \vec{v} \times \vec{B} \right). \tag{23}$$

The laws of transformation of energetic and power factors of EMF action:

$$W' = W - \vec{v} \cdot \vec{G}, \quad \vec{G}' = G, \quad w' = w + (1 + 1/\varepsilon) \vec{v} \cdot \vec{G}, \quad Q' = Q - \vec{F}_0 \cdot \vec{v} + \Delta Q,$$
$$\vec{F}'_0 = \vec{F}_0, \quad \vec{\mathcal{F}}' = \vec{\mathcal{F}}' + \vec{\nabla} \left(\vec{v} \cdot \vec{G} \right) / \varepsilon, \quad \vec{N}' = N, \quad P'_{\alpha\alpha} = P_{\alpha\alpha} + v_\alpha G_\alpha + \vec{v} \cdot \vec{G} / \varepsilon,$$
$$P'_{\alpha\beta} = P_{\alpha\beta} + v_\beta G_\alpha + D_\alpha \left(\vec{v} \times \vec{B} \right)_\beta \quad (\alpha \neq \beta), \tag{24}$$

where

$$\Delta Q = 0{,}5\mu_0 \left[\vec{H} \cdot \left(\vec{v} \cdot \vec{\nabla} \right) \vec{M} - \vec{M} \cdot \left(\vec{v} \cdot \vec{\nabla} \right) \vec{H} \right]. \tag{25}$$

The laws of transformation of thermomechanical values:

$$T' = T\,, \quad \rho' = \rho\,, \quad \vec{u}' = \vec{u}\,, \quad \vec{v}' = d\vec{u}/dt\,, \quad \hat{e}' = \hat{e}\,, \quad \hat{\sigma}' = \hat{\sigma}\,. \tag{26}$$

The balance relations of mass and momentum:

$$\frac{d\rho}{dt} + \vec{\nabla} \cdot \left(\rho \frac{d\vec{u}}{dt} \right) = 0\,, \quad \frac{d}{dt}\left(\rho \frac{d\vec{u}}{dt} \right) = \vec{\nabla} \cdot \left(\hat{\sigma} + \hat{P} + \Delta \hat{P} \right) - \frac{d\vec{G}}{dt}\,. \tag{27}$$

The heat conduction equation:

$$\kappa \Delta T + Q - \vec{F}_0 \cdot \vec{v} + \Delta Q = \rho C_e \frac{dT}{dt} + \frac{\alpha_t E_p}{1 - 2\nu_p} T \frac{de}{dt} = \rho C_\sigma \frac{dT}{dt} + \alpha_t T \frac{d\sigma}{dt}\,, \tag{28}$$

and the expression for ΔQ is given by (25).

To the system of equations of electromagnetothermomechanics (3)-(28) it is necessary to add the corresponding electrodynamic, thermal and mechanical initial, boundary and contact conditions that correspond to a particular case.

2. The initial system of equations of electromagnetothermomechanics in dimensionless form

We confine ourselves to the situation when the frequency of external EMF does not get in the neighborhood of the natural frequencies of the body, i. e. we do not consider the resonances phenomena. Using the method of similarity and dimensional theory [28, 29], we introduce the relevant characteristic values of the considered magnitudes and reduce a system of equations of electromagnetothermomechanics (3)-(28) to a dimensionless form. These considerations apply both equations in the moving K' and not moving K frames of reference. However, all equations and relations in dimensionless values we write in the fixed laboratory frame of reference, since it holds all cases studied.

All dimensionless values (except time $\tau = \omega t$) will be denoted by an asterisk «*» below. Let l be a characteristic size of the body. Then for the dimensionless radius-vector and operators $\vec{\nabla}$ and Δ we have:

$$\vec{r}_* = \vec{r}/l\,, \quad \vec{\nabla}_* = l\vec{\nabla}\,, \quad \Delta_* = l^2 \Delta\,. \tag{29}$$

Dimensionless functions Z_* considered (scalar, vector and tensor) we introduce as:

$$Z = Z_0 + Z_1 Z_*\,, \tag{30}$$

where $Z = \left\{ \rho, q, W, Q, T, \vec{H}, \vec{B}, \vec{M}, \vec{E}, \vec{j}, \vec{F}, \vec{G}, \vec{N}, \vec{v}, \vec{u}, \hat{P}, \hat{\sigma}, \hat{e} \right\}$, Z_0 is the original (in the absence of an external magnetic field H_1) value of Z.

