УДК 539.3, 538.54

On determination the thermoelastic state of electroconducting nonferromagnetic layer in time-harmonic field

Oleksandr Hachkevych[1], Mykhaylo Solodiak[2], Mykola Makhorkin[3]

[1] DSc., professor, Pidstrygach Institute for Applied Problems of Mechanics and Mathematics of NASU, 3b Naukova str, Lviv, 79060; Technical University in Opole, Liuboshitska str., 3, Opole, Poland; e-mail: dept13@iapmm.lviv.ua, garlu@po.opole.pl
[2] Ph. D., Pidstrygach Institute for Applied Problems of Mechanics and Mathematics of NASU, 3b Naukova str., Lviv, 79060; e-mail: dept13@iapmm.lviv.ua
[3] Ph. D., Pidstrygach Institute for Applied Problems of Mechanics and Mathematics of NASU, 3b Naukova str., Lviv, 79060; e-mail: mahorkin@ukr.net

A calculation model of determination the thermoelastic state of electroconducting nonferromagnetic layer in time-harmonic field is offered. In the model the oscillating components of heat release and ponderomotive force are considered. And dynamic terms of thermoelasticity problem as well. The conditions under which one needs to use the refined calculation model are determined.

Keywords: time-harmonic magnetic field, oscillation components of heat release, dynamic terms of thermoelasticity problem, ponderomotive force.

Introduction. As is well known, in electroconducting bodies wich are in electromagnetic field (EMF) electric currents are induced that accompany the heat release and action of ponderomotive force, which in turn cause the body to warm up and initiation of mechanical stresses. In addition, they can exceed the permissible value (melting point or ultimate strength). To this end one has to develop the effective computational models in order to determine the EMF, heat release, temperature and stresses depending on external factors.

The study of thermoelastic state of nonferromagnetic electroconductive bodies in an external time harmonic EMF one can find in the fundamental monograph [1]. This paper presents a simplified method of determination the magnetoelectro elastic state which is as follows. At the first stage from the equations of electrodynamics the EMF is determined in a steady approximation. Then the averaged for a period of oscillations of external EMF, corresponding expressions for the power of heat releases and density of ponderomotive force are written. In addition their oscillatory (harmonic) components are neglected. At the second stage from the heat releases (where heat releases, determined at the first stage, are heat sources) the temperature field is determined. At the third stage from equations of quasi-static thermoelasticity [1, 2] (the dynamic terms, ponderomotive forces and connectedness of temperature and deformation fields are neglected) the displacements and stresses are calculated. In the equations of thermoelasticity the expressions for the energy density are taken from the first stage and the expressions for the temperature are taken from the second stage.

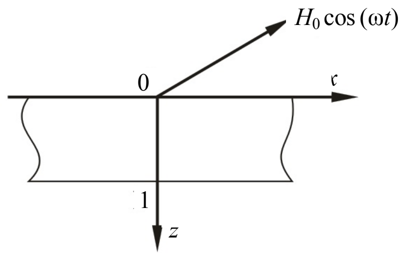

Fig. 1. Scheme of the electroconductive layer

For practical purposes it is necessary to develop a more refined computational model, which would take into account the oscillating components of the heat release and ponderomotive force and dynamic terms in quasi-static thermoelasticity problem as well. The criteria justification for the model is given in [3].

1. Statement of the problem

We consider the electroconductive layer of thickness, related to a rectangular dimensionless coordinate system (x, y, z) (Fig. 1). A tangent component of the external magnetic field H_y on the upper surface influences the layer

$$H_y(0;t) = H_0 \cos\omega t\,, \quad H_y(1;t) = 0\,. \tag{1}$$

here $\omega = 2\pi\nu$, ν is the frequency, t is time; H_0 is the amplitude of harmonic component of the magnetic field intensity. Later on index «*y*» is omitted.

The magnetic field intensity $H(z;t)$ in the layer region we find from Maxwell's equation, which in this case is

$$\frac{\partial^2 H}{\partial z^2} = \mu_0 \lambda l^2 \frac{\partial H}{\partial t}\,, \tag{2}$$

where μ_0 is permeability of free space, λ is the electrical conductivity.

Knowing the magnetic field in the layer region, the Joule heat power Q, the energy density W of EMF stored in the body and the density of ponderomotive force F we determine from

$$Q = \frac{1}{\lambda l^2}\left(\frac{\partial H}{\partial z}\right)^2, \quad W = \frac{1}{2}\mu_0 H^2, \quad F = -\frac{1}{l}\frac{\partial W}{\partial z}\,. \tag{3}$$

The last relation (3) allows us in this model instead of ponderomotive force F to use the energy density of EMF W, what will be used in future.

Defining the thermoelastic state, assume that on the upper base of the layer a convective heat transfer takes place with the environment, the temperature of which T_0 is equal to the temperature of the layer and the bottom one is insulated. Assume also that the base $z = 0$ is power load-free, and $z = 1$ is fixed rigidly with a dielectric half-space.

Then the temperature field is found from the equation

$$\frac{1}{a_T}\frac{\partial T}{\partial t}=\frac{1}{l^2}\frac{\partial^2 T}{\partial z^2}+\frac{Q}{\kappa} \tag{4}$$

for initial condition

$$T(z;0)=T_0, \tag{5}$$

and boundary conditions

$$\frac{\partial T(0;t)}{\partial z}=Bi\left[T(0;t)-T_0\right],\quad \frac{\partial T(1;t)}{\partial z}=0. \tag{6}$$

Here κ and a_T are thermal conductivity and thermal diffusivity factors; Bi is Bio criterion.

In the system of thermoelasticity equations we will have a non-zero component of displacements $u_z=u(z;t)$ and three diagonal components of tensor of mechanical stresses σ_{xx}, σ_{yy} and σ_{zz}. The equation of motion is written as follows

$$\rho\frac{\partial^2 u}{\partial t^2}=\frac{1}{l}\frac{\partial}{\partial z}(\sigma_{zz}-W) \tag{7}$$

for initial conditions

$$u(z;0)=\frac{\partial u(z;0)}{\partial t}=0, \tag{8}$$

and boundary condition

$$u(1;t)=0,\quad \sigma_{zz}(0;t)=0. \tag{9}$$

In formulas (4) and (7) in the classical theory [1] the values Q and W are averaged for the period of oscillations of the external EMF.

