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Thermal instability and failure prediction of structural elements with transversely isotropic nanocomposite material

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This paper is dedicated to the failure prediction for nanocomposite structural elements due to thermal instability under cyclic loading. It is based on the model of monoharmonic approximation and concept of complex moduli. The temperature and amplitude dependent complex moduli is used for the investigation of the dissipative heating influence on mechanical stability of polymeric nanocomposite rod subjected to both static and monoharmonic loading. Also, the influence of amplitude of the excitation loading and the volume fraction of nanofibers on thermal stability of the polymeric nanocomposite rod is studied.

Keywords: complex moduli, polymeric nanocomposite, thermal stability, dissipative heating.

Introduction. In recent years, polymeric nanocomposites have been widely used as structural elements in different areas of engineering. The levels of vibro-heating (or dissipative heating) can be high enough the critical thermal states to occur that lead to thermal instability effects under cyclic (monoharmonic) loading [1, 2, 3]. If critical values of some parameters characterizing the behavior of the body are exceeded, there is a significant growth of temperature with time. This growth of temperature due to dissipation of mechanical energy causes the decrease in stiffness. As a result, the strength of materials is significantly reduced [1]. It is the main mechanism of thermal failure for plastics and polymeric composites [3]. In this paper, an approach to investigation of the dissipative heating influence on mechanical and thermal stability of polymeric nanocomposite element based on temperature and amplitude dependent complex moduli is proposed.

1. Simulation of nanocomposite response under monoharmonic loading

The objective of this section is to present a model for description of the macroscopically inelastic behavior of transversely isotropic nanocomposite systems. This material consists of a macroscopically isotropic polymeric matrix filled with unidirectionally aligned nanofibers. The complete set of the constitutive equations for nonlinear polymeric materials and the procedures to evaluate the storage and loss moduli for nanocomposite constituents under monoharmonic deformation based on the concept of complex moduli were developed and published in previous papers of authors [4, 5, 6] in whole details.

2. Problem statement and simulation technique

It is well known, the most interesting effect under quasi-static deformation is thermal instability for polymer and nanocomposite bodies, which often leads to thermal failure of the elements [1, 3]. In this paper, the particular problem of the vibro-heating of a nanocomposite rod with a circular cross section is under consideration. The length of the rod and the cross sectional radius are L and R, respectively. The nanocomposite rod is subjected to both static compressive and low amplitude harmonic loading, $P(t)=P_0+P_1 cos\omega t$, applied at the free end. It is assumed that excitation frequency, ω, is small enough to prevent any parametric or sub- as well as superharmonic resonances to occur. For the loading forces and amplitudes, the inequality $P_1<<P_0<P_{cr}$ holds where P_{cr} is the Eulerian critical load so that we can apply the quasi static theory of viscoelasticity. The relation between stress and strain in the viscoelastic rod is of the well-known form: $\sigma(t)=\sigma_0+\sigma_1 cos\omega t,\quad \varepsilon(t)=\sigma_0 J^\infty+\sigma_1 Re[\tilde{J}ei\omega t],$ where J^∞ is equilibrium creep compliance and $\tilde{J}=J'-iJ''$ is complex creep compliance; σ_0 and σ_1 are magnitudes of the steady compressive and harmonic stress, respectively. The period-averaged energy dissipation rate, $\langle D'\rangle$, (see Eq. (1)) is used as a heat source in the transient heat conduction equation for one dimension problem to predict the temperature evolution over the viscoelastic rod. Therefore, the equation takes the form Eq. (1). Also, the temperature is considered to have uniform distribution along the length of the rod.

$$\frac{d\theta^*}{dt}=-\frac{\alpha_t S}{C_V V}\theta^*+\frac{\alpha_t S}{C_V V}+\frac{\omega}{2C_V\theta_0}J''\sigma_1^2,\quad \langle D'\rangle=\frac{\omega}{2\pi}\int_t^{t+\frac{2\pi}{\omega}}\sigma(\xi)\dot{\varepsilon}(\xi)d\xi=\frac{\omega}{2}J''\sigma_1^2, \tag{1}$$

where $\theta^*=\theta/\theta_0$ is normalized temperature. Coefficients C_V and α_t are specific heat capacity for constant volume and heat transfer coefficient through the rod surface, respectively; S, V and θ_0 are the lateral surface area, volume of the rod and temperature of surrounding media, respectively. It is important to notice that to calculate the loss creep compliance moduli, J'', by using values of complex moduli at different conditions of cyclic loading which were obtained with the use of the procedures presented in [5], approximate equation $J''=J''_0\theta^{*\beta}$ can be used where J''_0 shows initial value of the loss creep compliance under harmonic loading and β is constant of materials. Indeed, these constants can be calculated with making use of data obtained in the frame of simplified monoharmonic approach described in [5]. The loss creep compliance modulus depends on the value of harmonic loading amplitude, P_1, and temperature caused by vibro-heating, while J''_0 depends on temperature. Eq. (1) can be rewritten in the form

$$\frac{d\theta^{*}}{d\tau} = -\theta^{*} + 1 + \mu\theta^{*\beta}, \quad \mu = \frac{\omega}{2C_V\theta_0} J''_0\sigma_1^2, \quad \lambda = \frac{\alpha_t S}{C_V V}, \tag{2}$$

