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Peculiarities of motion of the system reservoir – liquid on pendulum suspension under external harmonic force

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Angular motion of cylindrical reservoir on pendulum suspension, partially filled with ideal liquid, under external harmonic force is considered for different frequencies of external loading. It is shown that resonant frequencies of combined motion considerably differ from partial frequencies of liquid sloshing and oscillations of pendulum. Some peculiarities of the system behavior in different ranges of frequencies of disturbance are discussed.

Keywords: cylindrical tank, free surface liquid, angular motion of reservoir, nonlinear oscillations, sloshing.

Introduction. Problems of dynamics liquid in reservoir on pendulum suspension is one of classical problems of fluid – structure interaction. Investigation of sloshing liquid in reservoir, which performs angular motion, represents complicated problem due to awkwardness of problem statement. Especially complicated is the problem of combined motion of reservoir on pendulum suspension and liquid with a free surface. Analysis showed that due to combined character of motion of system components normal frequencies of this system differ from partial frequencies. Therefore, for confirmation of variation of system resonant properties we investigate system behavior in different frequency ranges.

1. Object of investigation

We consider motion of the system cylindrical reservoir – liquid on pendulum suspension under external harmonic loading. Initially system is at a rest state in equilibrium position. Liquid is supposed to be ideal, incompressible, homogeneous, reservoir walls are supposed to be absolutely rigid. For more complete description of the system real properties, we take into account viscous properties of liquid according to technique of generalize dissipation, suggested by G. N. Mikishev [1]. Suspension point is immovable. If we select reference frame with origin at suspension point, reservoir can perform only angular motion. We consider the Cauchy problem for this system, therefore, the system has transient stage of motion and later it tends to steady mode of motion. General scheme of the system is given in Fig. 1.

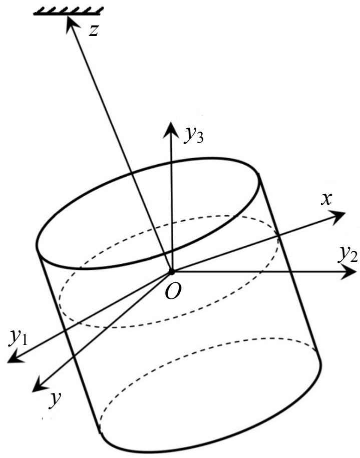

Fig. 1. General scheme of the system reservoir – liquid on pendulum suspension

2. Mathematical model

For construction of mathematical model we used the approach, described in publication [2]. Model was constructed on the basis of the Hamilton-Ostrogradskiy variational principle. According to this approach problem statement should be considered as aggregate of kinematical boundary conditions (constraints) and variational principle, which includes motion equations and dynamical boundary conditions. For description of motion of bounded liquid volume we shall introduce the following denotations: τ is the domain, occupied by liquid, S is the free surface of liquid, Σ is moistening surface of reservoir. Let us introduce potential of velocity as

$$\phi = \phi_0 + \dot{\vec{\varepsilon}} \cdot \vec{r} + \vec{\omega} \cdot \vec{\Omega}, \tag{1}$$

here ϕ is potential velocity of liquid, ϕ_0 is potential velocity of liquid wavy motion, $\vec{\varepsilon}$ is the vector of translational motion of the reservoir, $\vec{\omega}$ is the vector of angular velocity of the reservoir, $\vec{\Omega}$ is the Stokes-Zhukovskiy velocity potential. System motion occurs under the presence of the following kinematical constraints

$$\Delta\phi = 0, \quad \Delta\vec{\Omega} = 0 \quad \text{in} \quad \tau,$$

$$\left.\frac{\partial\phi}{\partial n}\right|_{\Sigma} = \dot{\vec{\varepsilon}} \cdot \vec{n} + \vec{\omega} \cdot \left.\frac{\partial\vec{\Omega}}{\partial n}\right|_{\Sigma}, \quad \left.\frac{\partial\phi}{\partial n}\right|_{S} = \dot{\vec{\varepsilon}} \cdot \vec{n} + \vec{\omega} \cdot \left.\frac{\partial\vec{\Omega}}{\partial n}\right|_{\Sigma} + \frac{\dfrac{\partial\xi}{\partial t}}{\sqrt{1 + (\vec{\nabla}\xi)^2}}, \tag{2}$$

It is noteworthy that due to selection of origin of reference frame at the point of pendulum suspension of the reservoir and since suspension point is immovable, translational motions in the considered problem are absent and it is possible to describe motion of the re dimension. Elements of these matrix and vector are determined as quadratures from normal modes of oscillations ψ_i and the Stokes-Zhukovskiy vector potential, which is determined analytically for cylindrical reservoir [3].