In this case, the non-zero initial values the density of the body ρ_0 and the absolute temperature T_0. All values Z_1 we enumerate on the amplitude of the magnetic field H_1 as follows:

$$B_1 = \mu_0 \mu H_1, \quad M_1 = \chi H_1, \quad w_1 = \mu_0 H_1^2, \quad W_1 = P_1 = \sigma_1 = \mu_0 \mu H_1^2,$$

$$E_1 = \frac{H_1}{\lambda l}, \quad D_1 = \frac{\varepsilon_0 \varepsilon H_1}{\lambda l}, \quad j_1 = \frac{H_1}{l}, \quad q_1 = \frac{\varepsilon_0 \varepsilon H_1}{\rho_0 \lambda l^2}, \quad Q_1 = \frac{H_1^2}{\lambda l^2}, \quad F_1 = \frac{\mu_0 \mu H_1^2}{l},$$

$$T_1 = \frac{H_1^2}{\kappa\lambda}, \; G_1 = \frac{\varepsilon \mu H_1^2}{\lambda l C^2}, \; e_1 = \frac{Co}{2}, \; u_1 = \frac{lCo}{2}, \; \rho_1 = \frac{\rho_0 Co}{2}, \; v_1 = \frac{\omega l Co}{2}. \tag{31}$$

Here μ and $\chi = \mu - 1$ are the initial relative magnetic permeability and susceptibility of the medium; $Co = \left(\frac{v_A}{c_2}\right)^2 = \frac{2(1+\nu_\nu)}{E_p}\mu_0 \mu H_1^2 = \left(\frac{H_1}{H_{Co}}\right)^2$ is the second Kaulinha parameter, $v_A = \sqrt{\frac{\mu\mu_0}{\rho_0}} H_1$ is Alfvena velocity; $c_2 = \sqrt{\frac{E_p}{2\rho_0(1+\nu_p)}}$ is the of propagation speed of transverse waves; $H_{Co} = \sqrt{\frac{E_p}{2(1+\nu_\nu)\mu\mu_0}} = \sqrt{\frac{\rho_0}{\mu\mu_0}} c_2$.

Taking into account the choice of definitions (29)-(31), we rewrite the basic relations of electromagnetothermomechanics (2)-(28) in dimensionless values. Then we obtain the following.

General relations of electrodynamics:

$$\vec{\nabla}_* \times \vec{H}_* = \varepsilon_\varepsilon \frac{\partial \vec{E}_*}{\partial \tau} + j_*, \quad \vec{\nabla}_* \times \vec{E}_* = -2\gamma^2 \frac{\partial \vec{B}_*}{\partial \tau}, \quad \vec{\nabla} \cdot \vec{B}_* = 0,$$

$$\vec{\nabla}_* \cdot \vec{E}_* = \left(1 + \frac{Co}{2}\rho_*\right) q_*, \quad \varepsilon_\varepsilon \frac{\partial}{\partial \tau}\left[\left(1 + \frac{Co}{2}\rho_*\right) q_*\right] + \vec{\nabla}_* \cdot \vec{j}_* = 0,$$

$$\frac{d}{d\tau} = \frac{\partial}{\partial \tau} + \frac{Co}{2}\vec{v}_* \cdot \vec{\nabla}_*, \quad \vec{v}_* = \frac{d\vec{u}_*}{d\tau}, \quad \vec{H}'_* = \vec{H}_* - \frac{\varepsilon_0}{2} Co \vec{v}_* \times \vec{D}_*,$$

$$\mu \vec{B}_* = \vec{H}_* + \chi \vec{M}_*, \quad \vec{D}_* = \vec{E}_* + \gamma^2 Co \vec{v}_* \times \vec{B}_*, \quad \vec{E}'_* = \vec{E}_* + \gamma^2 Co \vec{v}_* \times \vec{B}_*,$$

$$\vec{j}_* = \frac{\varepsilon_\varepsilon}{2} Co \left(1 + \frac{Co}{2}\rho_*\right) q_* \vec{v}_* + \vec{E}_* + \gamma^2 Co \vec{v}_* \times \vec{B}_* + \vec{\nabla}_* \left(\frac{H_1}{H_{\varphi T}} T_* + \frac{H_1}{H_{\varphi\sigma}} \sigma_* - \varepsilon_{\varphi q} q_*\right). \tag{32}$$