From Hooke's law we obtain the relations

$$\sigma_{xx}=\sigma_{yy}=\frac{E_p}{1-2\nu_p}\left[\frac{\nu_p}{1+\nu_p}\frac{1}{l}\frac{\partial u}{\partial z}-\alpha_t\left(T-T_0\right)\right], \tag{10}$$

$$\sigma_{zz}=\frac{E_p}{1-2\nu_p}\left[\frac{1-\nu_p}{1+\nu_p}\frac{1}{l}\frac{\partial u}{\partial z}-\alpha_t\left(T-T_0\right)\right], \tag{11}$$

that allow you to express the components σ_{xx} and σ_{yy} in terms of the normal component σ_{zz}

$$\sigma_{xx}=\sigma_{yy}=\frac{1}{1-\nu_p}\left[\nu_p\sigma_{zz}-\alpha_t E_p\left(T-T_0\right)\right], \tag{12}$$

where E_p, ν_p is elasticity modulus and Poisson's ratio; α_T is the coefficient of linear expansion.

Substituting formula (11) in relations (7) and (9), we obtain the equations for displacements

$$\frac{\partial^2 u}{\partial z^2} - \frac{l^2}{c_1^2}\frac{\partial^2 u}{\partial t^2} = \frac{1+\nu_p}{1-\nu_p} l \frac{\partial}{\partial z}\left(\alpha_T T + \frac{1-2\nu_p}{E_p} W \right) \tag{13}$$

and the boundary conditions

$$u(1;t) = 0\,, \quad \frac{1}{l}\frac{\partial u(0;t)}{\partial z} = \frac{1+\nu_p}{1-\nu_p}\alpha_T \left[T(0;t) - T_0 \right]. \tag{14}$$

Here $c_1 = \sqrt{\dfrac{(1-\nu_p)E_p}{\rho(1+\nu_p)(1+2\nu_p)}}$ is the velocity of longitudinal waves, ρ is the body density.

We go to the solution of the problem stated (1)-(14).

2. Methods of solving the electrodynamics problem

Solution of the problem of electrodynamics (1), (2) is presented in the form

$$H(z;t) = \frac{H_0}{2}\left[h(z)e^{i\omega t} + \tilde{h}(z)e^{-i\omega t} \right]. \tag{15}$$

Here tilde «$\sim$» over the value indicates its complex conjugate value; i is imaginary unit.

Substituting the presentation (15) in relations (1), (2) for function $h(z)$ we obtain the following ordinary differential equation and boundary conditions:

$$\frac{d^2 h}{dz^2} = 2i\gamma^2 h\,, \quad h(0) = 1\,, \quad h(1) = 0\,, \tag{16}$$

where $\gamma = l\sqrt{\pi\lambda\mu_0\nu}$, $\delta = 1/\gamma$ is the relative depth of penetration of the magnetic field in the environment.

Solution of the problem (16) will be as follows:

$$h(z) = \mathrm{sh}(1+i)\gamma(1-z)\left[\mathrm{sh}(1+i)z \right]^{-1}. \tag{17}$$

Based on the formula (3), for the heat release Q and energy density of EMF W stored in the body we will have

$$\Psi(z;t) = \overline{\Psi}(z) + \Psi_2(z)e^{2i\omega t} + \widetilde{\Psi}_2(z)e^{-2i\omega t}\,, \tag{18}$$

where $\Psi \equiv \{Q;W\}$, $\overline{\Psi}(z)$ are the averaged values for the oscillation period of EMF; $\Psi_2(z)$ are their second harmonics.

Write the specific expressions of these values

- averaged values:

$$\bar{Q}(z) = \frac{H_0^2}{2\lambda l^2}\frac{dh(z)}{dz}\frac{d\tilde{h}(z)}{dz} = \frac{\gamma^2 H_0^2}{\lambda l^2}\frac{\operatorname{ch}2\gamma(1-z)+\cos 2\gamma(1-z)}{\operatorname{ch}2\gamma-\cos 2\gamma},$$

$$\bar{W}(z) = \frac{\mu_0 H_0^2}{4}h(z)\tilde{h}(z) = \frac{\mu_0 H_0^2}{4}\frac{\operatorname{ch}2\gamma(1-z)-\cos 2\gamma(1-z)}{\operatorname{ch}2\gamma-\cos 2\gamma}; \qquad (19)$$

- harmonic components:

$$Q_2(z) = \frac{H_0^2}{4\lambda l^2}\left(\frac{dh(z)}{dz}\right)^2 = \frac{i\gamma^2 H_0^2}{2\lambda l^2}\frac{\operatorname{ch}2(1+i)\gamma(1-z)+1}{\operatorname{ch}2(1+i)\gamma-1},$$

$$W_2(z) = \frac{\mu_0 H_0^2}{8}h^2(z) = \frac{\mu_0 H_0^2}{8}\frac{\operatorname{ch}2(1+i)\gamma(1-z)-1}{\operatorname{ch}2(1+i)\gamma-1}. \qquad (20)$$

The obtained expressions (19), (20), considering presentations (18), are included in the heat conduction equation (4) and the equation for the displacements (13).

3. Solution of the problem of thermoelasticity

The temperature field we seek from the heat conduction problem (4)-(6) where the expression for the heat release Q is given by the formulas (18)-(20). Using the Laplace time transform t [4], for function $T(z;t)$ we will have:

$$T(z;t) = T_0 + \bar{T}(z) + T_2(z)e^{2i\omega t} + \tilde{T}_2(z)e^{-2i\omega t} + T_*(z;t), \qquad (21)$$

where $\bar{T}(z)$ is a component of the temperature caused by the averaged heat release $\bar{Q}(z)$; $T_2(z)$ is a harmonic component caused by oscillatory heat release $Q_2(z)$; $T_*(z;t)$ is a component that describes the transition temperature regime.

Write down the specific expressions for them

$$\bar{T}(z) = T_m\left(\operatorname{ch}2\gamma-\cos 2\gamma\right)^{-1}\left[a_0-\operatorname{ch}2\gamma(1-z)+\cos 2\gamma(1-z)\right], \qquad (22)$$

$$T_2(z) = T_m\frac{(\varepsilon-2)d_1+\varepsilon d_1\operatorname{ch}2(1+i)\gamma(1-z)-Bi_1\operatorname{ch}\sqrt{2\varepsilon}(1+i)\gamma(1-z)}{4\varepsilon(\varepsilon-2)d_1\operatorname{sh}^2(1+i)\gamma}, \qquad (23)$$

$$T_*(z;t) = -T_m\sum_{n=1}^{\infty}T_n\cos\mu_n(1-z)e^{-a_T\mu_n^2 t/l^2}. \qquad (24)$$

