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On the statement of the problem of electrodynamics for nonferromagnetic layer subjected to bilateral induction heating

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A problem on determining the electromagnetic field parameters in an electroconductive nonferromagnetic layer subjected to bilateral induction heating is formulated. The coupled problem for three areas was reduced to the boundary value problem for the layer, a solution of which is obtained for the given time-harmonic inductor current. The energy and power factors of the action of the field on the body are written down.

Keywords: electromagnetic field, nonferromagnetic bodies, induction heating, Joule heat, ponderomotive force

Introduction. In many branches of industry for heat treatment of parts with complex shape or structural elements, the non-contact method of heating the products made of electroconductive materials by electric currents, induced by alternating magnetic field of inductor, is often used. Various issues of industrial application of induction heating as well as calculation and design of inductive plants were described in full in [1-7].

Induction heating is carried out as follows. Electroconductive (made of metal or graphite) billet (preferably a cylindrical rod) is placed in inductor in the form of a coil with current (solenoid) (Fig. 1). Through its wires (most often copper) using a special generator the powerful currents of certain frequencies (from ten Hz to several MHz) are conducted, whereupon around the inductor electromagnetic field (EMF) arises. If the length of the solenoid is much longer than its diameter, the field inside the coil is practically homogeneous and directed along the main axis. EMF induces in the billet eddy currents, which warm it up under the action of Joule heat.

The system "inductor-billet" is a transformer without a core, in which the inductor is the primary winding and the billet is a short closed secondary winding. The magnetic flux between the windings closes by air. At high frequency the eddy currents are displaced by a magnetic field created by them in thin surface layers of the billet, whereupon their density increases dramatically and the billet warms up. The layers of material located below the surfaces warm up due to thermal conductivity.

The structural elements are very often in the form of plates which are modeled

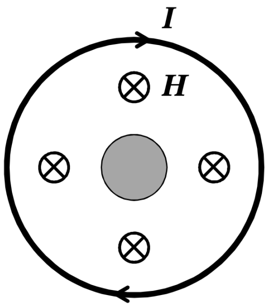

Fig. 1

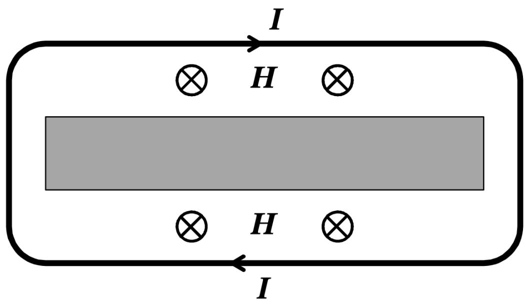

Fig. 2

by the layers. Such issues are investigated in [8-10], where electroconductive bodies subjected to unilateral induction heating are considered. Fig. 1 illustrates the presence of a magnetic field everywhere around the lateral surface. If imaginary to deform (to stretch) the cross-section of the rod, turning it into the plate, the magnetic field will be on both sides of the plate (Fig. 2). Therefore it would be logical to consider a bilateral induction heating. Further the plate we shall model as a layer. So, the problem of electrodynamics for a layer subjected to bilateral induction heating is formulated.

1. Formulation of the problem

The electroconductive layer of thickness l, referred to the dimensionless rectangular coordinate system (x, y, z) (Fig. 3), is considered. The layer is contained in inductor, that is modeled by two infinitely thin current-carrying planes parallel to the layer's surfaces, which are located at the distance z_0 from the layer's surfaces. The index $k = (1, 2)$ denotes the corresponding half-spaces over and under the layer. Their physical characteristics are accepted in vacuum approximation. The electric current densities in the upper and lower plates of inductor have non-zero x-components only, that are defined in this way:

$$j_x^{(1)}(z,t) = j_0 j(t)\delta(z - z_0 - 1), \qquad j_x^{(2)}(z,t) = -j_0 j(t)\delta(z + z_0). \tag{1}$$