The factors of EMF action:

$$\vec{G}_* = \vec{D}_* \times \vec{B}_*, \quad W_* = \left(\vec{H}_* \cdot \vec{B}_* + \varepsilon_\mu \vec{E}_*^2 - \varepsilon_\varepsilon Co \vec{v}_* \cdot \vec{G}_*\right)/2,$$

$$w_* = \frac{1}{2}\left(\chi^2 \vec{M}_*^2 - \vec{H}_*^2\right) - \frac{\mu \varepsilon_\mu}{2\varepsilon}\left[\vec{E}_*^2 - (\varepsilon + 1) Co \vec{v}_* \cdot \vec{G}_*\right],$$

$$Q_* = \vec{j}_* \cdot \vec{E}_* + \frac{\chi\gamma^2}{\mu}\left(\vec{H}_* \cdot \frac{d\vec{M}_*}{d\tau} - \vec{M}_* \cdot \frac{d\vec{H}_*}{d\tau}\right) - \gamma^2 Co \vec{v}_* \cdot \vec{F}_{0*},$$

$$\vec{F}_{0*}=\vec{j}_*\times\vec{B}_*+\varepsilon_\mu\left(1+Co\,\rho_*/2\right)q_*\,\vec{E}_*\,,\qquad \vec{\mathscr{F}}_*=\chi\left[\left(\vec{M}_*\cdot\vec{\nabla}_*\right)\vec{B}_*+\vec{M}_*\times\left(\vec{\nabla}_*\times\vec{B}_*\right)\right]+$$

$$+\left(1-1/\varepsilon\right)\varepsilon_\mu\left[\left(\vec{E}_*\cdot\vec{\nabla}_*\right)+\vec{E}_*\times\left(\vec{\nabla}_*\times\vec{E}_*\right)\right]+\varepsilon_\varepsilon\,Co\vec{\nabla}_*\left(\vec{v}_*\cdot\vec{G}_*\right)\Big/(2\varepsilon)\,,$$

$$P_{*\alpha\alpha}=H_{*\alpha}B_{*\alpha}+\frac{1}{2\mu}\left(\chi^2\,\vec{M}_*^2-\vec{H}_*^2\right)+\varepsilon_\mu\left(E_{*\alpha}^2-\frac{1}{2\varepsilon}\vec{E}_*^2\right)+0{,}5\varepsilon_\varepsilon\,Co\left(v_{*\alpha}G_{*\alpha}+\vec{v}_*\cdot\frac{\vec{G}_*}{\varepsilon}\right),$$

$$P_{*\alpha\beta}=H_{*\alpha}B_{*\alpha}+\varepsilon_\mu E_{*\alpha}E_{*\beta}+0{,}5\varepsilon_\varepsilon Co\,v_{*\beta}\,G_{*\alpha}\quad(\alpha\neq\beta).\qquad(33)$$

Balance equations:

$$\vec{F}_*=\vec{\nabla}_*\cdot\hat{P}_*-\varepsilon_\varepsilon\frac{\partial\vec{G}_*}{\partial\tau},\quad \frac{d\rho_*}{d\tau}+\vec{\nabla}_*\left[\left(1+\frac{Co\,\rho_*}{2}\right)\vec{v}_*\right]=0\,,$$

$$\varepsilon_\rho\frac{d}{d\tau}\left[\left(1+\frac{Co\,\rho_*}{2}\right)\vec{v}_*\right]=\vec{\nabla}_*\cdot\left(\hat{\sigma}_*+\hat{P}_*\right)-\varepsilon_\varepsilon\frac{d\vec{G}_*}{dt}\,.\qquad(34)$$

Relations of thermomechanics:

$$\hat{e}_*=\frac{1}{2}\left(\vec{\nabla}_*\otimes\vec{u}_*+\vec{u}_*\otimes\vec{\nabla}_*+\frac{Co}{2}\vec{\nabla}_*\otimes\vec{u}_*\cdot\vec{u}_*\otimes\vec{\nabla}_*\right),$$

$$\hat{\sigma}_*=\hat{e}_*+\left(\frac{\nu_p}{1-2\nu_p}e_*-\varepsilon_{T\sigma}\,T_*\right)\hat{I}\,,$$

$$\Delta_*T_*+Q_*=Pe_e\left(1+\frac{Co}{2}\rho_*\right)\frac{dT_*}{d\tau}+\frac{1+\nu_p}{1-2\nu_p}\cdot2\gamma^2\varepsilon_T\left(1+e_{Te}\,T_*\right)\frac{d\,e_*}{d\tau}=$$

