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Investigation of magneto-thermomechanical processes in ferrite bodies in external time-periodic magnetic field with bias

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Using the methods of similarity and dimensional theory, a system of equations of electrothermomechanics of electricity-nonconductive bodies that are in external time-periodic magnetic field with bias is reduced to a dimensionless form. A quantative analysis of characteristic dimensionless criteria for 200 HH ferrite is carried out. The effects for which characteristic dimensionless parameters are less than 1 % are neglected. A simplified system of equations of the model to find the magnetic field, temperature, displacements and mechanical stresses in ferrite bodies is obtained. A quasistatic approximation of equations of electrodynamics, known in literature, is substantiated.

Keywords: electromagnetic field, magneto-thermomechanics, ferrites, methods of similarity and dimensional theory, quasi-static approximation.

Introduction. This article is a continuation of the work [1], including the formulation of the problem, notations, definitions and basic relations. We mention only the main differences in ferrite bodies and external influences.

We assume that the body is exposed to an external electromagnetic field (EMF), given by the vector of magnetic field on the surface of the body

$$\vec{H}^{(ext.)}(r,t)=\vec{H}_0(\vec{r}_1)+\vec{H}_1(\vec{r}_1)\cos(\omega t), \quad (1)$$

where $\vec{H}_0(\vec{r}_1)$ and $\vec{H}_1(\vec{r}_1)$ are the value of constant field and amplitude of time-harmonic magnetic field; $\vec{r}_1$ is the radius-vector of the surface point; ν is frequency, t is time.

1. Mathematical model of magnetothermomechanics in its native frame of reference

We assume that in the body there are no extraneous charges and currents. We confine ourselves to the study of nonconducting (ferrite) bodies with small coefficient of conductivity

$\left(\lambda = \left(10^{-3} - 10^{-14}\right) \text{А}/\left(\text{В} \cdot \text{м}\right)\right)$, which is negligible compared to the electroconducting bodies and therefore we do not take into account the conduction currents and the resulting free electric charge (i.e. $q' = 0,\ j' = 0$) [2, 3].

In electrical devices and equipment the systems of coils, through which the time-variable (harmonic circularly polarized) currents pass, perpendicular to the external magnetic field, are commonly used. Under such conditions there occurs ferromagnetic resonance — intense absorption of electromagnetic energy by the body at certain frequencies of magnetic field. Therefore, in the systems such as magnetic circuits magnetostatic waves arise. In this case, the vector of magnetic field strength and the vector of magnetization as well realize oscillations, which are described by equation of gyromagnetic oscillations in the Hilbert form or in the Landau-Lifshitz form [4-6]

$$\frac{\partial \vec{M}'}{\partial t'} = -\gamma_s \vec{M}' \times \vec{H}' + \frac{\alpha_s}{M'} \vec{M}' \times \frac{\partial \vec{M}'}{\partial t'} = -\frac{\gamma_s}{1+\alpha_s^2} \left[\vec{M}' \times \vec{H}' + \frac{\alpha_s}{M'} \vec{M}' \times \left(\vec{M}' \times \vec{H}' \right) \right], \quad (2)$$

where $M' = \left| \vec{M}' \right|$ is the module of vector $\vec{M}'$; $\gamma_s = g_0 \mu_0 e_e / (2 m_e)$ is magnetomechanical (gyromagnetic) relation; g_0 is Lande factor; e_e and m_e, respectively, the charge and mass of an electron; α_s is a loss parameter.

From equation (2), as a result, the relation follows [4, 5]

$$\frac{\partial \left(\vec{M}' \right)^2}{\partial t'} = 0 \qquad \text{or} \qquad M' = M + \frac{2\varepsilon_0 \varepsilon}{M} \vec{v} \cdot \vec{E} \times \vec{M}\ , \quad (3)$$

which means that the length of the magnetization vector at its random changes (oscillations) remains fixed. If the material is magnetized to saturation, then $M = M_S$, where M_S is the saturation magnetization. Note that in the derivation of relation (3) the formula (23) from work [1] is used.

