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Effect of inclination of tank walls on liquid sloshing by the example of conic and cylindrical tanks with free surfaced liquid

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Effect of inclination of tank walls on generation of surface waves in tanks is under investigation. By the example of waves in conic and cylindrical tanks generated by short time force impulse, we analyze specificity of redistribution of contribution of normal modes amplitudes on free surface elevation. It was shown that inclination of tank walls promotes excitation of nonlinear mechanisms, mostly caused by geometrical nonlinearities.

Keywords: tank with liquid, nonlinear oscillations, inclination of tank walls, geometrical nonlinearities.

Introduction. Problems of liquid sloshing in non-cylindrical tanks are much more complicated than in the case of reservoirs with vertical walls [1-3]. In the case of tanks with inclined walls, it is necessary to modify considerably the mathematical description of the problem, because for non-cylindrical tanks it contains considerable contradiction in physical and mathematical sense of the problem. First of all, the problem statement does not contain information about behavior of tank walls above undisturbed position of liquid free surface. However, in the case of nonlinear problem liquid should follow tank walls for certain height, defined by amplitudes of excitation of waves. It was shown that this property is a part of requirements of solvability conditions for the boundary value of the problem of liquid sloshing [3]. Further investigations showed that for solving the problem of oscillation of liquid in tanks of non-cylindrical shape it is necessary to use non-cylindrical parameterization of the domain, occupied with liquid. At the same time, new types of nonlinearities, predetermined by geometry, are manifested strongly for this type of problems. For example, the first normal mode in conic reservoir with right cone angle should be plain. However, in the case of nonlinear problem such normal mode violates conservation law for mass and demands compensation by vertical displacement of middle point of liquid free surface.

Objective of investigation of the present article is determination of influence of inclination of tank walls on formation of waves in tanks of cylindrical and conic shapes and analysis of distinctive properties of generation of waves, caused of geometric factors.

1. Mechanical object and mathematical model of the system

We consider absolutely rigid cylindrical tank and inversed conic tank with right cone angle. Tanks are filled with ideal incompressible homogeneous liquid. Initially system is at rest state. According to the Lagrange theorem its further motion will be vortex free. We consider that reservoir performs only horizontal motion. Motion is caused by force, applied to tank in horizontal direction, $F_x = \left(M_l + M_t\right)k$, applied only during time t_0 (so force has character of rectangular impulse), where M_l and M_t are masses of liquid and the tank correspondingly, k is the force factor, which is chosen so that liquid oscillations hit nonlinear range of amplitudes of oscillations.

Mathematical model was constructed on the basis of variational algorithm [3], designed for tanks of revolution and aimed at investigation of transient processes of combined motion of the system «tank – liquid». It is known that liquid motion is completely defined by motion of tank and motion of a free surface of liquid. Therefore, system of amplitudes of excitation of normal modes of oscillations of a liquid free surface a_i and amplitude parameters of translational motion of tank ε_i forms complete and independent (minimal) system of parameters, whose number coincides with the number of degrees of freedom of the system

Finally mathematical model of combined motion of the system «liquid – tank» has the following form

$$\sum_{n=1}^{N} p_{rn}\left(a_k,t\right)\ddot{a}_n + \sum_{n=N+1}^{N+3} p_{rn}\left(a_k,t\right)\ddot{\varepsilon}_{n-N} = q_n\left(a_k,\dot{a}_k,t\right), \qquad r = \overline{1,N+3}\,.$$

Here N is the number of normal modes of oscillations of a liquid free surface. Accepted in the model, p_{rn} and q_n are determined by algebraic forms from the first to third order with coefficients, determined as quadratures from normal modes of oscillations of a free surface of liquid (coordinate functions). For construction of coordinate functions we used method of auxiliary domain, which in contrast to the classical method takes into account realization of nonflowing condition above level of undisturbed free surface of liquid. Results of determination of relative error of fulfillment of nonflowing condition (ratio of violation of nonflowing condition to maximal elevation on a free surface of liquid) for cylindrical tank and for inversed cone testify that error of realization of nonflowing condition on tank walls below liquid free surface does not exceed 10^{-6}, and above liquid free surface till height 0,2 of radius of a free surface it does not exceed 10^{-3}, which is completely acceptable for applied investigations. It is important that requirement of fulfillment of these conditions with high precision is connected with realization of solvability conditions of the Newman problem for the Laplace equation, by which the problem of determination of normal modes of oscillations of a free surface is described.