The system of equations (5) is linear relative to the second derivatives of unknowns; this enables transformation of the system to the Cauchy normal form and performing integration of it by time using the Runge-Kutta method.

2. System behavior depending on frequency of loading

We considered motion of the cylindrical reservoir of R=1 m on pendulum suspension with $l = R$ length. A part of the obtained results were compared qualitatively with laboratory equipment, therefore we used the Reynolds number $k_{\upsilon} = 46$ for analogy with laboratory sample on determination of viscosity factor according to the approach of publication [3].

Significant property of combined motion of the system reservoir – liquid consists in changing of its resonant properties. Partial frequencies for angular oscillations physical pendulum and the first (antisymmetric) normal mode of liquid oscillations are ω_1 = 2,55 and ω_2 = 4,14, correspondingly (frequencies are given in 1/sec). The corresponding normal frequencies for combined oscillations of the system become ω_1^* = 2,28 and ω_2^* = 7,19. There variation is considerable, especially for liquid. For confirmation of this changes in resonant properties of the system we investigated system behavior for frequencies of external loading in close vicinity of every of these system frequencies. So, we expect that for combined modes of system motion partial frequencies will not be resonant, however system will manifest resonant properties for loading with frequencies, which correspond to combined modes of motion. It is necessary to note that resonance manifests sharply for ω_1^*, while for frequency ω_2^* it is practically impossible to hit the resonant domain. Investigations showed this is caused by high fidelity of resonant curve in vicinity of this resonance. Let us show how these properties are realized practically for the accepted system reservoir – liquid.

For external loading on frequency ω_1^* = 2,28 it is practically impossible to get numerical results because of sharp manifestation of increase of amplitudes of the first normal mode of liquid and angular oscillations. So, this can be considered as confirmation of the presence of resonance on this frequency.

If we consider external loading with frequency lower than this resonant frequency ω = 2, system behavior shows tendency to steady mode of motion. Fig. 2 shows variation in time of the first axisymmetric normal mode of oscillations. It is know that amplitude of this mode can be considered as degree of manifestation of nonlinearity in the system and it is responsible for significant property of nonlinear surface waves, which consists in exceeding of wave crest height over depth of wave foot. Graph in Fig. 2 shows shift of values of amplitude in positive direction, which is just correspond to providing of this nonlinear properties. On the stage of transient mode of system motion nonlinearities are stronger that on motion of the system on the stage of motion by inertia. At the same time we see also decrease of amplitudes of oscillations caused by the presence of viscosity and considerable manifestation of modulation of oscillations. So, this mode of motion can be named as steady mode of oscillations only conventionally.