Maxwell's equations (7) [1] can be reduced to a system of equations for function $\vec{H}$ or $\vec{E}$. For this purpose we act by the rotor operation on the first equation of system (7) using the second equation of the system and the formula of vector analysis $\vec{\nabla} \times \left(\vec{\nabla} \times \vec{a} \right) = \vec{\nabla} \left(\vec{\nabla} \cdot \vec{a} \right) - \Delta \vec{a}$, we obtain the following equation for the magnetic field strength

$$\Delta' \vec{H}' - \vec{\nabla}' \left(\vec{\nabla}' \cdot \vec{H}' \right) = \frac{\varepsilon}{c^2} \frac{\partial^2}{\partial t'^2} \left(\vec{H}' + \vec{M}' \right), \quad (4)$$

A similar equation we obtain for the electric field strength

$$\Delta' \vec{H}' - \vec{\nabla}' \left(\vec{\nabla}' \cdot \vec{H}' \right) = \frac{\varepsilon}{c^2} \frac{\partial^2}{\partial t'^2} \left(\vec{H}' + \vec{M}' \right). \quad (5)$$

2. The initial system of equations of magnetothermomechanics in dimensionless form

We pass to the dimensionless form of writing the equations of magnetothermomechanics, that is the proposed functions Z we present in the form (30) [1]. In this case the non-zero initial values have besides the density of the body ρ_0 and absolute temperature T_0 still further characteristics of stable magnetic field: $\vec{H}_0$ — the strength of given magnetic field; $\vec{M}_S$ — the vector of saturation magnetization (characteristics of the material), and the magnitude of the magnetic field induction as well

$$\vec{B}_0 = \mu_0\left(\vec{H}_0 + \vec{M}_S\right). \tag{6}$$

Thus, we assume that the vectors $\vec{M}_S$ and $\vec{H}_0$ are parallel, i.e. [4, 6]

$$\vec{M}_S \times \vec{H}_0 \equiv 0 \quad \text{or} \quad \vec{M}_S = \chi \vec{H}_0, \tag{7}$$

where $\chi \equiv M_S / H_0$ is magnetic susceptibility.

Different from zero in the initial state will be also the Maxwell tension tensor

$$\hat{P}_0 = \vec{H}_0 \otimes \vec{B}_0 + w_0 \hat{I}, \qquad w_0 = \mu_0\left(M_S^2 - H_0^2\right). \tag{8}$$

All other values, that are discussed in this paper, are absent in the initial state.

Since $\lambda = 0$ and, normalizing, it can not be used, then the appropriate values we propose as follows

$$\begin{aligned}
&B_1 = \mu_0 H_1, \quad M_1 = H_1, \quad E_1 = \mu_0 \omega l H_1, \quad D_1 = \varepsilon\omega l H_1/c^2, \quad Q_1 = \mu_0 \omega H_1^2, \\
&T_1 = \mu_0 \omega l^2 H_1^2/\kappa, \quad F_1 = \mu_0 H_1^2/l, \quad P_1 = \sigma_1 = N_1 = \mu_0 H_1^2, \\
&G_1 = \varepsilon\mu_0 \omega l^2 H_1^2/c^2, \quad e_1 = Co/2, \quad u_1 = l\,Co/2, \quad v_1 = \omega l\,Co/2.
\end{aligned} \tag{9}$$

Here Co is the Kaulinha second parameter , defined in [1].

Given the choice of definitions [4], we write the system of equations of magnetothermomechanics [1] in dimensionless values.