In formulas (21)-(24) we introduce the notations: $T_m \equiv H_0^2(4\kappa\lambda)^{-1}$ — characteristic temperature; $\varepsilon \equiv (\mu_0 a_T\lambda)^{-1}$ — dimensionless parameter; $Bi_1 \equiv (\varepsilon-2)Bi+\varepsilon d_0$;

$$a_0 \equiv \operatorname{ch}2\gamma-\cos 2\gamma+\frac{2\gamma}{Bi}\left(\operatorname{sh}2\gamma+\sin 2\gamma\right),$$

$$d_0 \equiv Bi\operatorname{ch}2(1+i)\gamma+2(1+i)\gamma\operatorname{sh}2(1+i)\gamma,$$

$$d_1 \equiv Bi\,\mathrm{ch}\sqrt{2\varepsilon}(1+i)\gamma + \sqrt{2\varepsilon}(1+i)\gamma\,\mathrm{sh}\sqrt{2\varepsilon}(1+i)\gamma ,$$

$$T_n \equiv 8\gamma^2 A_n \left\{ \left(\mathrm{ch}\,2\gamma - \cos 2\gamma\right)^2 z_n(\gamma)\left[\mu_n^2 + Bi\left(Bi+1\right)\right]\cos\mu_n \right\}^{-1},$$

$$A_n \equiv \left(\mathrm{ch}\,2\gamma - \cos 2\gamma\right)\left(64\gamma^4 + \mu_n^4\right)\left(16\varepsilon\gamma^4 + \mu_n^4\right)a_n(\gamma) + \left(16\gamma^4 - \mu_n^4\right)\times$$

$$\times\left\{\mu_n^2\left[2(\varepsilon+2)\gamma^2\mu_n^2\xi_1(\gamma) - \left(32\varepsilon\gamma^4 - \mu_n^4\right)\xi_2(\gamma)\right] - Bi\left(64\gamma^4 + \mu_n^4\right)b_n(\gamma)\right\},$$

$$z_n(\gamma) \equiv \left(16\gamma^4 - \mu_n^4\right)\left(64\gamma^4 + \mu_n^4\right)\left(16\varepsilon\gamma^4 + \mu_n^4\right),$$

$$a_n(\gamma) = \left(4\gamma^2 - \mu_n^2\right)\left(Bi\,\mathrm{ch}\,2\gamma + 2\gamma\,\mathrm{sh}\,2\gamma\right) - \left(4\gamma^2 + \mu_n^2\right)\left(Bi\cos 2\gamma - 2\gamma\sin 2\gamma\right),$$

$$b_n(\gamma) \equiv 4\varepsilon\gamma^2\left(1 - \mathrm{ch}\,2\gamma\cos 2\gamma\right) - \mu_n^2\,\mathrm{sh}\,2\gamma\sin 2\gamma ,$$

$$\xi_1(\gamma) \equiv Bi\left(\mathrm{ch}\,4\gamma + \cos 4\gamma - 2\,\mathrm{ch}\,2\gamma\cos 2\gamma\right) +$$

$$+2\gamma\left[\mathrm{sh}\,4\gamma - \sin 4\gamma + 2\left(\mathrm{ch}\,2\gamma\sin 2\gamma - \mathrm{sh}\,2\gamma\cos 2\gamma\right)\right],$$

$$\xi_2(\gamma) \equiv Bi\,\mathrm{sh}\,2\gamma\sin 2\gamma - \gamma\left[\mathrm{sh}\,4\gamma + \sin 4\gamma - 2\left(\mathrm{sh}\,2\gamma\cos 2\gamma + \mathrm{ch}\,2\gamma\sin 2\gamma\right)\right],$$

and μ_n are the roots of the equation $Bi\cos\mu_n = \mu_n\sin\mu_n$.

Note that in the case of simplified methods [1] the expression (22) for $\overline{T}(z)$ remains the same, $T_2(z) \equiv 0$, and for $T_*(z;t)$ we obtain

$$T_{*kl}(z;t) = -\frac{8\gamma^2 T_m}{\left(\mathrm{ch}\,2\gamma - \cos 2\gamma\right)}\sum_{n=1}^{\infty}\frac{a_n(\gamma)\cos\mu_n(1-z)e^{-a_T\mu_n^2 t/l^2}}{\left(16\gamma^4 - \mu_n^4\right)\left[\mu_n^2 + Bi\left(Bi+1\right)\right]\cos\mu_n}. \qquad (25)$$

We turn to finding the displacements. Considering the presentations (18) for energy density of EMF and (21) — for the temperature, in the same form can be written also the form for the displacements

$$u(z;t) = \overline{u}(z) + u_2(z)e^{2i\omega t} + \tilde{u}_2(z)e^{-2i\omega t} + u_*(z;t). \qquad (26)$$

Note that each term in the representation (26) has the same interpretation as for the temperature in the representation (21).

Solving the problem (13), (14), with this in mind, we obtain the following expressions for the components of displacements (26):

$$\overline{u}(z) = \frac{u_m}{2\gamma\left(\mathrm{ch}\,2\gamma - \cos 2\gamma\right)}\left\{(1-\varepsilon_\sigma)\left[\mathrm{sh}\,2\gamma(1-z) - \sin 2\gamma(1-z)\right] - 2\gamma a_1(1-z)\right\}, \qquad (27)$$

$$u_2(z) = \frac{2c_1(\gamma)\left(\sqrt{2\varepsilon}\varepsilon_T\gamma\right)^{-1}\sin 4\varepsilon\varepsilon_T\gamma^2(1-z) - (1-i)c_2(z;\gamma)\cos 4\varepsilon\varepsilon_T\gamma^2}{8\varepsilon\gamma\left[1 - 2i\left(\varepsilon\varepsilon_T\gamma\right)^2\right]\left[1 - 4i\varepsilon\varepsilon_T^2\gamma^2\right]z_0\cos 4\varepsilon\varepsilon_T\gamma^2}u_m, \qquad (28)$$

$$u_*(z;t) = u_m \sum_{n=1}^{\infty} u_n(z) e^{-a_T \mu_n^2 t / l^2} \ , \tag{29}$$

$$u_n(z) = T_n \left(\mu_n^2 + \varepsilon_n^2\right)^{-1} \left[\varepsilon_n \cos\mu_n \operatorname{ch}^{-1}\varepsilon_n \operatorname{sh}\varepsilon_n(1-z) + \mu_n \sin\mu_n(1-z)\right]. \tag{30}$$

In formulas (27)-(30), the following notations are introduced: $u_m \equiv \dfrac{1+\nu_p}{1-\nu_p}\alpha_T l T_m$ are characteristic displacements; $l_T = a_T c_1^{-1}$ is characteristic thermal size; $\varepsilon_T = l_T^2 l^{-2}$, $\varepsilon_n \equiv \varepsilon_T \mu_n^2$ are characteristic thermal parameters; $\varepsilon_\sigma \equiv \left(1-2\nu_p\right)\mu_0 \kappa\lambda \left(\alpha_T E_p\right)^{-1}$ is the parameter that characterizes the influence of power stresses compared to temperature stresses on the thermoelastic state of the body.