$$=Pe_\sigma\left(1+\frac{Co}{2}\rho_*\right)\frac{dT_*}{d\tau}+2\gamma^2\varepsilon_T\left(1+\varepsilon_{Te}T_*\right)\frac{d\sigma_*}{d\tau}\,,$$

$$C_e\left(T\right)=C_\sigma-\Delta C\frac{1+\varepsilon_{Te}T_*}{1+Co\,\rho_*/2},\quad \Delta C\equiv\frac{2\alpha_t^2E_p\,T_0}{\left(1-2\nu_p\right)\rho_0}\,.\qquad(35)$$

In formulas (32), (33) the following notations are introduced: $\varepsilon_\varepsilon=\omega/\omega_\varepsilon$, $\varepsilon_\mu=\left(l_\mu/l\right)^2$, $\varepsilon_{\varphi q}=\left(\dfrac{l_\varepsilon}{l}\right)^2$, where $\omega_\varepsilon=\dfrac{\lambda}{\varepsilon\varepsilon_0}$, $l_\mu=\dfrac{1}{\lambda}\sqrt{\dfrac{\varepsilon\varepsilon_0}{\mu\mu_0}}$, $l_\varepsilon=\sqrt{\dfrac{\varepsilon\varepsilon_0\varphi_0}{3q_n\,\rho_0}}$ is characteristic frequency and sizes of bias currents; $Rm=2\gamma^2=\lambda\mu\mu_0\omega l^2$ is magnetic Reynolds number; $\gamma=1/\delta$, δ is the relative depth of penetration of EMF; $\varepsilon_\rho=0{,}5\left(\omega l/c_2\right)^2$ is a parameter that characterizes the contribution of inertial forces compared to electromagnetic forces; $Pe_e=\omega l^2/a_e$ and $Pe_\sigma=\omega l^2/a_\sigma$ are the Pecle numbers corresponding to the thermal diffusivity at constant strains $a_e=\kappa/\left(\rho_0\,C_e\right)$ and stresses (pressure) $a_\sigma=\kappa/\left(\rho_0\,C_\sigma\right)$,

$\varepsilon_T = \alpha_t T_0$, $\varepsilon_{T\sigma} = \dfrac{\alpha_t E_p}{(1-2\nu_p)\kappa\lambda\mu\mu_0}$, $\varepsilon_{Te} = (H_1/H_{Te})^2$ and also the characteristic magnetic field intensities: $H_{Te} = \sqrt{\kappa\lambda T_0}$, $H_{\varphi T} = \dfrac{\kappa}{\alpha_t \varphi_0}$, $H_{\varphi\sigma} = \dfrac{3E_p}{(1-2\nu_p)\lambda\mu\mu_0\varphi_0}$.

We estimate the criteria when the material of the body is technically pure iron. As characteristics of material we take the following [30-32]: $\varepsilon = 1$, $\mu = 250$, $\lambda = 1{,}02\cdot10^7\ \mathrm{A/(V\cdot m)}$, $\rho_0 = 7{,}87\cdot10^3\ \mathrm{кg/m^3}$, $E_p = 2{,}1\cdot10^{11}\ \mathrm{N/m^2}$, $\nu_p = 0{,}28$, $\alpha_T = 1{,}22\cdot10^{-5}\ \mathrm{K^{-1}}$, $\kappa = 74\ \mathrm{W/(m\cdot K)}$, $C_\sigma = 460\ \mathrm{J/(кg\cdot K)}$, $T_0 = 300\mathrm{K}$, $\varphi_0 = 1{,}4086\ \mathrm{V}$, $q_n = 2{,}42\cdot10^6\ \mathrm{C/кg}$, $\varepsilon_0 = 8{,}854\cdot10^{-12}\ \mathrm{C^2/(N\cdot m^2)}$, $\mu_0 = 4\pi\cdot10^{-7}\ \mathrm{N/A^2}$, $c = 3\cdot10^8\ \mathrm{m/s}$.