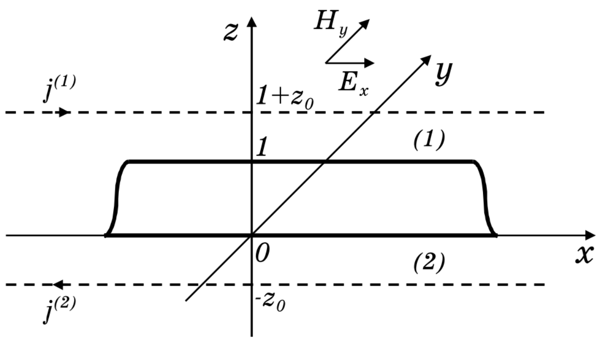

Fig. 3

Here j_0 is the amplitude of external current density; $j(t)$ is the function which describes its change with time t; $\delta(z)$ is Dirac delta-function [11]. Such currents pattern induces nonzero y-components of magnetic field strength in the body $H \equiv H_y(z,t)$, as well as in vacuum $H^{(k)} \equiv H_y^{(k)}(z,t)$, and also x-components of elecric field strength $E \equiv E_x(z,t)$ in the body and $E^{(k)} \equiv E_x^{(k)}(z,t)$ in vacuum, respectively. In future the subscripts « x » and « y » we shall omit.

For material medium the dependencies between the conduction current density j and the electric field strength E (Ohm's law), and also between the magnetic flux density B and the elecric field induction D and corresponding magnetic H and elecric E field strengths are taken in the form

$$j = \lambda E, \qquad B = \mu_0 \mu H, \qquad D = \varepsilon_0 \varepsilon E. \tag{2}$$

Here λ is electric conductivity; μ and ε are relative magnetic and dielectric permittivities of medium; μ_0 and ε_0 are magnetic and dielectric constants.

For vacuum in the formulas (2) it is required to put

$$\lambda = 0, \qquad \mu = 1, \qquad \varepsilon = 1. \tag{3}$$

EMF is considered as external action, which is manifested in the medium by the energy and power impact factors. The problem about finding the magnetic and electric field strengths in the layer, subjected to EMF, created by inductor and also the impact factors of this field (Joule heat and ponderomotive force) is formulated.

From the system of equations of electrodynamics (Maxwell's equations [12-14]) for magnetic and electric field strengths in the layer we will have:

$$-\frac{1}{l}\frac{\partial H}{\partial z} = \varepsilon_0 \varepsilon \frac{\partial E}{\partial t} + \lambda E, \qquad \frac{1}{l}\frac{\partial E}{\partial z} = -\mu_0 \mu \frac{\partial H}{\partial t}. \tag{4}$$

Should be noted, that the system of equations (4) can be reduced to the equation for magnetic or electric field strength

$$\frac{1}{l^2}\frac{\partial^2 H}{\partial z^2} = \frac{\varepsilon\mu}{c_0^2}\frac{\partial^2 H}{\partial t^2} + \lambda \mu_0 \mu \frac{\partial H}{\partial t}, \qquad \frac{1}{l^2}\frac{\partial^2 E}{\partial z^2} = \frac{\varepsilon\mu}{c_0^2}\frac{\partial^2 E}{\partial t^2} + \lambda \mu_0 \mu \frac{\partial E}{\partial t}, \tag{5}$$

where $c_0 = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}}$ is the velocity of electromagnetic wave propagation in vacuum.

The system of equations of the type (4) for vacuum (for the half-spaces over and under the layer, respectively) will read:

$$-\frac{1}{l}\frac{\partial H^{(k)}}{\partial z} = \varepsilon_0 \frac{\partial E^{(k)}}{\partial t} + j^{(k)}, \qquad \frac{1}{l}\frac{\partial E^{(k)}}{\partial z} = -\mu_0 \frac{\partial H^{(k)}}{\partial t}. \tag{6}$$