2. Results of numerical experiments

We use this model for determination of elevation of a liquid free surface on tank wall for the same problem of force disturbance of conic and cylindrical tanks. For numerical examples we make use of the following parameters of the system and its loading: $H = R$, where H is height of liquid, $R = 1$ m, $M_t = 0{,}2M_l$, $t_0 = 1$ s, $k = 1$. So, we analyze system behavior on active stage of its motion and on interval of system motion by inertia.

The suggested approach makes it possible to determine elevation of a free surface of liquid at every point of tank cross-section, variation in time of every amplitude of oscillation of a free surface a_i, amplitude parameters of translational motion of tank ε_i and liquid force response on tank walls.

Fig. 1 shows variation in time of elevation of liquid on tank wall on time interval. Here and after solid line corresponds to conic tank, dashed line corresponds to cylindrical tank. For every graph we consider time interval, which corresponds approximately to 5 periods of normal oscillations, and exceeds considerably time of loading action. Time is given in seconds.

As it is seen from Fig. 1, elevation of a free surface of liquid for conic tank exceeds elevation of a free surface for cylindrical tank especially for liquid motion after removing external loading. On interval of transient mode of system motion this difference is weaker, but it is present also. If we analyze law of variation of these two curves, we can draw conclusion that for conic tank the presence of higher normal modes is considerable. To analyze nature of this effect let us show variation in time of the first antisymmetric a_2 (Fig. 2) and the first axisymmetric a_3 (Fig. 3) normal modes.

As it is seen from figures difference of amplitudes of the first antisymmetric normal modes is weak, but they differ in frequencies of variation. If we analyze variation in time of the first axisymmetric normal modes for these two shapes of tanks, we can conclude that this difference is strong both in variation of amplitudes and frequencies. It is necessary to note that in contrast to amplitudes of the first antisymmetric normal modes the first axisymmetric normal modes are excited only due to nonlinear mechanisms.

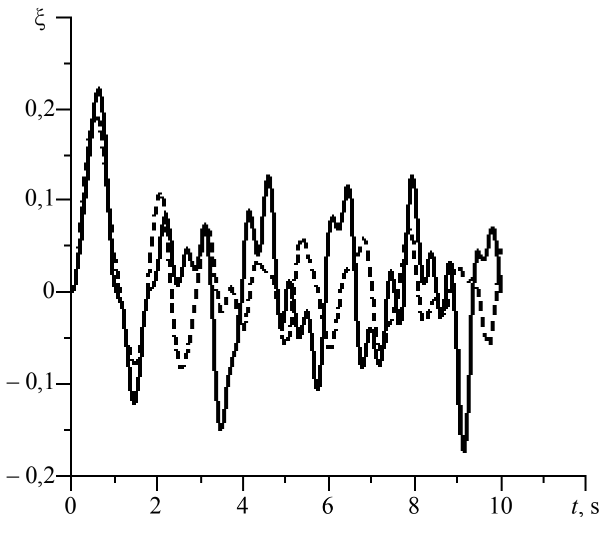

Fig. 1. Variation in time of elevation of liquid on tank wall

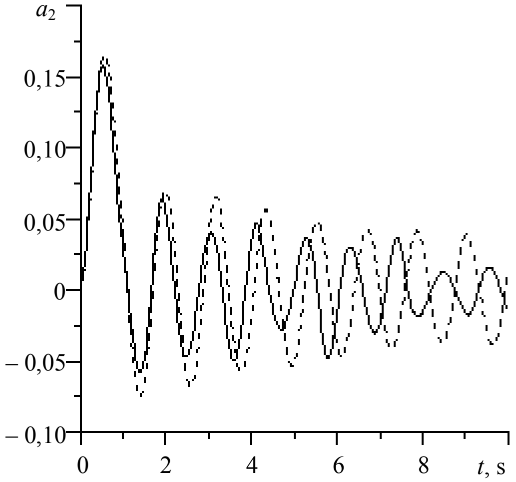

Fig. 2. Variation in time of amplitude of the first antisymmetric normal mode of oscillations of free surface of liquid