We find first the values that do not depend on the frequency ν and amplitude H_1 of the external magnetic field and also the characteristic size of the body l. We have: $\omega_\varepsilon = 1{,}15\cdot10^{18}\ \mathrm{Hz}, l_\mu = 1{,}65\cdot10^{-11}\ \mathrm{m}, l_\varepsilon = 1{,}48\cdot10^{-11}\ \mathrm{m}, c_2 = 3228{,}5\ \mathrm{m/s^2}, \Delta C = 5{,}42\ \mathrm{J/(кg\cdot K)}$, $\Delta C/C_\sigma = 1{,}18\cdot10^{-2}$, $C_e = 454{,}6\ \mathrm{J/(кg\cdot K)}$, $a_e = 2{,}07\cdot10^{-5}\ \mathrm{m^2/s}$, $a_\sigma = 2{,}04\cdot10^{-5}\ \mathrm{m^2/s}$, $\varepsilon_T = 3{,}66\cdot10^{-3}$, $\varepsilon_{T\sigma} = 24{,}57$, $H_{Te} = 4{,}76\cdot10^5\ \mathrm{A/m}$, $H_{\varphi T} = 4{,}31\cdot10^6\ \mathrm{A/m}$, $H_{\varphi\sigma} = 3{,}17\cdot10^8\ \mathrm{A/m}$, $H_{Co} = 1{,}62\cdot10^7\ \mathrm{A/m}$.

Note that the characteristic frequency ω_ε reaches the values of X-rays, and typical dimensions l_μ and l_ε — the atomic sizes.

Up to 1,18 % we will ignore the difference between the specific heat C_e and C_σ, and hence between the coefficients of thermal diffusivity a_e and a_σ. In this case, as the value C we take the value found experimentally by using the thermodynamic approach [27, 30, and others]. Sometimes $C = (C_\sigma + C_e)/2 \cong 457{,}3\ \mathrm{J/(кg\cdot K)}$ and, respectively, $a = (a_\sigma + a_e)/2 \cong 2{,}055\ \mathrm{m^2/s}$ are taken as such.

Large value of parameter $\varepsilon_{T\sigma}$ indicates that in the Duhamel-Neumann law (20) thermal stresses are much higher than the power ones [1, 27], so the law is not changed.

You can ignore the parameter ε_T that is in the heat conduction equation (35). But there it enters into combinations with parameter γ^2 as a product $\gamma^2\varepsilon_T$ (and it needs more research).

The maximum temperature at which a ferromagnet loses its magnetic properties is Curie temperature (T_k = 1043 K for iron). The intensity of magnetic field corresponding to given temperature is $H_{1k} = \sqrt{\kappa\lambda(T_k - T_0)} = 7{,}49\cdot10^5\ \mathrm{A/m}$. So, for the frequency range, the body size and magnetic field intensity under consideration we obtain: $\varepsilon_\varepsilon \approx 10^{-16} \div 10^{-6}$, $\varepsilon_\mu \approx 10^{-14} \div 10^{-22}$, $\varepsilon_{\varphi q} \approx 10^{-13} \div 10^{-21}$, $Co \approx 0 \div 2{,}14\cdot10^{-3}$, $H_1/H_{\varphi\sigma} \approx 0 \div 2{,}36\cdot10^{-3}$.

Thus, in the dimensionless system of equations of electrothermoelasticity (32)-(35), we reject the parameters (which means that the terms at which they are also) in magnitude less than 1 % (< 0,01). Then the values ε_ε, ε_μ, $\varepsilon_{\varphi q}$, Co and $H_1/H_{\varphi\sigma}$, will be neglected always, especially where there is their product (e. g. $\varepsilon_\varepsilon Co$ or $\varepsilon_\mu Co$). This means that for electroconducting bodies the currents of displacement can be neglected, that is a term $\varepsilon_0\varepsilon\partial\vec{E}/\partial t$ of the first equation of system (7) and change of electric charge in time $\partial(\rho q)/\partial t$ in the last equation of the system. Although the electric charge we leave in the fourth equation of system (7), but in all other formulas we will reject it. Difference between the substantial and local derivatives disappears, that is $d/dt = \partial/\partial t$. The density of mass distribution can be considered constant, i. e. $\rho = \rho_0$. In the expression $\vec{E}_s$ we reject the dependence on σ and q (we have a small dependence on temperature). In factors of EMF action (formulas (24) and (25)) and the balance equations (18) we neglect the terms associated with the electric field intensity and pulse density of EMF. Relation (19) between the strain tensor and displacement vector we take to be linear.