$$a_1 \equiv a_0 - \varepsilon_\sigma\left(\operatorname{ch}2\gamma - \cos 2\gamma\right),\ z_0 \equiv 2\sqrt{2\varepsilon}\left(\varepsilon-2\right)d_1 \operatorname{sh}^2\left(1+i\right)\gamma\,,$$
$$z_1 \equiv \left(\varepsilon-2\right)d_1 + \varepsilon d_1 \operatorname{ch}2\left(1+i\right)\gamma - Bi_1 \operatorname{ch}\sqrt{2\varepsilon}\left(1+i\right)\gamma\,,\ \chi \equiv \varepsilon_\sigma\left(\varepsilon-2\right),$$
$$c_1\left(\gamma\right) \equiv c_0\left(\gamma\right) - \left[1-2i\left(\varepsilon\varepsilon_T\gamma\right)^2\right]\left[1-4i\varepsilon\varepsilon_T^2\gamma^2\right]z_1,$$
$$c_0\left(\gamma\right) \equiv \varepsilon\left(1+\chi\right)\left[1-4i\varepsilon\varepsilon_T^2\gamma^2\right]d_1 \operatorname{ch}2\left(1+i\right)\gamma - Bi_1\left[1-2i\left(\varepsilon\varepsilon_T\gamma\right)^2\right]\operatorname{ch}\sqrt{2\varepsilon}\left(1+i\right)\gamma,$$
$$c_2\left(z;\gamma\right) \equiv \varepsilon\sqrt{2\varepsilon}\left(1+\chi\right)\left[1-4i\varepsilon\varepsilon_T^2\gamma^2\right]d_1 \operatorname{sh}2\left(1+i\right)\gamma\left(1-z\right) -$$
$$-2Bi_1\left[1-2i\left(\varepsilon\varepsilon_T\gamma\right)^2\right]\operatorname{sh}\sqrt{2\varepsilon}\left(1+i\right)\gamma\left(1-z\right). \tag{31}$$

Note that in the case of simplified methods [1] the expression (27) remains unchanged, and $u_2(z) \equiv 0$ and $u_*(z;t) \equiv 0$.

Substituting the presentations (21) and (26) in formulas (10)-(12), for stresses we obtain the same representations as for the temperature and displacements with a similar interpretation of terms. Write down the appropriate expressions

$$\bar{\sigma}_{xx}(z) = \bar{\sigma}_{yy}(z) = \sigma_m\left(\operatorname{ch}2\gamma - \cos 2\gamma\right)^{-1}\left\{\left[1-\nu_p\left(2-\varepsilon_\sigma\right)\right]\left[\operatorname{ch}2\gamma(1-z) -\right.\right.$$
$$\left.\left. -\operatorname{ch}2\gamma - \cos 2\gamma(1-z) + \cos 2\gamma\right] - \left(1-2\nu_p\right)2\gamma\left(\operatorname{sh} + \sin 2\gamma\right)Bi^{-1}\right\},$$
$$\bar{\sigma}_{zz}(z) = \frac{\left(1-\nu_p\right)\varepsilon_\sigma\sigma_m}{\operatorname{ch}2\gamma - \cos 2\gamma}\left[\operatorname{ch}2\gamma(1-z) - \operatorname{ch}2\gamma - \cos 2\gamma(1-z) + \cos 2\gamma\right];$$
$$\sigma_{xx,2}(z) = -\sigma_m z_2^{-1} C_{xx,2}\left(z;\gamma\right), \quad \sigma_{zz,2}(z) = -\left(1-\nu_p\right)\sigma_m z_2^{-1} C_{xx,2}\left(z;\gamma\right),$$
$$z_2 \equiv \left[1-2i\left(\varepsilon\gamma\right)^2\right]\left[1-4i\varepsilon\varepsilon_T^2\gamma^2\right]4\varepsilon\left(\varepsilon-2\right)d_1 \cos\left(4\varepsilon\varepsilon_T\gamma^2\right)\operatorname{sh}^2\left(1+i\right)\gamma\,,$$
$$C_{xx,2}\left(z;\gamma\right) \equiv \nu_p c_1\left(\gamma\right)\cos 4\varepsilon\varepsilon_T\gamma^2\left(1-z\right) + \left(1-\nu_p\right)\left[1-2i\left(\varepsilon\varepsilon_T\gamma\right)^2\right]\times$$

$$\times\left(1-4i\varepsilon\varepsilon_T^2\gamma^2\right)\left(\varepsilon-2\right)d_1\cos 4\varepsilon\varepsilon_T\gamma^2+\left(1-4i\varepsilon\varepsilon_T^2\gamma^2\right)\left\{\left(1-\nu_p\right)\left[1-2i\left(\varepsilon\varepsilon_T\gamma\right)^2\right]\times\right.$$

$$\left.\times\cos 4\varepsilon\varepsilon_T\gamma^2-\nu_p\left(1+\chi\right)\right\}\varepsilon d_1\,\mathrm{ch}\,2\left(1+i\right)\gamma\left(1-z\right)+\left[1-2i\left(\varepsilon\varepsilon_T\gamma\right)^2\right]\times$$

$$\times Bi_1\,\mathrm{ch}\sqrt{2\varepsilon}\left(1+i\right)\gamma\left(1-z\right)\left\{\nu_p-\left(1-\nu_p\right)\left(1-4i\varepsilon\varepsilon_T^2\gamma^2\right)\cos 4\varepsilon\varepsilon_T\gamma^2\right\},$$

$$C_{zz,2}\left(z;\gamma\right)\equiv c_1\left(\gamma\right)\cos 4\varepsilon\varepsilon_T\gamma^2\left(1-z\right)+\left[1-2i\left(\varepsilon\varepsilon_T\gamma\right)^2\right]\times$$

$$\times\left(1-4i\varepsilon\varepsilon_T^2\gamma^2\right)\left(\varepsilon-2\right)d_1\cos 4\varepsilon\varepsilon_T\gamma^2+\left(1-4i\varepsilon\varepsilon_T^2\gamma^2\right)\times$$

$$\times\left\{\left[1-2i\left(\varepsilon\varepsilon_T\gamma\right)^2\right]\cos 4\varepsilon\varepsilon_T\gamma^2-\left(1+\chi\right)\right\}\varepsilon d_1\,\mathrm{ch}\,2\left(1+i\right)\gamma\left(1-z\right)+$$