It can also be reduced to the equation on the $H^{(k)}$ or $E^{(k)}$:

$$\frac{1}{l^2}\frac{\partial^2 H^{(k)}}{\partial z^2}=\frac{1}{c_0^2}\frac{\partial^2 H^{(k)}}{\partial t^2}-\frac{1}{l}\frac{\partial j^{(k)}}{\partial z}, \qquad \frac{1}{l^2}\frac{\partial^2 E^{(k)}}{\partial z^2}=\frac{1}{c_0^2}\frac{\partial^2 E^{(k)}}{\partial t^2}+\mu_0\frac{\partial j^{(k)}}{\partial t}. \qquad (7)$$

We shall assume, that at the initial moment of time $t=0$ EMF in the body and in vacuum, and also the external electric currents are absent. Then the initial conditions will be written in the form

$$j^{(k)}(z,0)=0,\quad E^{(k)}(z,0)=0,\quad H^{(k)}(z,0)=0,\quad E(z,0)=0,\quad H(z,0)=0. \qquad (8)$$

At the body-vacuum separation boundary the contact conditions are satisfied [10, 15-17]:

$$E(1,t)=E^{(1)}(1,t),\qquad H(1,t)=H^{(1)}(1,t);$$

$$E(0,t)=E^{(2)}(0,t),\qquad H(0,t)=H^{(2)}(0,t). \qquad (9)$$

Herewith the surface charges and currents are not considered [10].

The conditions of radiation at infinity in the areas of vacuum will be [10, 17]:

$$\lim_{z\to\pm\infty}\left[\frac{1}{l}\frac{\partial H^{(k)}(z,t)}{\partial z}+\frac{1}{c_0}\frac{\partial H^{(k)}(z,t)}{\partial t}\right]=0,$$

$$\lim_{z\to\pm\infty}\left[\frac{1}{l}\frac{\partial E^{(k)}(z,t)}{\partial z}+\frac{1}{c_0}\frac{\partial E^{(k)}(z,t)}{\partial t}\right]=0. \qquad (10)$$

Should be noted, that the sign «$+$» under the symbol «lim» refers to the upper ($k=1$) and the sign «$-$» – to the lower half-space ($k=2$).

The problem (1)-(10) allows to determine uniquely the EMF parameters both in the body and in the areas of vacuum depending on the particular form of function $j(t)$. This problem can be reduced to a boundary-value problem only for a body with equivalent to the original boundary conditions on its surfaces. We will perform this procedure for the time-periodic function $j(t)=\sin\omega t$, where $\omega=2\pi\nu$, ν is frequency.

2. Temporal presentations of solutions

We shall assume, that in inductor flows a time-harmonic electric current. In this case, the formulas (1) will have such form:

$$j^{(1)}(z,t)=\frac{j_0}{2i}\left(e^{i\omega t}-e^{-i\omega t}\right)\delta(z-z_0-1),$$

$$j^{(2)}(z,t)=-\frac{j_0}{2i}\left(e^{i\omega t}-e^{-i\omega t}\right)\delta(z+z_0), \qquad (11)$$

where i is imaginary unit. The magnetic and electric field strengths in the layer as well as in the areas of vacuum will be presented in a similar manner:

$$H(z,t)=\frac{j_0 l}{2}\left[h(z)e^{i\omega t}+\tilde{h}(z)e^{-i\omega t}\right],\quad E(z,t)=\frac{j_0}{2}\left[\frac{e(z)}{\lambda+i\varepsilon_0\varepsilon\omega}e^{i\omega t}+\frac{\tilde{e}(z)}{\lambda-i\varepsilon_0\varepsilon\omega}e^{-i\omega t}\right],$$

$$H^{(k)}(z,t)=\frac{j_0 l}{2}\left[h^{(k)}(z)e^{i\omega t}+\tilde{h}^{(k)}(z)e^{-i\omega t}\right],$$

$$E^{(k)}(z,t)=\frac{j_0}{2i\varepsilon_0\omega}\left[e^{(k)}(z)e^{i\omega t}-\tilde{e}^{(k)}(z)e^{-i\omega t}\right]. \tag{12}$$

Here the tilde «$\sim$» over the quantity means its complex conjugate value. The field's normalization in the expressions (12) was selected so that their harmonics would be dimensionless.

