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Parametric oscillations of liquid with a free surface in reservoir of conic shape

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The problem about parametric oscillations of liquid with a free surface in reservoir of conic shape (the Faraday problem) is under investigation. Liquid is considered as ideal, incompressible, reservoir is a rigid body of conic shape. Investigation is done on the basis of nonlinear model of combined motion of liquid with reservoir with representation of liquid motion as decomposition with respect to normal modes of oscillations. Resolving mathematical model is obtained in the form of a system of ordinary differential equations relative to parameters of motion of reservoir and amplitudes of excitation of normal modes of oscillation of liquid with a free surface. It is shown how tank walls inclination and initial conditions influence development of dynamic processes.

Keywords: liquid with a free surface, conic reservoir, parametric oscillations.

Introduction. For the first time the problem of parametric resonance in reservoir of cylindrical shape, partially filled by liquid, was investigated by Faraday in 1831 [1]. Recent investigations showed that the Faraday classical problem is to the great extent idealized and for better correspondence with practice a number of additional factors should be taken into account. Certain part of them is caused by engineering problem of longitudinal oscillations of rockets. In the Faraday classical problem reservoir motion was perturbed according to the given law, which corresponds to infinite mass of reservoir. Otherwise in applied problems mass of liquid considerably exceeds mass of reservoir, therefore, it is important to investigate force disturbance of system oscillations in combined statement. Until now the Faraday problem was considered only for reservoirs of cylindrical shape.

Objective of the present article is statement of the Faraday problem about parametric oscillations of liquid in reservoir of non-cylindrical shape and investigation of effect of initial conditions and inclinations of reservoir walls on development of parametric resonance.

1. Mathematical model of the system

Let us consider the problem about oscillations of the mechanical system, which consists of two interacting components, i.e., conic reservoir and liquid with a free surface. We suppose reservoir as absolutely rigid body. Liquid is supposed to be ideal, homogeneous and at initial time its motion is vortex-free. We neglect effect of capillarity on liquid oscillations.

Motion equations of the system reservoir–liquid in amplitude parameters a_i and parameters of translational motion of the reservoir, which carry liquid, ε_i are written as [2]:

$$\sum_{n=1}^{N} p_{rn}\left(a_k,t\right)\ddot{a}_n + \sum_{n=N+1}^{N+3} p_{rn}\left(a_k,t\right)\ddot{\varepsilon}_{n-N} = q_r\left(a_k,\dot{a}_l,t\right), \quad r=\overline{1,N+3}. \tag{1}$$

Expressions for p_{rn} and q_r, where p_{rn} is quadratic matrix, and q_r is vector of *N+3* dimension, are represented as algebraic forms from zero to third order of amplitude parameters of free surface of liquid oscillations a_i and displacements of the reservoir (explicit dependence of the matrix p_{rn} on *t* takes place only under the presence of liquid outflow from the reservoir). For construction of the system of coordinate function we used the method of auxiliary domain [2], which in contrast to classical approach takes into account realization of non-flowing condition on tank walls above the level of unperturbed free surface of liquid. Results of determination of relative error of fulfillment of non-flowing conditions (ratio of violation of non-flowing condition to maximum of elevation of a liquid free surface) for inversed truncated cone show that below level of undisturbed free surface of liquid error does not exceeds 10^{-6}, and on prolongation of lateral surface above undisturbed free surface of liquid until the height 0,2 of radius of a liquid free surface it does not exceed 10^{-3}, which is acceptable for applied investigations. Let us note that the necessity of fulfillment with high precision of these conditions is connected with fulfillment of solvability conditions of the Newman boundary value problem for the Laplace equation, by which the problem of determination of normal modes of oscillations of a liquid free surface is described.

Further we assume that reservoir performs oscillatory motion only in vertical direction, a liquid free surface at initial time instant has perturbation only with respect to the first normal mode of oscillations, the system performs oscillations due to application of harmonic force in vertical (longitudinal) direction $F_z = A\cos\left(\Omega t\right)$. We investigated system motion in the case, when frequency of external disturbance was twice greater than eigenfrequency of the first antisymmetric normal mode of oscillations.