$$+\left[1-2i\left(\varepsilon\varepsilon_T\gamma\right)^2\right]\left\{1-\left(1-4i\varepsilon\varepsilon_T^2\gamma^2\right)\cos 4\varepsilon\varepsilon_T\gamma^2\right\}Bi_1\,\mathrm{ch}\sqrt{2\varepsilon}\left(1+i\right)\gamma\left(1-z\right),$$

$$\sigma_{xx*}\left(z;t\right)=\sigma_{yy*}\left(z;t\right)=\sigma_m\sum_{n=1}^{\infty}\sigma_{xxn}\left(z\right)e^{-a_T\mu_n^2 t/l^2},$$

$$\sigma_{zz*}\left(z;t\right)=\left(1-\nu_p\right)\sigma_m\sum_{n=1}^{\infty}\sigma_{zzn}\left(z\right)e^{-a_T\mu_n^2 t/l^2},$$

$$\sigma_{xxn}(z)\equiv -T_n\left(\mu_n^2+\varepsilon_n^2\right)^{-1}\left\{\nu_p\varepsilon_n^2\cos\mu_n\,\mathrm{ch}^{-1}\varepsilon_n\,\mathrm{ch}\,\varepsilon_n\left(1-z\right)-\right.$$

$$\left.-\left[\left(1-2\nu_p\right)\mu_n^2+\left(1-\nu_p\right)\varepsilon_n^2\right]\cos\mu_n\left(1-z\right)\right\},$$

$$\sigma_{zzn}\equiv -\varepsilon_n^2 T_n\left(\mu_n^2+\varepsilon_n^2\right)^{-1}\left[\cos\mu_n\,\mathrm{ch}^{-1}\varepsilon_n\,\mathrm{ch}\,\varepsilon_n\left(1-z\right)-\cos\mu_n\left(1-z\right)\right]. \quad (32)$$

Here $\sigma_m\equiv E_p\alpha_T T_m\left(1-\nu_p\right)^{-1}\left(1-2\nu_p\right)^{-1}$. Formulas (15)-(32) will be used for specific numerical studies in this problem.

4. Analysis of the solutions obtained

We estimate the effect of oscillating components of temperature, displacements and mechanical stresses on the thermoelastic state of nonferromagnetic layer in a time harmonic magnetic field compared to the relevant time-averaged components. Below the calculations were performed for a copper layer, whose characteristics are such [5-7]: $\rho=8,92\cdot 10^3\ \mathrm{kg/m^3}$, $E_p=1,29\cdot 10^{11}\ \mathrm{N/m^2}$, $\nu_p=0,35$, $\alpha_T=1,18\cdot 10^{-4}\ \mathrm{m^2/s}$, $\kappa=406\ \mathrm{W/(m\cdot K)}$, $\alpha_T=1,72\cdot 10^{-5}\ \mathrm{1/K}$, $\lambda=5,88\cdot 10^7\ \mathrm{A/(V\cdot m)}$, $\mu_0=4\pi\cdot 10^{-7}\ \mathrm{N/A^2}$, $c_1=4,818\cdot 10^3\ \mathrm{m/s}$.

Suppose also that $T_0=300\ \mathrm{K}$ and take $Bi=0,2$. Since the frequency ν varies within the limits $\left(50\div 10^{10}\right)\mathrm{Hz}$, and the thickness l — $\left(10^{-6}\div 1\right)\mathrm{m}$, then the parameter γ will vary within the limits $10^{-4}\div 10^4$.

We estimate first the values that do not depend on the parameter γ. Since $\varepsilon_\sigma \cong 4,05 \cdot 10^{-3}$ (is negligibly small compared with a unity), then we will always ignore it. This means that in the formulas (27) and (31) it will be rejected [1].

For this case $\varepsilon \cong 114,75$, $l_T \cong 245 \cdot 10^{-8}\,\text{m}$. Later on we well neglect the quantities less than 1% $(0,01)$ compared with a unit. In this case the dimensionless parameter $\varepsilon_T = (l_T / l)^2 < 10^{-2}$ for the thicknesses is $l > 2,45 \cdot 10^{-6}\,\text{m}$, that is it can be ignored always. In formulas (28) and (31) such dimensionless quantities as $\varepsilon\varepsilon_T^2\gamma^2$ i $(\varepsilon\varepsilon_T\gamma)^2$ are included, their maximum values are, respectively, $9,59 \cdot 10^{-9}$ and $1,10 \cdot 10^{-6}$. So they also can be neglected always.

Further for convenience we will work with dimensionless temperature, displacement and time, which we introduce so:

$$\Theta(0;\tau) = \frac{T(0;\tau) - T_0}{T_m}, \quad \mathrm{v}(0;\tau) = \frac{u(0;\tau)}{u_m}, \quad \tau = \frac{a_T}{l^2} t. \tag{33}$$

One goal of this paper is to study the influence of amplitude of oscillatory component of temperature $A_\Theta(z) = \sqrt{\Theta_2(z)\tilde{\Theta}_2(z)}$ and displacements $A_\mathrm{v}(z) = \sqrt{\mathrm{v}_2(z)\tilde{\mathrm{v}}_2(z)}$ on thermoelastic state of nonferromagnetic conductive bodies compared with their established values — $\overline{\Theta}(z)$, $\overline{\mathrm{v}}(z)$. This effect will be characterized by relations $S_\Theta(z) = A_\Theta(z) / \overline{\Theta}(z)$ and $S_\mathrm{v}(z) = A_\mathrm{v}(z) / \overline{\mathrm{v}}(z)$. The oscillatory components of these values will be considered under the condition:

$$S_\Theta > 10^{-2}, \quad S_\mathrm{v} > 10^{-2}. \tag{34}$$

The Table 1 shows the values $S_\theta(z)$ and $S_\mathrm{v}(z)$ for different values γ, z.