Table 1 shows the values of seven magnitudes depending on the magnitudes ν , l and H_1 (see the third column), the range of values of which is very wide: from those where they can be neglected to such when they are much higher than others. In the fourth column are given the values of parameters ν , l and H_1 (or rather their combinations) at which the proposed seven variables can be neglected. In the last column as an example a commercial frequency is considered.

The Table shows, the relatively small frequency range ν and intensity of the external field H_1, as well as body size l, for which you can ignore the values considered (at the so called low-frequency induction heating [1]). As for the general case we will consider them in the original equation.

Let us make one more remark relatively the electroconductive nonferromagnetic material for which there is no Curie temperature. Then, as the basis we need to take other characteristic. The most logical temperature is the melting point temperature T_{melt} above which the body is in the liquid phase (melt). For the technically pure iron it is $T_{melt} = 1835\,\text{K}$.

Table 1

№	The value	The value range of given magnitude	The values of parameters ν, *l* and H_1 for which the magnitude considered is less than 1%	The previous column is for commercial freguency ν = 50 Hz
1	γ^2	$5{,}03\cdot10^{-3}\div1{,}01\cdot10^{16}$	$\nu l^2 < 10^{-6}\ \text{m}^2/\text{s}$	$l < 1{,}42\cdot10^{-4}$ m
2	$\gamma^2\varepsilon_T$	$1{,}84\cdot10^{-5}\div3{,}68\cdot10^{13}$	$\nu l^2 < 2{,}72\cdot10^{-4}\ \text{m}^2/\text{s}$	$l < 2{,}33\cdot10^{-3}$ m
3	$\gamma^2 Co$	$0\div2{,}16\cdot10^{13}$	$\nu l^2 H_1^2 < 2{,}60\cdot10^8\ \text{A}^2/\text{s}$	$lH_1 < 2{,}28\cdot10^4$ A
4	ε_ρ	$4{,}73\cdot10^{-11}\div1{,}89\cdot10^{18}$	$\nu l < 72{,}7\ \text{m/s}$	$l < 1{,}45$ m
5	P_e	$1{,}53\cdot10^{-2}\div3{,}06\cdot10^{12}$	$\nu l^2 < 32{,}72\ \text{m}^2/\text{s}$	$l < 0{,}65$ m
6	$H_1/H_{\varphi T}$	$0\div0{,}17$	$H_1 < 4{,}31\cdot10^2$ A/m	
7	ε_{Te}	$0\div2{,}48$	$H_1 < 7{,}49\cdot10^4$ A/m	

Then $H_{1\,melt}=\sqrt{\kappa\lambda\left(T_m-T_0\right)}=1{,}08\cdot 10^6$ A/m and $Co=4{,}43\cdot 10^{-3}$. Although the parameter *Co* slightly increased, but the overall picture has not changed. So the criteria listed in the Table will not be neglected.

3. A simplified mathematical model of magnetothermomechanics

If you disregard the above criteria, the original system of equations of electromagnetothermomechanics (3)-(28) will be of simplified form.
Output relations of electrodynamics:

$$\vec{\nabla}\times\vec{H}=\vec{j}\,,\quad \vec{\nabla}\times\vec{E}=-\partial\vec{B}/\partial t\,,\quad \vec{\nabla}\cdot\vec{B}=0\,,\quad \vec{\nabla}\cdot\vec{E}=\rho q/\left(\varepsilon_0\,\varepsilon\right),\quad \vec{\nabla}\cdot\vec{j}=0\,,$$
$$\vec{j}=\lambda\left(\vec{E}+\vec{v}\times\vec{B}+\alpha_T\varphi_0\vec{\nabla}T\right),\quad \vec{B}=\mu_0\,\mu\left(H\right)\vec{H}\,. \tag{36}$$

Factors of EMF action:

$$W=0{,}5\vec{H}\cdot\vec{B}\,,\quad w=0{,}5\mu_0\left(\vec{M}^2-\vec{H}^2\right),$$
$$Q=\vec{j}\cdot\vec{E}+\frac{\mu_0}{2}\left(\vec{H}\cdot\frac{\partial\vec{M}}{\partial t}-\vec{M}\cdot\frac{\partial\vec{H}}{\partial t}\right)-\vec{v}\cdot\vec{F}_0,\quad \vec{F}_0=\vec{j}\times\vec{B}\,,$$
$$\vec{\mathcal{F}}=\left(\vec{M}\cdot\vec{\nabla}\right)\vec{B}+\vec{M}\times\left(\vec{\nabla}\times\vec{B}\right),\quad \vec{F}=\vec{\nabla}\cdot\hat{P}\,,\quad \hat{P}=\vec{H}\otimes\vec{B}+w\hat{I}\,. \tag{37}$$