Substituting the representations (12) into relations (4), for harmonics of the magnetic and electric field strengths we will obtain such ordinary differential equations:

$$-\frac{dh(z)}{dz}=e(z),\qquad \frac{de(z)}{dz}=-i\mu_0\mu(\lambda+i\varepsilon_0\varepsilon\omega)\omega l^2 h, \tag{13}$$

which can be reduced to equations of the second order for functions $h(z)$ and $e(z)$:

$$\frac{d^2h(z)}{dz^2}=k^2h(z),\qquad \frac{d^2e(z)}{dz^2}=k^2e(z), \tag{14}$$

where $k^2=2i\gamma^2-\varepsilon\mu k_0^2$, $k_0^2=(\omega l/c_0)^2$, k and k_0 are dimensionless wave electromagnetic numbers, concerning the medium and vacuum, respectively; $\gamma^2=\frac{1}{2}\lambda\mu_0\mu\omega l^2$, $\gamma=\frac{1}{\delta}$, δ is the relative penetration depth of EMF in the medium.

Similarly, from the system of equations (6) for vacuum, considering the representation (12), we will obtain such system of differential equations for harmonics of the magnetic and electric field strengths:

$$-\frac{dh^{(1)}(z)}{dz}=e^{(1)}(z)-i\delta(z-z_0-1),\qquad \frac{de^{(1)}(z)}{dz}=k_0^2h^{(1)}(z),$$

$$-\frac{dh^{(2)}(z)}{dz}=e^{(2)}(z)+i\delta(z+z_0),\qquad \frac{de^{(2)}(z)}{dz}=k_0^2h^{(2)}(z), \tag{15}$$

which we will reduce to the uncoupled second order system of equations for functions $h^{(k)}(z)$ and $e^{(k)}(z)$:

$$\frac{d^2h^{(1)}(z)}{dz^2}+k_0^2h^{(1)}(z)=i\frac{d\delta(z-z_0-1)}{dz},\ \frac{d^2e^{(1)}(z)}{dz^2}+k_0^2e^{(1)}(z)=ik_0^2\delta(z-z_0-1),$$
$$\frac{d^2h^{(2)}(z)}{dz^2}+k_0^2h^{(2)}(z)=-i\frac{d\delta(z+z_0)}{dz},\ \frac{d^2e^{(2)}(z)}{dz^2}+k_0^2e^{(2)}(z)=-ik_0^2\delta(z+z_0). \quad (16)$$

The contact conditions (9) will be such:

$$h(1)=h^{(1)}(1),\qquad e(1)=\frac{\lambda+i\varepsilon_0\varepsilon\omega}{i\varepsilon_0\omega}e^{(1)}(1),$$
$$h(0)=h^{(2)}(0),\qquad e(0)=\frac{\lambda+i\varepsilon_0\varepsilon\omega}{i\varepsilon_0\omega}e^{(2)}(0), \quad (17)$$

and the conditions of radiation at infinity (10) will look as:

$$\lim_{z\to\infty}\left[\frac{dh^{(1)}(z)}{dz}+ik_0h^{(1)}(z)\right]=0,\qquad \lim_{z\to\infty}\left[\frac{de^{(1)}(z)}{dz}+ik_0e^{(1)}(z)\right]=0;$$
$$\lim_{z\to\infty}\left[-\frac{dh^{(2)}(z)}{dz}+ik_0h^{(2)}(z)\right]=0,\qquad \lim_{z\to\infty}\left[-\frac{de^{(2)}(z)}{dz}+ik_0e^{(2)}(z)\right]=0. \quad (18)$$