General relations of electrodynamics

$$\begin{aligned}
&\vec{\nabla}'_* \times \vec{H}'_* = \varepsilon_\omega \frac{\partial \vec{E}'_*}{\partial \tau}, \quad \vec{\nabla}'_* \cdot \vec{B}_* = 0, \quad \vec{\nabla}'_* \cdot \vec{E}'_* = -\frac{\partial \vec{B}'_*}{\partial \tau}, \quad \vec{\nabla}'_* \cdot \vec{E}'_* = 0, \\
&\vec{B}'_* = \vec{H}'_* + \vec{M}'_*, \quad \vec{D}'_* = \vec{E}'_*, \quad \vec{E}'_* = \vec{E}_* + Co\,\vec{v}_* \times \vec{B}_*/2, \\
&\vec{H}'_* = \vec{H}'_* - \varepsilon_\omega Co\,\vec{v}_* \times \vec{E}_*/2, \quad \vec{M}'_* = \vec{M}_* + \varepsilon_\omega Co\,\vec{v}_* \times \vec{E}_*/2.
\end{aligned} \tag{10}$$

Equations of gyromagnetic oscillations

$$\omega \frac{d\vec{M}'_*}{d\tau} = \gamma_S \vec{H}_0 \times \left(\vec{M}'_* - \chi \vec{H}'_*\right) - \gamma_S H_1 \vec{M}'_* \times \vec{H}'_* + \frac{\alpha_S \omega}{M'}\left(\chi \vec{H}_0 + H_1 \vec{M}'_*\right) \times \frac{d\vec{M}'_*}{d\tau}. \tag{11}$$

Equations for the magnetic (4) and electric (5) fields strength

$$\Delta'_* \vec{H}'_* - \vec{\nabla}'_*\left(\vec{\nabla}'_* \cdot \vec{H}'_*\right) = \varepsilon_\omega \frac{d^2}{d\tau^2}\left(\vec{H}'_* + \vec{M}'_*\right), \quad \Delta'_* \vec{E}'_* = \varepsilon_\omega \frac{d^2 \vec{E}'_*}{d\tau^2} + \vec{\nabla}'_* \times \frac{d\vec{M}'_*}{d\tau}. \qquad (12)$$

Heat release

$$Q_* = \frac{1}{2}\left[\frac{\vec{H}_0}{H_1} \cdot \frac{d}{d\tau}\left(\vec{M}'_* - \chi \vec{H}'_*\right) + \vec{H}'_* \cdot \frac{d\vec{M}'_*}{d\tau} - \vec{M}'_* \cdot \frac{d\vec{H}'_*}{d\tau}\right]. \qquad (13)$$

Definition of ponderomotive force

$$\vec{F}' = \left(1 - 1/\varepsilon\right)\varepsilon_\omega\left[\left(\vec{E}'_* \cdot \vec{\nabla}'_*\right)\vec{E}'_* + \vec{E}' \times \left(\vec{\nabla}'_* \cdot \vec{E}'_*\right)\right] +$$

$$+\left[\left(\frac{\chi}{H_1}\vec{H}_0 + \vec{M}'_*\right) \cdot \vec{\nabla}'_*\right]\vec{B}'_* + \left(\frac{\chi}{H_1}\vec{H}_0 + \vec{M}_*\right) \times \left(\vec{\nabla}'_* \times \vec{B}'_*\right) + \frac{\varepsilon_\omega}{2} Co\, \vec{\nabla}'_*\left(\vec{v}_0 \cdot \vec{G}_0\right). \qquad (14)$$

Relation between ponderomotive force and Maxwell tension tensor

$$\vec{F}'_* = \vec{\nabla}'_* \cdot \hat{P}'_* - \varepsilon_\omega \frac{d\vec{G}_*}{d\tau}, \quad \vec{G}'_* = \vec{E}'_* \times \left(\frac{\vec{B}_0}{\mu_0 H_1} + \vec{B}'_*\right). \qquad (15)$$

Expressions for Maxwell tension tensor

$$\hat{P}'_* = \varepsilon_\omega \vec{E}'_* \otimes \vec{E}'_* + \left(\frac{\vec{H}_0}{H_1} + \vec{H}'_*\right) \otimes \left(\frac{\vec{B}_0}{\mu_0 H_1} + \vec{B}'_*\right) + w'_* \hat{I},$$

$$w'_* = \frac{1}{2}\left[\left(\frac{\chi}{H_1}\vec{H}_0 + \vec{M}'_*\right)^2 - \left(\frac{\vec{H}_0}{H_1} + \vec{M}'_*\right)\right] - \frac{\varepsilon_\omega}{2\varepsilon}\left(E'_*\right)^2. \qquad (16)$$