Balance equations:

$$\rho=\rho_0\,,\quad \rho_0\frac{\partial\vec{v}}{\partial t}=\vec{\nabla}\cdot\left(\hat{\sigma}+\hat{P}\right),\quad \vec{v}=\frac{\partial\vec{u}}{\partial t}\,. \tag{38}$$

Thermomechanics relations:

$$\hat{e}=0{,}5\left(\vec{\nabla}\otimes\vec{u}+\vec{u}\otimes\vec{\nabla}\right),$$
$$\hat{\sigma}=\frac{E_p}{\left(1+\nu_p\right)\left(1-2\nu_p\right)}\left\{\left(1-2\nu_p\right)\hat{e}+\left[\nu_p e-\left(1+\nu_p\right)\alpha_t\left(T-T_0\right)\right]\hat{I}\right\},$$
$$\Delta T+\frac{Q}{\kappa}=\frac{1}{a}\frac{\partial T}{\partial t}+\frac{\alpha_T\,E_p}{\left(1-2\nu_p\right)\kappa}T\frac{\partial e}{\partial t}=\frac{1}{a}\frac{\partial T}{\partial t}+\frac{\alpha_T}{\kappa}T\frac{\partial\sigma}{\partial t}\,. \tag{39}$$

Conclusions. Thus, the original system of equations of electromagnetothermomechanics (3)-(28) is reduced to system of equations of magnetothermomechanics (36)-(39), which is a working one to find EMF, temperature distribution and mechanical stresses in ferromagnetic conductive bodies under the action of an external time-harmonic EMF specified by (1).

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References

[1] Термоупругость электропроводных тел / *Я. С. Подстригач, Я. И. Бурак, А. Р. Гачкевич, Л. В. Чернявская*. — Киев: Наук. думка, 1997. — 248 с.

[2] *Бурак Я. Й., Галапац Б. П., Гнідець Б. М.* Фізико-механічні процеси в електропровідних тілах. — Киев: Наук. думка, 1978. — 232 с.

[3] *Кудрявцев Б. А, Партон В. З.* Магнитотермоупругость // Механика деформ. твердого тела. Итоги науки и техники. — 4.– Москва: ВИНИТИ, 1981. — С. 3-59.

[4] *Подстригач Я. С., Бурак Я. И., Кондрат В. Ф.* Магнитотермоупругость электропроводных тел. — Киев: Наук. думка, 1982. — 296 с.

[5] *Поливанов К. М.* Электродинамика движущихся тел. — Москва: Энергоиздат, 1982. — 192 с.

[6] *Ландау Л. Д., Лифшиц Е. М.* Теория поля. — Москва: Наука, 1973. — 504 с.

[7] *Ландау Л. Д., Лифшиц Е. М.* Электродинамика сплошных сред. — Москва: Наука, 1982. — 624 с.

[8] *Седов Л. И.* Механика сплошной среды: в 2 т. — Москва: Наука, 1976. — Т. 1. — 526 с.

[9] *Седов Л. И., Цыпкин А. Г.* Основы макроскопических теорий гравитации и электромагнетизма. — Москва: Наука, 1989. — 272 с.

[10] *Толмачев В. В., Головин А. М., Потапов В. С.* Термодинамика и электродинамика сплошной среды. — Москва: Изд-во МГУ, 1988. — 232 с.

[11] *Богородский А. Ф.* Всемирное тяготение. — Киев: Наук. думка, 1971. — 352 с.

[12] *Джексон Д. Ж.* Классическая электродинамика. — Москва: Мир, 1965. — 702 с.

[13] *Пановский В., Филипс М.* Классическая электродинамика. — Москва: Физматгиз, 1963. — 432 с.

[14] *Тамм И. Е.* Основы теории электричества. — Москва: Наука, 1976. — 616 с.

[15] *Терлецкий Я. П., Рыбаков Ю. П.* Электродинамика. — Москва: Высш. школа, 1990. — 352 с.

[16] *Толмен Р.* Относительность, термодинамика и космология. — Москва: Наука, 1974. — 520 с.

[17] *Солодяк М. Т., Івасько Р. О.* Рівняння Мінковського в нелінійних намагнічуваних і поляризованих рухомих середовищах // Мат. методи і фіз.-мех. поля. — 1999. — Т. 42, № 1. — С. 50-54.