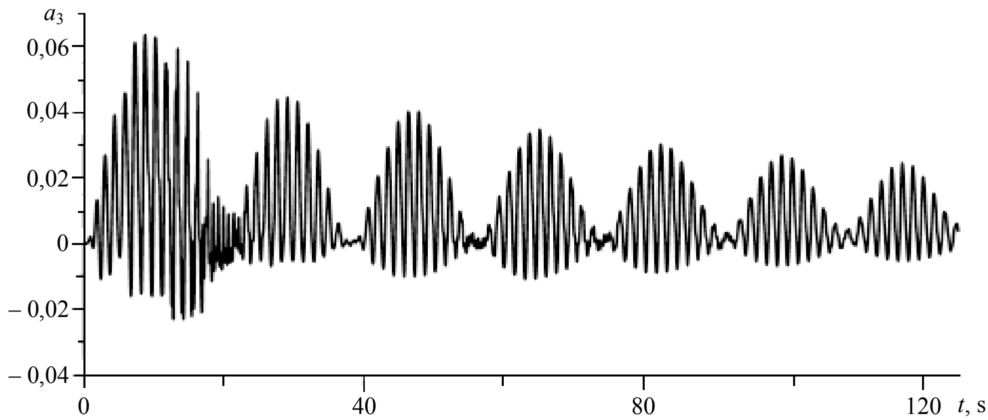

Fig. 2. Variation in time of amplitude the first axisymmetric normal mode

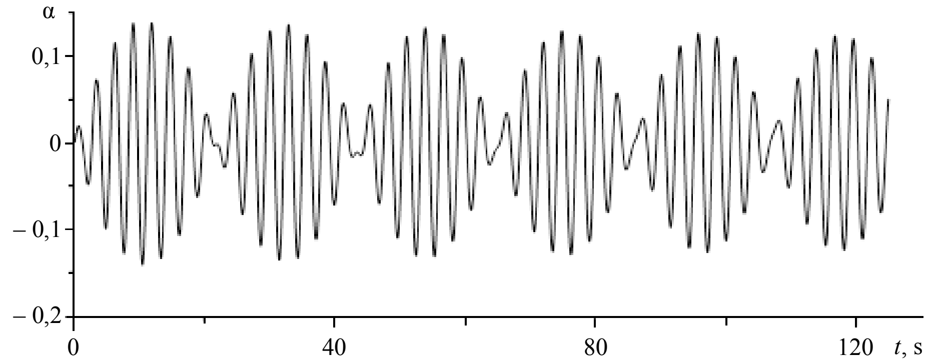

Fig. 3. Variation in time of amplitude of angle of inclination of pendulum

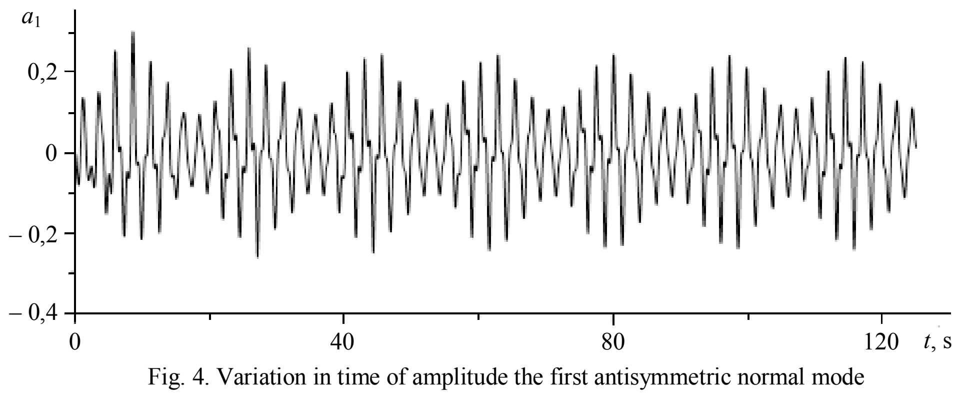

Fig. 4. Variation in time of amplitude the first antisymmetric normal mode

In Fig. 3 we show variation in time of angle of inclination of pendulum for frequency of external loading $\omega = 2{,}1$. It is significant to note that angle of inclination practically does not decrease in time, but law of its variations shows strong modulation.

If we investigate system behavior on the first partial frequency $\omega = 2{,}55$, we can see that now resonant properties manifest. Fig. 4 shows variation in time of amplitude the first antisymmetric normal mode. The presence of modulation and influence of high-frequency normal modes are seen in the graph. Fig. 5 shows variation in time of amplitude the first axisymmetric normal mode. Law of variation of amplitude of this mode differs considerably from Fig. 2, but systematic shift of graph in positive direction and modulation take place also.

If we investigate behavior of the system on the first partial frequency of liquid oscillation ω = 4,14, we see in variation of amplitude of the first antisymmetric normal mode (Fig. 6) that after short transient mode system passes into quasi-steady mode of motion. Analysis of variation on time of the first axisymmetric normal mode (Fig. 7) shows that nonlinear effects mostly present on transient stage of motion and steadily disappeared in time. So, no resonant effects present on oscillations on partial frequency, which corresponds to the first normal mode of liquid.