2. Analysis of numerical results

We considered for comparison the following variants of initial conditions $a_2(0)=0,02$; $a_2(0)=0,05$; $a_2(0)=0,1$ for radius of bottom $R = 0$ (cone) and $R = 1$ (cylinder). System motion for initial perturbation of the first normal mode with amplitude 0,02 under harmonic force loading with amplitude 0,5 and with frequency, which is approximately equal to doubled eigenfrequency of system normal mode. In this case oscillations of a liquid free surface for $R = 0$ (Fig. 1*a*) and $R = 1$ (Fig. 1*b*) obtain modulation.

System motion for initial perturbation 0,05 for $R = 0$ (Fig. 2*a*) and $R = 1$ (Fig. 2*b*) will have similar behavior.

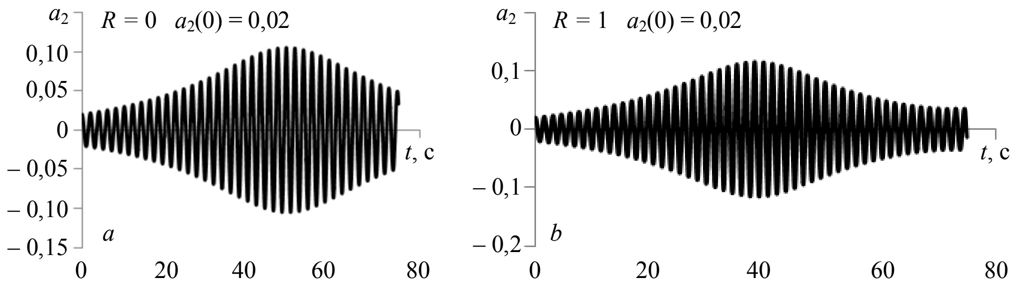

Fig. 1. Parametric oscillations of liquid free surface for $a_2(0) = 0{,}02$

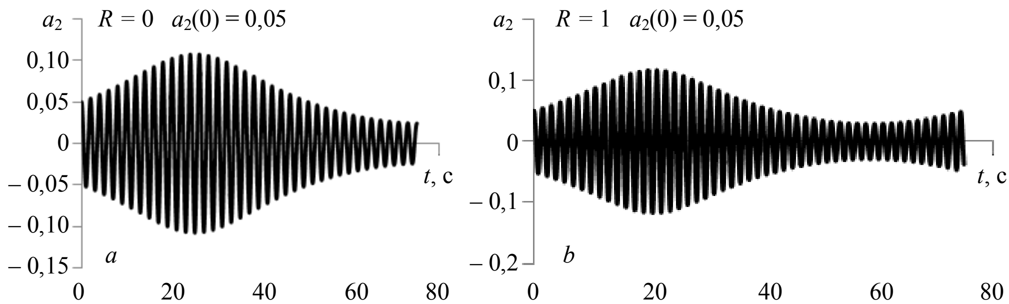

Fig. 2. Parametric oscillations of liquid free surface for $a_2(0) = 0{,}05$

For initial perturbation 0,1 system motion for $R = 0$ (Fig. 3*a*) and $R = 1$ (Fig. 3*b*) variation of amplitude is shown in Fig. 3.

On the whole as it is seen from figures magnitude of initial perturbation and inclination of reservoir walls affect weakly on maximal value of amplitude variation. Here maximal values of amplitudes in all cases are approximately equal to 0,15 of radius of a liquid free surface. In all cases tending to steady mode of motion is not manifested.

Let us consider table of maximums of wave heights (these columns are denoted by «max») and time instants, when these maximums are manifested, (denoted by $t_{\max}$).