Table 1

γ	0.001	0.01	0.1	1	10	100
$S_\Theta(0)$	0.990	0.97	$4.09 \cdot 10^{-2}$	$5.58 \cdot 10^{-4}$	$7.69 \cdot 10^{-5}$	$7.69 \cdot 10^{-6}$
$S_\Theta(0.25)$	0.990	0.97	$4.09 \cdot 10^{-2}$	$4.55 \cdot 10^{-4}$	$5.91 \cdot 10^{-7}$	$1.71 \cdot 10^{-27}$
$S_\Theta(0.50)$	0.990	0.97	$4.08 \cdot 10^{-2}$	$3.78 \cdot 10^{-4}$	$3.99 \cdot 10^{-9}$	$3.29 \cdot 10^{-49}$
$S_\Theta(0.75)$	0.990	0.97	$4.07 \cdot 10^{-2}$	$3.58 \cdot 10^{-4}$	$2.69 \cdot 10^{-11}$	$6.36 \cdot 10^{-71}$
$S_\Theta(1)$	0.990	0.97	$4.06 \cdot 10^{-2}$	$3.55 \cdot 10^{-4}$	$7.18 \cdot 10^{-13}$	$4.86 \cdot 10^{-92}$
$-S_\mathrm{v}(0)$	0.990	0.97	$4.08 \cdot 10^{-2}$	$4.42 \cdot 10^{-4}$	$3.70 \cdot 10^{-5}$	$4.01 \cdot 10^{-6}$
$-S_\mathrm{v}(0.25)$	0.990	0.97	$4.07 \cdot 10^{-2}$	$5.35 \cdot 10^{-4}$	$4.00 \cdot 10^{-5}$	$4.04 \cdot 10^{-6}$
$-S_\mathrm{v}(0.50)$	0.990	0.97	$4.07 \cdot 10^{-2}$	$5.89 \cdot 10^{-4}$	$4.00 \cdot 10^{-5}$	$4.05 \cdot 10^{-6}$
$-S_\mathrm{v}(0.75)$	0.990	0.97	$4.06 \cdot 10^{-2}$	$6.17 \cdot 10^{-4}$	$4.01 \cdot 10^{-5}$	$4.05 \cdot 10^{-6}$
$-S_\mathrm{v}(1)$	0.990	0.97	$4.06 \cdot 10^{-2}$	$6.25 \cdot 10^{-4}$	$4.01 \cdot 10^{-5}$	$4.05 \cdot 10^{-6}$

Note that $S_{\text{v}}(z) < 0$, since the displacement is $\bar{u}(z) < 0$ through the whole layer thickness.

Analysis of the data presented in the Table 1 and formulas (22), (23) and (27), (28) as well shows that for relatively small γ (in the limiting case $\gamma \to 0$) these relations are equal to a unit (minus unit), regardless of z. When γ increases, they decrease and at infinity vanish. From the definition (33) and conditions (34) it follows that $\gamma \le 0{,}1$ must be considered as a constant component of temperature and displacement, and their oscillating components also. At $\gamma \ge 1$ the values $S_{\theta}(z)$ and $S_{\text{v}}(z)$ are so negligible, that there is no need to explore them.

In the Table 2a the values of the frequencies ν and thicknesses l are presented, which must take into account the oscillating components of temperature and displacements.

Table 2a

ν, Hz	50	10^{2}	10^{3}	10^{4}	10^{5}	10^{6}
l, m	$0.93\cdot10^{-3}$	$6.57\cdot10^{-4}$	$2.08\cdot10^{-4}$	$6.57\cdot10^{-5}$	$2.08\cdot10^{-5}$	$6.57\cdot10^{-6}$

Thus, for a very wide frequency range the condition (34) is suitable only for a very small thickness (thin films).

Note that for frequencies $\nu < 50\,\text{Hz}$ (see Table 2b), which exist, for example, in biological systems [8, 9] the oscillating components of thermoelastic characteristics for thicknesses in the range $l = 1\,\text{mm} \div 1\,\text{m}$ should be considered.

Table 2b

l, m	1	10^{-1}	10^{-2}	10^{-3}
ν, Hz	$4.31\cdot10^{-5}$	$4.31\cdot10^{-3}$	0.43	43.12

In the Figs. 2 and 3 are the graphs of dimensionless temperature $\Theta(0;\tau)$ and displacements $\text{v}(0;\tau)$ versus dimensionless time τ for the same values γ, that are listed in the Table 1, on the upper surface $z = 0$. The solid lines correspond to the temperature $\Theta(0;\tau)$ calculated according to formulas (21)-(24) (displacements $\text{v}(0;\tau)$ calculated according to formulas (26)-(30)), what corresponds to a refined calculation model and the dotted lines correspond to the temperature in the case of the simplified methods when $\bar{\Theta}(0)$ is calculated by the formula (22), $\Theta_2(0;\tau) = 0$, and $\Theta_{*kl}(0;\tau)$ — by the formula (25) (displacement $\bar{\text{v}}(0)$ is calculated by the formula (27), and $\text{v}_2(z) = 0$ and $\text{v}_*(z;\tau) = 0$).

For the parameter $\gamma = 10^{-2}$ (Figs. 2*a* and 3*a*) the temperature and displacements have a clearly oscillating nature. They oscillate relatively the equilibrium position, which is the steady temperature $\bar{\Theta}(0;\tau)$ (displacement $\bar{\text{v}}(0)$). For $\gamma = 10^{-1}$ it is seen that the amplitude of oscillation is clearly reduced. For larger γ (Figs. 2*b*, *c* and 3*b*, *c*) oscillations are practically absent. In these cases you can talk only about microoscillations.