Using the equations of electrodynamics for the body (13) and vacuum (15), the contact conditions (17) can be expressed in the terms of the magnetic field strength only

$$h(1)=h^{(1)}(1),\qquad \frac{dh(1)}{dz}=\frac{\lambda+i\varepsilon_0\varepsilon\omega}{i\varepsilon_0\omega}\frac{dh^{(1)}(1)}{dz};$$
$$h(0)=h^{(2)}(0),\qquad \frac{dh(0)}{dz}=\frac{\lambda+i\varepsilon_0\varepsilon\omega}{i\varepsilon_0\omega}\frac{dh^{(2)}(0)}{dz}; \quad (19)$$

or in the terms of the electric field strength only

$$e(1)=\frac{\lambda+i\varepsilon_0\varepsilon\omega}{i\varepsilon_0\omega}e^{(1)}(1),\qquad \frac{de(1)}{dz}=\frac{\mu(\lambda+i\varepsilon_0\varepsilon\omega)}{i\varepsilon_0\omega}\frac{de^{(1)}(1)}{dz};$$
$$e(0)=\frac{\lambda+i\varepsilon_0\varepsilon\omega}{i\varepsilon_0\omega}e^{(2)}(0),\qquad \frac{de(0)}{dz}=\frac{\mu(\lambda+i\varepsilon_0\varepsilon\omega)}{i\varepsilon_0\omega}\frac{de^{(2)}(0)}{dz}. \quad (20)$$

Thus, the problem of electrodynamics (11)-(18) has been reduced to the determination of EMF's parameters in the body from the equations (14) and in vacuum from the equations (16) taking into account the contact conditions (17) and the conditions of radiation at infinity (18). We shall reduce it to the boundary-value problem only for a body with equivalent to the original boundary conditions on its surfaces. To obtain such conditions we shall find first the solutions of the system of equations (16), using the method of variation of constants. Herewith, as in [18], it is possible to proceed from the known properties of Dirac delta-function [19, 20]:

$$\int_a^b f(x)\delta\left(x-x_0\right)dx=\begin{cases}0, & x_0\notin[a;b];\\ \frac{1}{2}f\left(x_0\right), & x_0=a\vee x_0=b;\\ f\left(x_0\right), & x_0\in[a;b];\end{cases} \quad (21)$$

$$\int_a^x f\left(\xi\right)\delta'\left(\xi-x_0\right)d\xi=-f'\left(x_0\right)S_+\left(x-x_0\right), \quad x_0>a; \quad (22)$$

$$S_+\left(x-x_0\right)=\begin{cases}0, & x<x_0;\\ 1, & x\ge x_0;\end{cases} \qquad S'_+\left(x-x_0\right)=\delta\left(x-x_0\right). \quad (23)$$

In this paper another approach was used – the approximation of Dirac delta-function by continuously differentiable function [11]:

$$\delta\left(x\right)=\lim_{\alpha\to\infty}\frac{\alpha}{\pi\left(1+\alpha^2x^2\right)}. \quad (24)$$

Then the general solutions of system (16) will be such:

$$h^{(1)}\left(z\right)=\left[A_1+\frac{i}{2\pi}e^{-ik_0\left(z_0+1\right)}\lim_{\alpha\to\infty}\operatorname{arctg}\alpha\left(z-z_0-1\right)\right]e^{ik_0z}+$$
$$+\left[A_2+\frac{i}{2\pi}e^{ik_0\left(z_0+1\right)}\lim_{\alpha\to\infty}\operatorname{arctg}\alpha\left(z-z_0-1\right)\right]e^{-ik_0z},$$
$$e^{(1)}\left(z\right)=\left[B_1+\frac{k_0}{2\pi}e^{-ik_0\left(z_0+1\right)}\lim_{\alpha\to\infty}\operatorname{arctg}\alpha\left(z-z_0-1\right)\right]e^{ik_0z}+$$
$$+\left[B_2-\frac{k_0}{2\pi}e^{ik_0\left(z_0+1\right)}\lim_{\alpha\to\infty}\operatorname{arctg}\alpha\left(z-z_0-1\right)\right]e^{-ik_0z};$$
$$h^{(2)}\left(z\right)=\left[C_1-\frac{i}{2\pi}e^{ik_0z_0}\lim_{\alpha\to\infty}\operatorname{arctg}\alpha\left(z+z_0\right)\right]e^{ik_0z}+$$
$$+\left[C_2-\frac{i}{2\pi}e^{-ik_0z_0}\lim_{\alpha\to\infty}\operatorname{arctg}\alpha\left(z+z_0\right)\right]e^{-ik_0z},$$
$$e^{(2)}\left(z\right)=\left[D_1-\frac{k_0}{2\pi}e^{ik_0z_0}\lim_{\alpha\to\infty}\operatorname{arctg}\alpha\left(z+z_0\right)\right]e^{ik_0z}+$$
$$+\left[D_2+\frac{k_0}{2\pi}e^{-ik_0z_0}\lim_{\alpha\to\infty}\operatorname{arctg}\alpha\left(z+z_0\right)\right]e^{-ik_0z}. \quad (25)$$