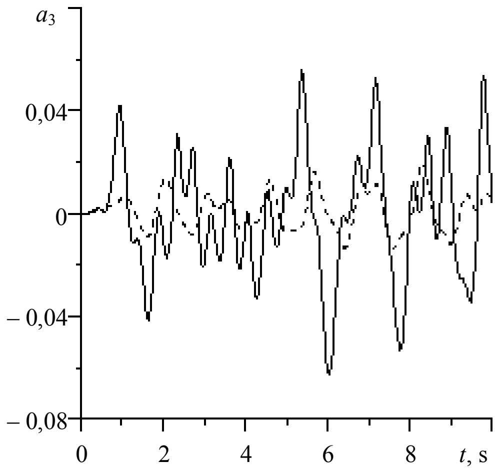

Fig. 3. Variation in time of amplitude of the first axisymmetric normal mode of oscillations of free surface of liquid

There are at least three reasons for increase of this difference. First, for conic reservoir (as for variant of non-cylindrical reservoir) potential energy of liquid is represented not only by quadratic terms relative to liquid elevation, but it contains terms of the third and fourth orders, which take into account inclination and curvature of tank walls in a vicinity of a free surface of liquid. Second, expansions of elevations of a free surface contain nonlinear term, which compensates variation of liquid volume due to absence of symmetry above and below undisturbed free surface of liquid. These terms depend again of inclination and curvature of tank walls in a vicinity of a free surface of liquid.

Third, increase of wave height is accompanied with two simultaneous effects, namely, wave increases both in vertical and horizontal direction due to inclination of tank wall and increase of wave causes lowering of middle point of a free surface for providing conservation law of liquid volume. First of all these effects promote excitation of axisymmetric normal modes and they are considerably caused by geometric nonlinearities of the system due to non-cylindrical shape of tank. Therefore, our study makes it possible to draw conclusion that nonlinear mechanisms manifest to greater extent in tank of non-cylindrical shape than in cylindrical tank.

Conclusion. We consider problem of modeling of liquid sloshing in conic and cylindrical tanks, caused by non-stationary loading. Analysis of generation of surface waves and contribution of separate normal modes into formation of liquid free surface motion make it possible to draw the following conclusions. Geometrical nonlinearities, caused by non-cylindrical shape of tank, mostly manifested by the presence of inclination of tank walls and their curvature in a vicinity of liquid free surface. This increases total manifestation of nonlinear effects in liquid sloshing. Moreover, mostly these effects are connected with axisymmetric normal modes of oscillations of liquid in tanks.

References

[1] *Нариманов Г. С., Докучаев Л. В., Луковский И. А.* Нелинейная динамика летательного аппарата с жидкостью. — Москва: Машиностроение, 1977. — 208 с.

[2] *Ibrahim R. A.* Liquid sloshing dynamics: theory and applications. — Cambridge: Cambridge University Press, 2005. — 950 p.

[3] *Limarchenko O. S.* Peculiarities of application of perturbation techniques in problems of nonlinear oscillations of liquid with a free surface in cavities of non-cylindrical shape // Укр. мат. журн. — 2007. — Т. 59, № 1. — С. 44-70.

Вплив нахилу стінок бака на коливання поверхні рідини на прикладі конічного та циліндричного баків з рідиною

Інна Кудзіновська

Досліджується вплив нахилу стінок бака на зародження поверхневих хвиль у баках. На прикладі хвиль у конічному та циліндричному баках, що виникають внаслідок дії короткотривалого силового імпульсу аналізується специфіка перерозподілу внеску форм коливань на збурення вільної поверхні. Показано, що нахил стінок бака сприяє збудженню нелінійних механізмів, які викликані переважно геометричними нелінійностями.

Влияние наклона стенок бака на колебания поверхности жидкости на примере конического и цилиндрического баков с жидкостью

Инна Кудзиновская

Исследуется влияние наклона стенок бака на зарождение поверхностных волн в баках. На примере волн в коническом и цилиндрическом баках, возникающих вследствие действия кратковременного силового импульса, анализируется специфика перераспределения вклада форм колебаний на возмущения свободной поверхности. Показано, что наклон стенок бака способствует возбуждению нелинейных механизмов, вызванных преимущественно геометрическими нелинейностями.

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