[18] *Колесников П. М.* Введение в нелинейную электродинамику. — Минск: Наука и техника, 1971. — 384 с.

[19] Моделювання механічних систем за дії електромагнітних навантажень / *О. Гачкевич, М. Солодяк, Р. Івасько, С. Шимура* // Машинознавство. — 2001. — № 12(54). — С. 8-14.

[20] Моделювання дії електромагнетного поля на термомеханічну поведінку деформівних твердих тіл / *О. Р. Гачкевич, М. Т. Солодяк, Р. О. Івасько , В. Я. Бойчук* // Фіз.-хім. механіка матеріалів. — 2009. — Т. 45, № 2. — С. 43-54.

[21] *Солодяк М. Т.* Термодифузійні процеси в електропровідних тілах у зовнішньому періодичному в часі магнетному полі // Фіз.-хім. механіка матеріалів. — 2000. — № 5. — С. 91-98.

[22] *Гроот С. Р., де Сатторп Л. Г.* Электродинамика. — Москва: Наука, 1982. — 560 с.

[23] *Hutter K.* Ware propagation and attenuation in paramagnetic and soft ferromagnetic materials. Part 2 // Ibid. — 1976. — Vol. 14.— P. 883-894.

[24] *Максвелл Дж. К.* Избранные сочинения по теории ЭМП. — Москва: Гостехиздат, 1954. — 688 с.

[25] *Поливанов К. М.* Вращающий момент в электромагнитных системах и тензор механических напряжений // Изв. вузов. Электромеханика. — 1975. — Т. 1. — С. 3-10.

[26] *Де Гроот С. П., Мазур П.* Неравновесная термодинамика. — Москва: Мир, 1964. — 456 с.

[27] *Коваленко А. Д.* Основы термоупругости. – Киев: Наук. думка, 1970. — 307 с.

[28] *Седов Л. И.* Методы подобия и размерности в механике. — Москва: Наука, 1977. — 438 с.

[29] *Филиппов А. Т.* Многоликий солитон. — Москва: Наука, 1986. — 224 с.

[30] Таблицы физических величин: справ.; под ред. *И. К. Кикоина*. — Москва: Атомиздат, 1976. — 1008 с.

[31] *Кей Дж., Леби Т.* Таблицы физических и химических постоянных / Под ред. *К. П. Яковлева.* — Москва: Физматгиз, 1962. — 247 с.

[32] *Лившиц Б. Г., Крапошин В. С., Линецкий Я. Л.* Физические свойства металлов и сплавов. — Москва: Металлургия, 1980. — 320 с.

До впливу рухомості середовища на зумовлені магнітним полем термомеханічні процеси у феромагнітних тілах

Олександр Гачкевич, Михайло Солодяк, Ростислав Терлецький, Любов Гаєвська

Викладено варіант кількісного опису взаємозв'язку електромагнітотермомеханічних процесів у електропровідних феромагнітних тілах у зовнішньому періодичному за часом магнітному полі. Використовуючи методи теорії подібності та розмірностей, систему рівнянь електродинаміки та термопружності зведено до безрозмірної форми. Проведено кількісний аналіз характерних безрозмірних критеріїв для технічно чистого заліза. Нехтується ефектами, для яких характерні безрозмірні параметри набагато менші порівняно з одиницею. Записано спрощену систему рівнянь моделі для знаходження магнітного поля, температури, переміщень і механічних напружень у феромагнітних тілах. Знайдено параметри, за яких потрібно враховувати рухомість середовища.

О влиянии подвижности среды на обусловленные магнитным полем термомеханические процессы в ферромагнитных телах

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Изложен вариант количественного описания взаимосвязи электромагнитотермомеханических процессов в электропроводных ферромагнитных телах во внешнем периодическом во времени магнитном поле. Используя методы теории подобия и размерностей, систему уравнений электродинамики и термоупругости сведено к безразмерной форме. Проведен количественный анализ характерных безразмерных критериев для технически чистого железа. Пренебрегается эффектами, для которых характерные безразмерные параметры намного меньше, по сравнению с единицей. Записана упрощенная система уравнений модели для нахождения магнитного поля, температуры, перемещений и механических напряжений в электропроводных ферромагнитных телах. Найдены параметры, при которых нужно учитывать подвижность среды.

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