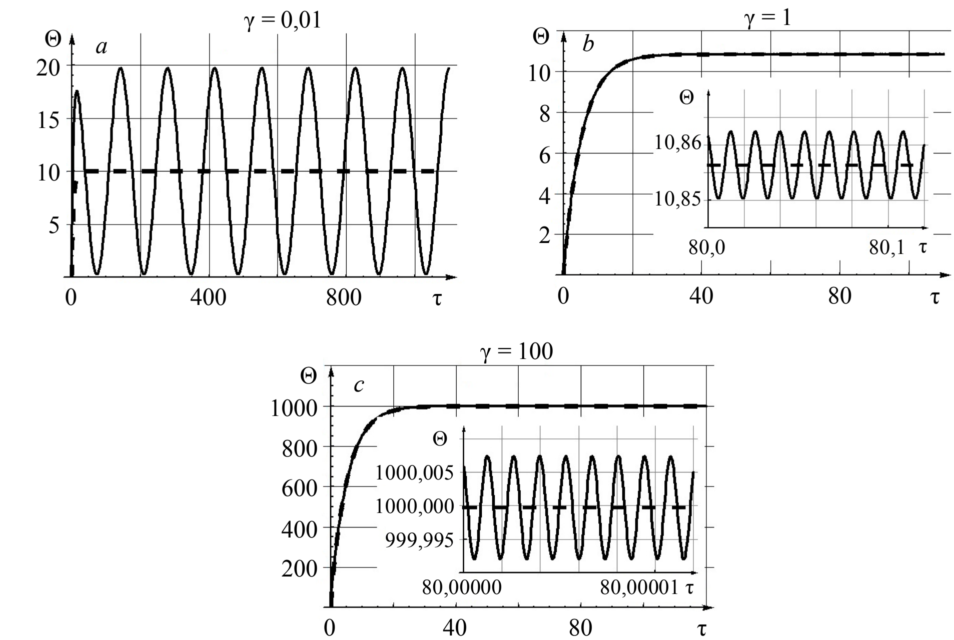

Fig. 2. Time dependence of temperature on the upper surface of the layer

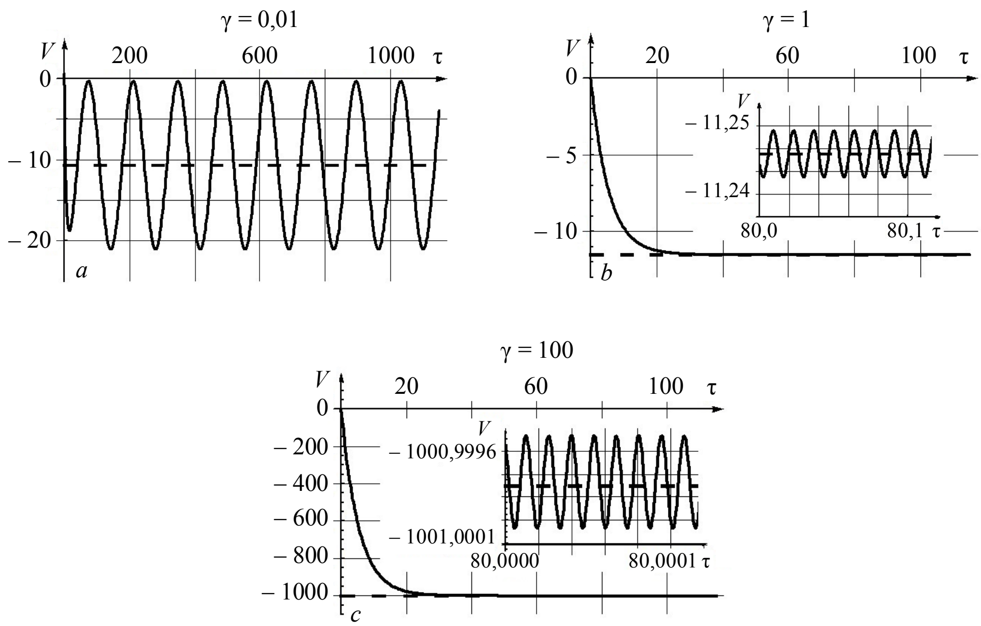

Fig. 3. Time dependence of displacements on the upper surface of the layer

Table 3

γ	0.001	0.01	0.1	1	10	100
A_Θ	10.0	9.6	0.40	0.050	0.008	0.0080
A_v	10.7	10.0	0.45	0.005	0.004	0.0004

The Table 3 presents the values of the amplitudes of oscillations (microoscillations) of temperature A_Θ and displacements A_v for the case shown in Figs. 2 and 3.

Note that for low frequencies ($\gamma < 1$) the temperature and displacements in the steady state mode do not dependent on γ and are equal to ± 10, and for high frequencies ($\gamma > 10$) are proportional to γ.

For all the considered cases achieving the steady thermal mode and mode of displacements occurs at the same time of order $\tau = 20$ and is independent of γ. At the stable mode microoscillations occur with amplitude A_Θ — temperature and A_v — displacement.

Distribution of dimensionless temperature $\bar{\Theta}(z)$ (solid line) in the steady thermal mode and amplitude $A_\Theta(z)$ (dash line) of its oscillating component (Fig. 4) through the coordinate of thickness is quite uniform. There is too weak quadratic dependence which with increasing the parameter initially reduces and then disappears (is linear). For small

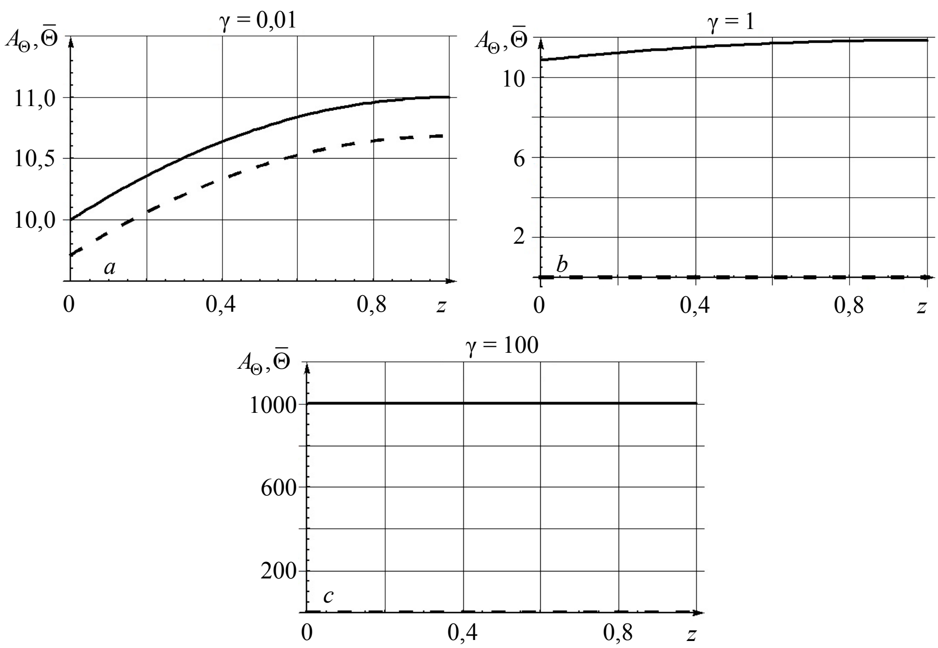

Fig. 4. Distribution of temperature $\bar{\Theta}(z)$ (solid line) in the steady temperature mode and amplitude $A_\Theta(z)$ (dash line) of its oscillatory component through the thickness coordinate

$\gamma\ \left(\gamma \le 10^{-3}\right)$ the amplitude of oscillatory component $A_{\Theta}(z)$ is practically similar to the constant component $\overline{\Theta}(z)$ distribution. At $\gamma \cong 10^{-2}$ it decreases by 2%, and at $\gamma \cong 0{,}1$ the amplitude $A_{\Theta}(z)$ is only 4% of $\overline{\Theta}(z)$. In the case of $\gamma \ge 1$ it is almost negligible.

Note that the difference between the dimensionless stable temperature and the amplitude of its oscillatory component on the upper $z = 0$ and the bottom $z = 1$ surfaces is equal to one and does not depend on the value γ.