Ponderomotive force moment

$$\vec{N}'_* = \left(\frac{\chi}{H_1}\vec{H}_0 + \vec{M}'_*\right) \times \left(\frac{\vec{H}_0}{H_1} + \vec{H}'_*\right). \qquad (17)$$

Balance mass relation (the second formula (34) [1]) will not change and for momentum (the third formula (34) [1]) it is necessary to change dimensionless parameter ε_ε for ε_μ. The expressions for total time derivative and definition of deformation tensor according to Cauchy will not change as well. The Duhamel-Neumann law will take the form

$$\hat{\sigma}_* = \hat{e}_* + \left(\frac{\nu_p}{1 - 2\nu_p} e_* - \varepsilon_\kappa T_*\right)\hat{I}. \qquad (18)$$

Heat conduction equation

$$\Delta_* T_* + Q'_* = P e_e \left(1 + \frac{C_0}{2}\rho_*\right)\frac{dT_*}{d\tau} + \frac{1+\nu_p}{1-2\nu_p}\varepsilon_T\left(1+\varepsilon_{Te}T_*\right)\frac{de_*}{d\tau} =$$

$$= Pe_\sigma\left(1 + \frac{C_0}{2}\rho_*\right)\frac{dT_*}{d\tau} + \varepsilon_T\left(1+\varepsilon_{Te}T_*\right)\frac{d\sigma_*}{d\tau}. \qquad (19)$$

The formula for difference between the specific heat capacities at constant pressure and displacements will not change.

In formulas (10)-(19) the following notations are introduced: $\varepsilon_\omega = \varepsilon\left(\omega l/c\right)^2$ is the parameter characterizing displacement currents for nonelectroconducting bodies; $\varepsilon_\kappa = \omega l^2 / D_\kappa$ is the coefficient characterizing contribution of temperature stresses in comparison with the mechanical ones; $D_\kappa = \left(1-2\nu_p\right)\kappa / \left(\alpha_T E_p\right)$ is the heat diffusion coefficient; $D_T = \kappa T_0 / \left(\mu_0 H_1^2\right)$, $\varepsilon_T = \alpha_T T_0$, $\varepsilon_{Te} = \omega l^2 / D_T$ are characteristic coefficients. The rest dimensionless values are such as in [1].

A 200HH ferrite film (nickel-zinc spinel) of thickness $l = 2\cdot 10^{-5}$ (initial temperature $T_0 = 300\,\text{K}$) is considered, the characteristics of which are as follows [7-10]

$$\rho_0 = 4{,}8\cdot 10^3\ \text{kg/m}^3,\quad E_p = 1{,}2\cdot 10^{11}\ \text{N/m}^2,\quad \nu_P = 0{,}27,\quad \alpha_T = 1{,}1\cdot 10^{-5}\ \text{K}^{-1},$$

$$\kappa = 4{,}19\ \text{W}/\left(\text{m}\cdot\text{K}\right),\ C_\sigma = 712\ \text{J}/\left(\text{kg}\cdot\text{K}\right),\ e_e = 1{,}6\cdot 10^{-19}\,\text{C},\ m_e = 9{,}1\cdot 10^{-31}\ \text{kg},$$

$$\varepsilon_0 = 8{,}854\cdot 10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right),\quad \mu_0 = 4\pi\cdot 10^{-7}\,\text{N/A}^2,\quad c = 3\cdot 10^8\ \text{m/s},\quad \varepsilon = 10.$$

It is also assumed that

$$\alpha_S = 0{,}01,\quad g_0 = 2,\quad H_0 = 2\cdot 10^5\ \text{A/m},\quad H_1 = 0{,}22\cdot 10^3\ \text{A/m},$$

$$M_S = 1{,}65\cdot 10^6\ \text{A/m},\quad B_0 = 2{,}32\,\text{T},\quad \chi = 8{,}25.$$