Let us consider now system behavior on loading with frequency ω = 7,19. Graphs in Fig. 8 and Fig. 9 show variation in time of amplitude the first antisymmetric and axisymmetric normal modes correspondingly. As it is seen from figures after comparatively short transient stage system passes into quasi-steady mode of motion, when the amplitude of the first antisymmetric normal mode performs practically stable oscillations, while the amplitude of the first axisymmetric normal mode is small (low nonlinear effects) and its law of changing in time has strong modulation. At the same time amplitudes of the first axisymmetric normal mode has systematic shift, which corresponds

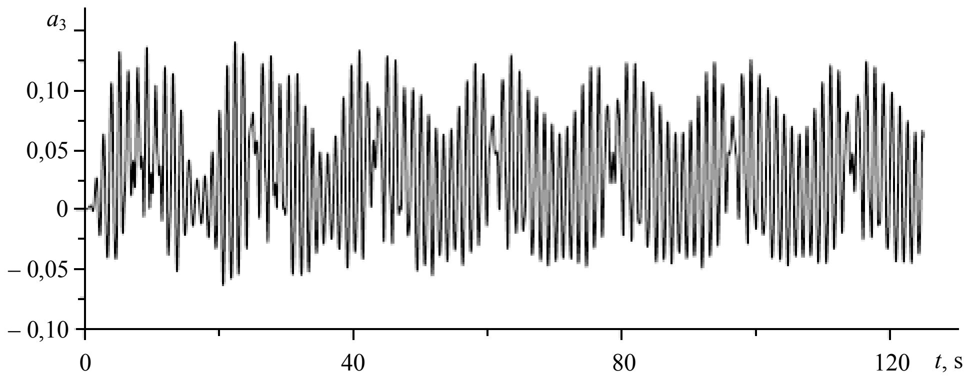

Fig. 5. Variation in time of amplitude the first axisymmetric normal mode

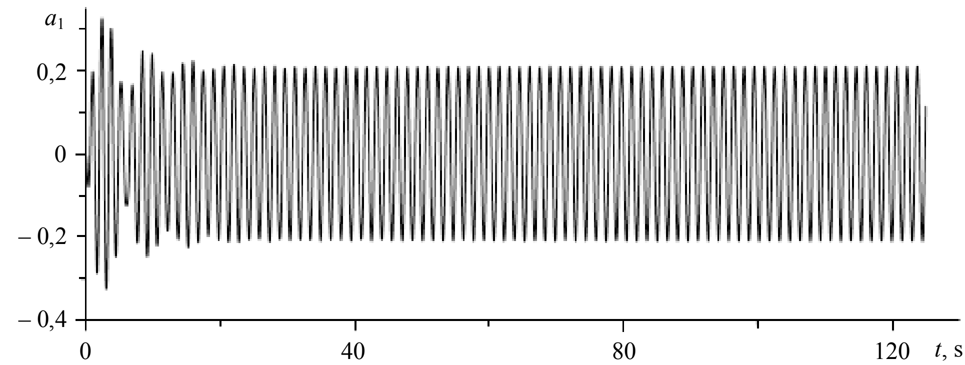

Fig. 6. Variation in time of amplitude the first antisymmetric normal mode

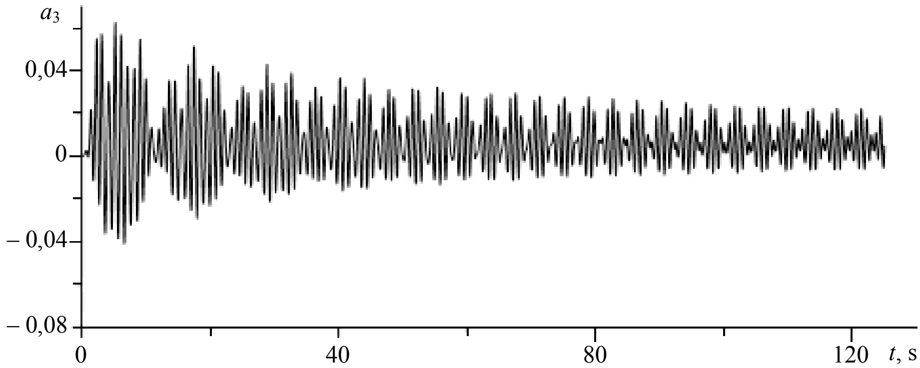

Fig. 7. Variation in time of amplitude the first axisymmetric normal mode

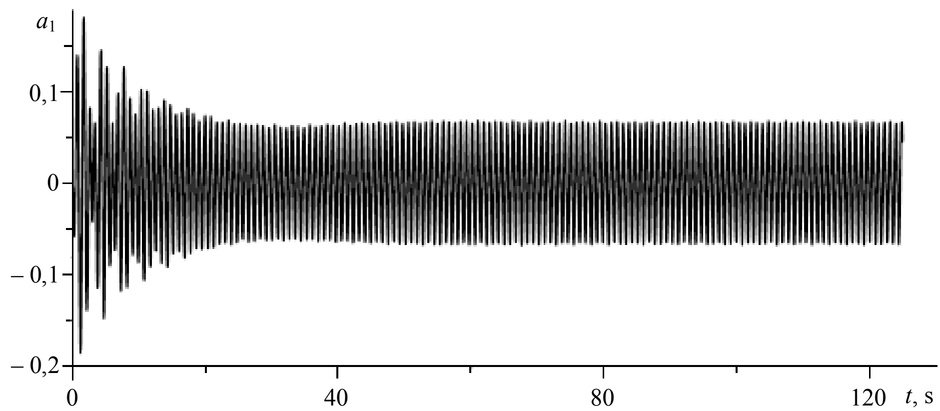

Fig. 8. Variation in time of amplitude the first antisymmetric normal mode

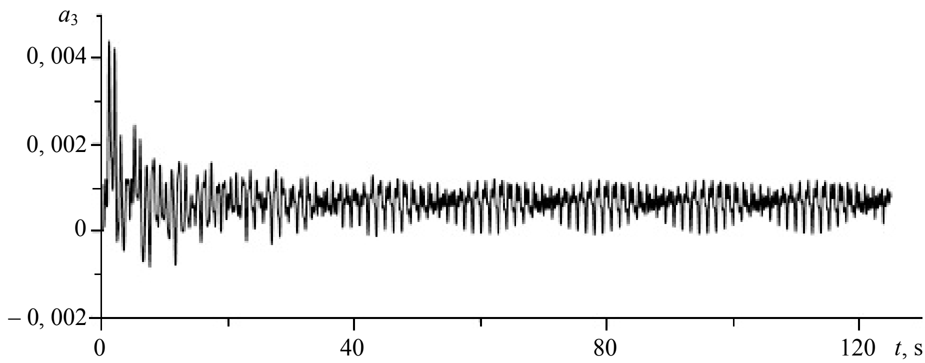

Fig. 9. Variation in time of amplitude the first axisymmetric normal mode

amplitudes of the first axisymmetric normal mode has systematic shift, which corresponds to non-symmetry of wave profile, which is registered in experiments. Therefore, in a vicinity of the second normal frequency of combined oscillations resonant effects are not manifested. This can be caused by high fidelity of resonant curves in this domain.

Conclusions. Investigation of combined motion of cylindrical reservoir, partially filled with liquid, on pendulum suspension under external harmonic loading showed some significant peculiarities of system behavior. First of all combined oscillations have normal frequencies, which considerably differ from partial frequencies of oscillations. On resonant frequency, which corresponds to domination of angular oscillations