In contrast to the temperature, the distribution of displacement in the steady mode $\overline{\mathrm{v}}(z)$ (solid line) and amplitude of its oscillatory component $A_{\mathrm{v}}(z)$ (dash line) have completely uniform character through the thickness coordinate (Fig. 5). On the upper surface $z = 0$ the displacements $\overline{\mathrm{v}}(z)$ adopt minimum and are uniformly increase with increasing z and according to the first boundary condition (9) take zero value on the lower surface $z = 0$.

Similar is the behavior of the amplitude $A_{\mathrm{v}}(z)$ compared to $\overline{\mathrm{v}}(z)$. So for small values $\gamma = 10^{-3}$ and $\gamma = 10^{-2}$ they coincide up to the sign, at $\gamma = 0{,}1$ the difference does not exceed 4% and with further increase of γ the influence of the amplitude $A_{\mathrm{v}}(z)$ is negligibly small.

The graphics for mechanical stresses we will not give as they repeat the temperature distribution up to a factor.

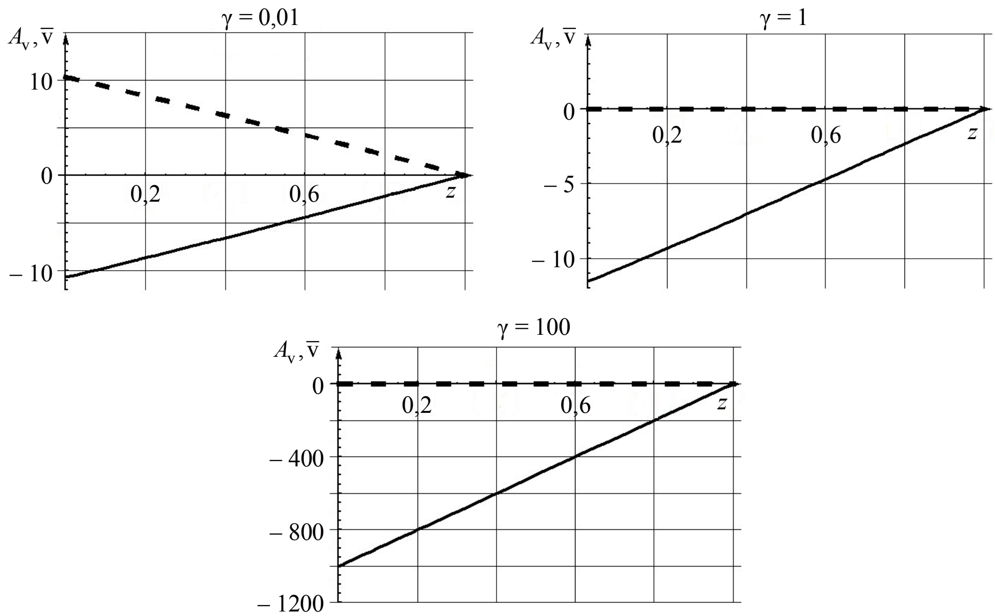

Fig. 5. Distribution of displacement $\overline{\mathrm{v}}(z)$ (solid line) and amplitude of its oscillatory component $A_{\mathrm{v}}(z)$ (dash line) through the coordinate of the layer thickness in the steady thermal mode

Conclusions. Thermoelastic state of nonferromagnetic conductive layer under the action of time-harmonic magnetic field strongly depends on the parameter H_0 and the relative depth of penetration of the magnetic field γ. This value in turn depends on the layer thickness *l*, the frequency of the external magnetic field ν and electrophysical characteristics of the material.

It is shown that the contribution of ponderomotive force in thermoelastic state of nonferromagnetic electrically conducting bodies should always be neglected in comparison with heat releases.

For small γ (up to 1% for $\gamma < 0{,}1$) it is necessary to consider both the constant part component of the temperature and its oscillating component. Then the temperature and displacement are clearly of oscillating nature. In oscillatory components of displacements a typical thermal size is included, which can always be ignored. Those frequency values ν and layer thickness *l* for which it is necessary to consider both constant component of the temperature and its fluctuating component are determined.

Temperature and displacement distribution are practically linear through the thickness coordinate.

References

[1] Термоупругость электропроводных тел / *Я. С. Подстригач, Я. И. Бурак, А. Р. Гачкевич, Л. В. Чернявская*. — Киев: Наук. думка, 1977. — 248 с.
[2] *Коваленко А. Д.* Термоупругость. — Киев: Вища шк., 1975. — 216.
[3] *Солодяк М. Т.* Термодифузійні процеси в електропровідних тілах у зовнішньому періодичному в часі магнетному полі // Фіз.-хім. механіка матеріалів. — 2000. — № 5. — С. 91-98.
[4] *Диткин В. А., Прудников В. А.* Справочник по операционному исчислению. — Москва: Высшая школа, 1965. — 465 с.
[5] Таблицы физических величин: Справ.; под ред. *И. К. Кикоина.* — Москва: Атомиздат, 1976. — 1008 с.
[6] *Кей Дж., Леби Т.* Таблицы физических и химических постоянных; под ред. *К. П. Яковлева..* — Москва: Физматгиз, 1962. — 247 с.
[7] *Лившиц Б. Г., Крапошин В. С., Липецкий Я. Л.* Физические свойства металлов и сплавов. — Москва: Металлургия, 1980. — 320 с.
[8] *Жиганин А. Д.* Техногенные физические поля // Природа. — 1993. — № 2. — С. 15-23.
[9] *Посудін Ю. І.* Фізика і біофізика навколишнього середовища. — Київ: Світ, 2000. — 304 с.

Про знаходження термопружного стану електропровідного неферомагнітного шару в гармонічному за часом магнітному полі

Олександр Гачкевич, Михайло Солодяк, Микола Махоркін

Запропоновано розрахункову модель знаходження термопружного стану електропровідного неферомагнітного шару в гармонічному за часом магнітному полі. У цій моделі враховуються коливні складники тепловиділень і пондеромоторної сили, а також динамічні члени задачі термопружності. Визначено умови, за яких необхідно користуватися уточненою розрахунковою моделлю.

О нахождение термоупругого состояния электропроводного неферромагнитного слоя в гармоническом во времени магнитном поле

Александр Гачкевич, Михаил Солодяк, Николай Махоркин

Предложено расчетную модель нахождения термоупругого состояния электропроводного неферромагнитного слоя в гармоническом во времени магнитном поле. В данной модели учитываются колебательные составляющие тепловыделений и пондеромоторной силы, а также динамические члены задачи термоупругости. Определены условия, при которых необходимо использовать уточненную расчетную модель.

Отримано 25.03.15