Then

$$\gamma_S = 2{,}21\cdot 10^5\ \text{A/C},\quad \omega_H = \gamma_S H_0 = 4{,}42\cdot 10^{10}\,\text{Hz},\quad H_{Co} = 1{,}94\cdot 10^8\ \text{A/m},$$

$$c_2 = 3137{,}3\ \text{m/s},\quad D_\kappa = 1{,}46\cdot 10^{-6}\,\text{m}^2/\text{s},\quad \varepsilon_T = 3{,}3\cdot 10^{-3},$$

$$\Delta C = 3{,}95\ \text{J}\left(\text{kg}\cdot\text{K}\right),\quad \Delta C / C_\sigma = 5{,}54\cdot 10^{-3},\quad C_e = 708{,}05\ \text{J}/\left(\text{kg}\cdot\text{K}\right),$$

$$a_\sigma = 1{,}226\cdot 10^{-6}\,\text{m}^2/\text{s},\quad a_e = 1{,}232\cdot 10^{-6}\,\text{m}^2/\text{s}.$$

The rest values are found for resonance frequency ω_H, given thickness l and magnetic field strength H_1. In addition we have obtained

$$Co = 1{,}29\cdot 10^{-12},\quad \varepsilon_\omega = 8{,}69\cdot 10^{-6},\quad \varepsilon_{Te} = 7{,}37\cdot 10^{-4},\quad \varepsilon_\rho = 3{,}97\cdot 10^4,$$

$$\varepsilon_\kappa = 1{,}21\cdot 10^7,\quad D_T = 2{,}4\cdot 10^4\,\text{m}^2/\text{s},\quad Pe_\sigma = 1{,}442\cdot 10^7,\quad Pe_e = 1{,}435\cdot 10^7.$$

Thus, as in [1], in dimensionless system of equations (10)-(19) we will drop the parameters less than 1 % in comparison with a unit. This means that such parameters as Co, ε_ω and ε_T and ε_{Te} we will neglect. That is, in this case we will neglect the effect of electric field on factors of EMF action, and hence on the thermoelastic state of ferrite films, as well as EMF pulse density. Moreover, these equations for the electric field intensity (the third and fourth equation in system (10)) can be left. As in [1] we do not consider bias currents in the equations of electrodynamics and the impact of mobility of the environment on the thermoelastic processes, and besides we assume that $\rho = \rho_0$, the definition of the Cauchy strain we take linear. Then substantial difference between substantive and local derivatives disappears. In the equation of heat conduction the heat release due to deformation is neglected ($\varepsilon_T \approx 0$), that is influence of mechanical processes on the thermal ones and difference between the specific heat capacities at constant pressure and deformation as well.

It is necessary to consider the parameter option ε_ρ: the contribution of inertia forces in thermoelastic state is essential as compared with the forces of electromagnetic origin. The parameter ε_κ characterizing the effect of temperature on the mechanical state, as well as the Pecle number ($Pe_\sigma \approx Pe_e = Pe$), which describes the dynamics of temperature elevation in the heat conduction equation, are essential.

3. Mathematical model of magnetothermomechanics in the laboratory system

We write the simplified input system of dimensional equations of magnetothermomechanics in the case if you can ignore the above criteria.