Substituting the expressions (25) in the corresponding conditions of radiation at infinity (18), we shall find the constants of integration:

$$A_1=-\frac{i}{4}e^{-ik_0\left(z_0+1\right)}, \qquad B_1=-\frac{k_0}{4}e^{-ik_0\left(z_0+1\right)};$$
$$C_2=-\frac{i}{4}e^{-ik_0z_0}, \qquad D_2=\frac{k_0}{4}e^{-ik_0z_0}. \quad (26)$$

Taking into account the expressions (26) the formulas (25) will read:

$$h^{(1)}(z) = A_2 e^{-ik_0 z} - \frac{i}{4} e^{ik_0(z-z_0-1)} + \frac{i}{\pi} \cos k_0 (z - z_0 - 1) \lim_{\alpha\to\infty} \operatorname{arctg} \alpha (z - z_0 - 1),$$

$$e^{(1)}(z) = B_2 e^{-ik_0 z} - \frac{k_0}{4} e^{ik_0(z-z_0-1)} + \frac{ik_0}{\pi} \sin k_0 (z - z_0 - 1) \lim_{\alpha\to\infty} \operatorname{arctg} \alpha (z - z_0 - 1);$$

$$h^{(2)}(z) = C_1 e^{ik_0 z} - \frac{i}{4} e^{-ik_0(z+z_0)} - \frac{i}{\pi} \cos k_0 (z + z_0) \lim_{\alpha\to\infty} \operatorname{arctg} \alpha (z + z_0),$$

$$e^{(2)}(z) = D_1 e^{ik_0 z} + \frac{k_0}{4} e^{-ik_0 z_0} - \frac{ik_0}{\pi} \sin k_0 (z + z_0) \lim_{\alpha\to\infty} \operatorname{arctg} \alpha (z + z_0). \quad (27)$$

The values of functions (27) and their derivatives we find on the surfaces $z = 0$ and $z = 1$ and substitute in the contact conditions (19) and (20). We obtain the system of eight algebraic equations, from which we exclude the constants of integration. Herewith the contact conditions (19) and (20) we reduce to the boundary conditions on the layer's surfaces for magnetic

$$h(1) + \frac{\kappa}{k}\frac{dh(1)}{dz} = -ie^{-ik_0 z_0}, \qquad h(0) - \frac{\kappa}{k}\frac{dh(0)}{dz} = -ie^{-ik_0 z_0} \quad (28)$$

or electric field strength

$$e(1) - \frac{i}{\mu k_0}\frac{de(1)}{dz} = \frac{ik}{\kappa} e^{-ik_0 z_0}, \qquad e(0) + \frac{i}{\mu k_0}\frac{de(0)}{dz} = -\frac{ik}{\kappa} e^{-ik_0 z_0}. \quad (29)$$

Here $\kappa \equiv \frac{k l_\varepsilon \lambda}{(\lambda + i\varepsilon_0 \varepsilon \omega) l}$, $l_\varepsilon \equiv \frac{1}{\lambda}\sqrt{\frac{\varepsilon_0}{\mu_0}}$ is characteristic size.

Should be noted, that the boundary conditions (29) for electric field strength can be obtained from the conditions (28), using herewith the equation (13).