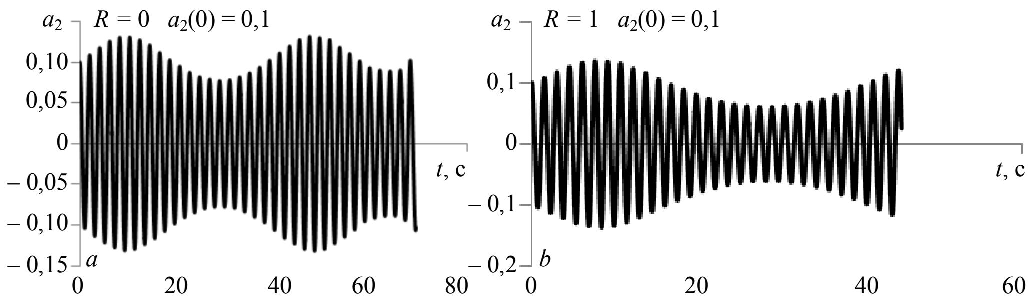

Fig. 3. Parametric oscillations of liquid free surface for $a_2(0) = 0{,}1$

Table

$a_2(0)$	Table of maximums									
	$R = 0$		$R = 0.25$		$R = 0.50$		$R = 0.75$		$R = 1$	
	max	t_{max}	max	t_{max}	max	t_{max}	max	t_{max}	max	t_{max}
0.02	0.105	48.42	0.1094	46.70	0.1101	43.17	0.1120	40.25	0.1149	38.12
0.05	0.1074	26.36	0.1118	24.38	0.1126	22.54	0.1146	22.63	0.1165	21.44
0.08	0.1188	14.31	0.1216	13.22	0.1226	12.21	0.1242	13.02	0.125	12.33
0.10	0.1303	10.31	0.1326	9.51	0.1334	8.78	0.1347	9.83	0.1356	7.75

The first column shows initial perturbation of the first normal mode a_2, the rest of columns correspond to different cases of cone truncation with different radiuses R of bottom.

Analysis of data, shown in Table, testifies monotonic dependence of maximums of wave heights on tank walls inclination. The greater is bottom of the reservoir, the higher is maximum wave height. However this dependence is rather weak. On the contrary time of manifestation of maximum wave considerably depends on initial perturbation of a liquid free surface. The greater is initial perturbation, the earlier maximum of wave height is reached. At the same time dependence of maximum wave height on magnitude of initial perturbation is rather weak, but also monotonic, i.e., wave amplitude weakly increases with increase of magnitude of initial perturbation.

Conclusion. We consider the problem of parametric oscillations of liquid with a free surface in reservoir of truncated conic shape. In contrast to the Faraday classical problem statement system motion is disturbed by longitudinal harmonic force, but transversal motion of reservoir is excluded. It was ascertained that on development of parametric oscillations in a vicinity of doubled normal frequency steady mode of oscillations are not manifested. Effect of inclination of reservoir walls is rather weak, however analysis shows that of maximum of wave height depends on inclination of tank walls monotonically. The greater is bottom size, the higher is maximum wave.

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Параметричні коливання рідини з вільною поверхнею в резервуарі конічної форми

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Розглядається задача про параметричні коливання рідини з вільною поверхнею в резервуарі конічної форми (задача Фарадея). Рідина вважається ідеальною, нестисливою, резервуар — абсолютне тверде тіло конічної форми. Дослідження виконується на основі нелінійної моделі сумісного руху рідини та резервуара з поданням руху рідини в вигляді розкладу за формами власних коливань. Розв'язувальну математичну модель одержано у вигляді нелінійної системи звичайних диференціальних рівнянь щодо параметрів руху резервуара й амплітуд збурення форм коливань вільної поверхні рідини. Показано як нахил стінок резервуара та початкові умови впливають на розвиток динамічних процесів.

Параметрические колебания жидкости со свободной поверхностью в резервуаре конической формы

Олег Лимарченко, Виктория Мельник

Рассматривается задача о параметрических колебаниях жидкости со свободной поверхностью в резервуаре конической формы (задача Фарадея). Жидкость предполагается идеальной, несжимаемой, резервуар — абсолютно твердое тело конической формы. Исследование выполнено на основе нелинейной модели совместного движения жидкости и резервуара с представлением движения жидкости в виде разложения по формам собственных колебаний. Разрешающую математическую модель получено в форме нелинейной системы обыкновенных дифференциальных уравнений относительно параметров движения резервуара и амплитуд возбуждения форм колебаний свободной поверхности жидкости. Показано как наклон стенок резервуара и начальные условия влияют на развитие динамических процессов.

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