Input relations of electrodynamics

$$\vec{\nabla} \times \vec{H} = 0, \quad \vec{\nabla} \cdot \vec{B} = 0, \quad \frac{\partial \vec{M}}{\partial t} = -\gamma_S \vec{M} \times \vec{H} + \frac{\alpha_S}{M} \vec{M} \times \frac{\partial \vec{M}}{\partial t},$$

$$\vec{\nabla} \cdot \vec{E} = -\mu_0 \partial \vec{M} / \partial t, \quad \vec{\nabla} \cdot \vec{E} = 0. \tag{20}$$

Factors of magnetic field action

$$Q = \frac{\mu_0}{2}\left(\vec{H} \cdot \frac{\partial \vec{M}}{\partial t} - \vec{M} \cdot \frac{\partial \vec{H}}{\partial t} \right), \quad F = \left(\vec{M} \cdot \vec{\nabla} \right) \vec{B} + \vec{M} \times \left(\vec{\nabla} \times \vec{B} \right), \quad \vec{N} = \vec{M} \times \vec{H},$$

$$\vec{F} = \vec{\nabla} \cdot \hat{T}, \quad \hat{T} = \vec{H} \otimes \vec{B} + w\hat{I}, \quad w = 0{,}5\mu_0 \left(\vec{M}^2 - \vec{H}^2 \right). \tag{21}$$

Balance relations

$$\rho = \rho_0, \quad \frac{d}{dt} = \frac{\partial}{\partial t}, \quad \rho_0 \frac{\partial^2 \vec{u}}{\partial t^2} = \vec{\nabla} \cdot \left(\hat{\sigma} + \hat{T} \right), \quad \hat{e} = \frac{1}{2}\left(\vec{\nabla} \otimes \vec{u} + \vec{u} \otimes \vec{\nabla} \right). \tag{22}$$

Duhamel-Neumann law

$$\hat{\sigma} = \frac{E_p}{\left(1 + \nu_p\right)\left(1 - 2\nu_p\right)} \left\{ \left(1 - 2\nu_p\right)\hat{e} + \left[\nu_p e - \left(1 + \nu_p\right)\alpha_T \left(T - T_0\right) \right] \hat{I} \right\}. \tag{23}$$

The heat conduction equation

$$\Delta T + \frac{Q}{\kappa} = \frac{1}{a}\frac{\partial T}{\partial t} . \qquad (24)$$

Conclusions. The input system of equations of magnetothermomechanics is reduced to simplified equations of electrodynamics (20), the factors of magnetic field action (21), and thermoelasticity (22)-(24).

Note that although the electric field has virtually no effect on the thermoelastic state of ferrite bodies it can be determined from the last two equations of system (20). The first two equations of (20) will be a quasi-static approximation of electrodynamics known in literature [4-6, 11, 12].

At the same time, such a simplified system of equations of magnetothermomechanics (20)-(24) allons to adopt such a scheme of its solution, which consists of three phases:

- at the first phase in the ferrite body from of the first three equations of (20) we find the magnetic field;
- from the system of equations (21) we find the factors of magnetic fields action (heat release, ponderomotive force and moment of force, and Maxwell tension tensor);
- at the third stage from the system of equations (22)-(24) we find the temperature fields, displacements and stresses that occur in the ferrite bodies at the given external action [1].

Remarks. In electrical engineering stronger magnetic fields up to $H_{кр.} \approx 10^7$ A/m are used. But even for them the Kaulinha parameter is very small ($Co \approx 2{,}65 \cdot 10^{-3} < 10^{-2}$) and it is always negligible, as well as the processes associated with it. This means that, unlike in [1] the influence of the environment mobility on thermomechanical behavior of ferrite bodies can always be neglected.

These studies can be carried out not only for thin films, but also for ordinary bodies when considering such phenomena as ultrasonic heating of dielectrics. The upper limit of the ultrasound frequencies is $\omega \approx 10^{13}$ Hz and, respectively, as the characteristic size of the body we can take $l = 1$. Then, $\varepsilon_\omega \approx 10^{10}$, and hence it cannot be neglected, that is, it is necessary to consider the bias currents and contribution of electric fields in the factors of EMF action. The product $\varepsilon_\omega Co \approx 10^7$, and thus it must be considered as well. The rest of the parameters ε_ρ, Pe, ε_κ and ε_{Te} will only increase their value, but they, as well as the processes for which they are responsible, were already taken into account.

We write again the input dimensional system of equations of magnetothermoelasticity for this case. Here only those equations that are not in the system (20)-(24) are neglected.