Thus, the coupled problem for three areas was reduced to the boundary-value problem for a layer only, which is described by equations (14) and boundary conditions (28) and (29).

Knowing the parameters of magnetic field in the area of layer, we shall write the impact factors of EMF – the power of Joule heat Q and the density of ponderomotive force F [21, 22]:

$$Q = \lambda E^2, \qquad F = \lambda \mu_0 \mu E H. \quad (30)$$

Substituting the representation (12) for magnetic and electric field strengths in the formulas (30), we shall obtain such temporal representation for the impact factors of EMF on the body:

$$\psi(z,t) = \bar{\psi}(z) + \psi_2(z) e^{2i\omega t} + \tilde{\psi}_2(z) e^{-2i\omega t}, \quad (31)$$

where $\psi \equiv \{Q, F\}$; $\bar{\psi}(z)$ is the quantity, averaged over the period of the EMF's oscillations; $\psi_2(z)$ is its second harmonic.

We shall provide the specific expressions for the Joule heat and the ponderomotive force, written in the terms of harmonics of magnetic and electric field strengths:

– *the averaged components*:

$$\bar{Q}(z)=\frac{\lambda j_0^2}{2\left(\lambda^2+\varepsilon_0^2\varepsilon^2\omega^2\right)}e(z)\tilde{e}(z),$$

$$\bar{F}(z)=\frac{\mu_0\mu\lambda j_0^2 l}{4\left(\lambda^2+\varepsilon_0^2\varepsilon^2\omega^2\right)}\left[\left(\lambda+i\varepsilon_0\varepsilon\omega\right)h(z)\tilde{e}(z)+\left(\lambda-i\varepsilon_0\varepsilon\omega\right)\tilde{h}(z)e(z)\right]; \qquad (32)$$

– *the harmonic components*:

$$Q_2(z)=\frac{\lambda j_0^2}{4\left(\lambda+i\varepsilon_0\varepsilon\omega\right)^2}e^2(z), \qquad F_2(z)=\frac{\mu_0\mu\lambda j_0^2 l}{4\left(\lambda+i\varepsilon_0\varepsilon\omega\right)}h(z)e(z). \qquad (33)$$

Conclusions. In this paper the expedience of formulation the problem about the bilateral induction heating of nonferromagnetic electroconductive bodies was substantiated. As an example, the problem of determining the characteristics of EMF and its impact factors in a nonferromagnetic electroconductive layer was considered. The layer is contained in inductor, that is modeled by two infinitely thin current-carrying planes parallel to the layer's surfaces with a given density of time-sinusoidal electric current. The equations of electrodynamics to determine the magnetic and electric field strengths in the area of layer and in the corresponding half-spaces over and under the layer, and also the initial, contact (at the interface of media) conditions and the conditions of radiation at infinity were written down. The coupled problem for three areas was reduced to the boundary-value problem for a layer only with the equivalent to the original boundary conditions on its surfaces. The results of the work will be used for finding the thermoelastic state's parameters of nonferromagnetic layer subjected to bilateral induction heating.

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Про постановку задачі електродинаміки для неферомагнітного шару за двостороннього індукційного нагрівання

Олександр Гачкевич, Михайло Солодяк, Роман Івасько

Зроблено постановку задачі про визначення параметрів електромагнітного поля в електропровідному неферомагнітному шарі за двостороннього індукційного нагрівання. Зв'язану задачу для трьох областей зведено до крайової задачі для шару, розв'язок якої отримано для заданого гармонічного за часом струму індуктора. Записано енергетичні та силові чинники дії поля на тіло.

О постановке задачи электродинамики для неферромагнитного слоя при двустороннем индукционном нагреве

Александр Гачкевич, Михаил Солодяк, Роман Ивасько

Осуществлена постановка задачи об определении параметров электромагнитного поля в электропроводном неферромагнитном слое при двустороннем индукционном нагреве. Связанная задача для трех областей сведена к краевой задаче для слоя, решение которой получено для заданного гармонического во времени тока индуктора. Записаны энергетические и силовые факторы воздействия поля на тело.

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