of pendulum, resonance manifests sharply. For resonant frequency, which corresponds to domination of sloshing oscillations, resonance effects practically do not manifest, probably because of high fidelity of resonant curves in close vicinity of this normal frequency. For all other frequencies including partial frequencies of system oscillations no resonant effect are observed. Liquid oscillations tend to quasi-steady mode of motion with weak decrease of oscillations due to effect of liquid viscosity and strong effect of modulation of oscillations. At the same time the higher is frequency of loading, the weaker is effect of modulation of oscillations. Therefore, under the presence of viscosity for frequencies of loading, which exceed partial frequency of sloshing according to the first normal mode, system tends to steady mode of motion.

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Особливості руху системи резервуар – рідина на маятниковому підвісі під дією зовнішньої гармонічної сили

Олег Лимарченко, Катерина Семенович

Для різних частот зовнішнього збудження досліджується кутовий рух циліндричного резервуара на маятниковому підвісі, частково заповненого рідиною. Показано, що резонансні частоти за сумісного руху суттєво відрізняються від парціальних частот коливань рідини та маятникових коливань. Обговорюються деякі особливості поведінки системи в різних діапазонах зміни частот збудження.

Особенности движения системы резервуар – жидкость на маятниковом подвесе под действием внешней гармонической силы

Олег Лимарченко, Екатерина Семенович

Для разных частот внешнего возбуждения исследуется угловое движение частично заполненного жидкостью цилиндрического резервуара на маятниковом подвесе. Показано, что резонансные частоты при совместном движении существенно отличаются от парциальных частот колебаний жидкости и маятниковых колебаний. Обсуждаются некоторые особенности поведения системы в разных диапазонах изменения частот возбуждения.

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