General relations of electrodynamics

$$\vec{\nabla}\times\vec{H}=\varepsilon_0\,\varepsilon\partial\vec{E}/\partial t\,,\quad \vec{H}'=\vec{H}-\varepsilon_0\varepsilon\vec{v}\times\vec{E}\,,\quad \vec{M}'=\vec{M}+\varepsilon_0\,\varepsilon\vec{v}\times\vec{E}\,,$$

$$\frac{\partial\vec{M}'}{\partial t}=-\gamma_S\vec{M}'\times\vec{H}'+\frac{\alpha_S}{M'}\times\frac{\partial\vec{M}'}{\partial t'}=-\frac{\gamma_S}{1+\alpha_S^2}\left[\vec{M}'\times\vec{H}'+\frac{\alpha_S}{M'}\vec{M}'\times\left(\vec{M}'\times\vec{H}'\right)\right].\quad (25)$$

Factors of EMF action

$$Q=\frac{\mu_0}{2}\left(\vec{H}'\cdot\frac{\partial\vec{M}'}{\partial t}-\vec{M}'\cdot\frac{\partial\vec{H}'}{\partial t}\right),$$

$$\vec{F}=\left(\vec{M}'\cdot\nabla\right)\vec{B}+\vec{M}'\times\left(\vec{\nabla}\times\vec{B}\right)+\varepsilon_0\left(\varepsilon-1\right)\left[\left(\vec{E}\cdot\vec{\nabla}\right)\vec{E}+\vec{E}\times\left(\vec{\nabla}\times\vec{E}\right)\right],$$

$$\vec{F}=\vec{\nabla}\cdot\hat{P}-\partial\vec{G}/\partial t\,,\quad \vec{G}=\varepsilon_0\,\varepsilon\vec{E}\times\vec{B}\,,\quad \vec{N}=\vec{B}\times\vec{H}'=\mu_0\vec{M}'\times\vec{H}'\,,$$

$$\hat{P}=\varepsilon_0\,\varepsilon\vec{E}\otimes\vec{E}+\vec{H}'\otimes\vec{B}+w\cdot\hat{I}\,,\quad w=0{,}5\left\{\mu_0\left[\left(\vec{M}'\right)^2-\left(\vec{H}'\right)^2\right]-\varepsilon_0\vec{E}^2\right\}.\quad (26)$$

Thus, at high frequencies and large sizes one must use the system of equations of electrodynamics (25)-(26) and thermoelasticity (20)-(24).

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Дослідження магнітотермомеханічних процесів у феритових тілах в зовнішньому періодичному за часом магнітному полі з підмагнічуванням

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Використовуючи методи теорії подібності та розмірностей, систему рівнянь електротермомеханіки неелектропровідних феритових тіл, що знаходяться в зовнішньому періодичному за часом магнітному полі з підмагнічуванням, зведено до безрозмірної форми. Проведено кількісний аналіз характерних безрозмірних критеріїв для фериту 200 НН. Нехтується ефектами, для яких характерні безрозмірні параметри менші за один відсоток. Записано спрощену систему рівнянь моделі для знаходження магнітного поля, температури, переміщень і механічних напружень у феритових тілах. Обґрунтоване відоме в літературі квазістатичне наближення рівнянь електродинаміки.

Исследование магнитотермомеханических процессов в ферритовых телах по внешнем периодическом по времени магнитном поле с подмагничиванием

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Используя методы теории подобия и размерностей, система уравнений электротермомеханики неэлектропроводных ферритовых тел, находящихся во внешнем периодическом по времени магнитном поле с подмагничиванием, приведена в безразмерной форме. Проведен количественный анализ характерных безразмерных критериев для феррита 200НН. Пренебрегается эффектами, для которых характерные безразмерные параметры меньше одного процента. Записано упрощенную систему уравнений модели для нахождения магнитного поля, температуры, перемещений и механических напряжений в ферритовых телах. Обосновано известное в литературе квазистатическое приближение уравнений электродинамики.

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