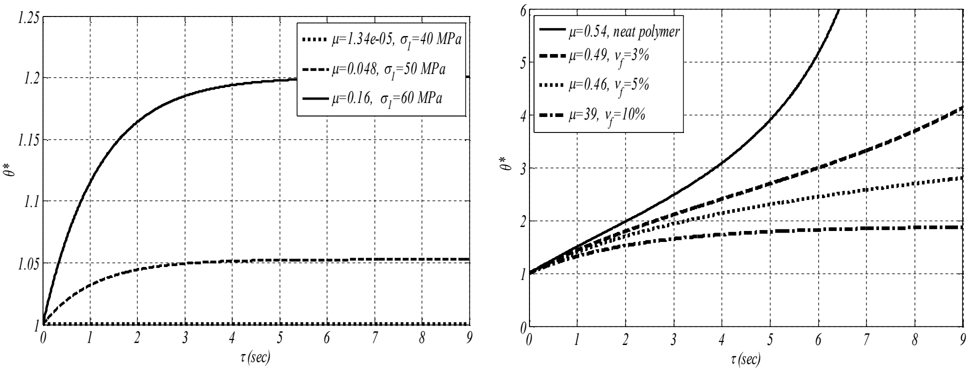
where μ and λ are loading parameter and time constant, respectively. As it follows from the Eq. (2), the stationary heat state can be presented as $\theta^{*} = 1 + \mu\theta^{*\beta}$. As mentioned above, the loss creep compliance modulus, J'', for most materials increases with temperature. If the loss creep compliance increases faster than according to the linear law, there exists a critical load parameter, μ^{*}, so that for $\mu > \mu^{*}$ there isn't a solution of equation $\theta^{*} = 1 + \mu\theta^{*\beta}$. Let us investigate the influence of the dissipative heating on the mechanical stability of viscoelastic rod. The Eulerian approach can be used to find the critical force, P_{cr}. Taking into account of temperature field uniformity and inequality $P_1 \ll P_0$, differential equation for rod axis and Eulerian critical load relation can be written as:

$$\frac{d^4W}{dz^4} + \xi_0^2\frac{d^2W}{dz^2} = 0, \ \xi_0 = \left(\frac{P_0 J^{\infty}}{I}\right)^{1/2}, \quad P_{cr} = \frac{\pi^2 I}{J^{\infty}(KL)} = \frac{\chi}{J^{\infty}}, \tag{3}$$

where K and L are effective length factor of the column and length of the rod, respectively; W and χ are deflection and constant which depends on the rod shape and end fixing conditions. For the most materials, J^{∞}, increases with the temperature growth. For certain critical temperature, θ^{*}_{cr}, Eulerian critical load, P_{cr}, can reach the value of applied compressive load, P_0, that initially was smaller than P_{cr}. The dependence of the equilibrium creep compliance moduli, J^{∞}, on temperature is formulated approximately by the expression $J^{\infty} = J_0^{\infty}\left(\theta^{*}\right)^{\alpha}$, where α is constant of material and J_0^{∞} denotes the initial value of equilibrium creep compliance under static loading. The normalized Eulerian critical stress, σ^{*}_{cr}, is defined by the expression $\sigma^{*}_{cr}=P_{cr}/P^{0}_{cr}=\sigma_{cr}/\sigma^{0}_{cr}=1/(\theta^{*})^{\alpha}$, where P^{0}_{cr} and σ^{0}_{cr} are the initial Eulerian critical load and stress. The variation of the load parameter, μ, with the normalized Eulerian critical load can be written as $\mu=[(\sigma^{*}_{cr})^{-\alpha}-1]\,(\sigma^{*}_{cr})^{-\beta/\alpha}$. In the present study, the rod length and the cross sectional radius are 0.5 and 0.03 m, respectively. To solve the transient heat conduction equation for one dimensional problem, the iterative process in combination with numerical method at the different load parameter is used. The loading frequency is assumed to be equal to *1Hz*.

3. Numerical Results of evaluation of critical parameters of structure

The dependence of the dimensionless temperature on time in the considered nanocomposite rod with 3% CNTs nanofibers under different amplitude of harmonic loading, $\sigma_1 = 40, 50$ and $60\ MPa$ are shown in Fig 1, (*a*). It is clearly observed that with increasing of excitation amplitude the loss creep compliance modulus and load parameters increase. Also, the dissipative heating does not lead to thermal instability under excitations considered.

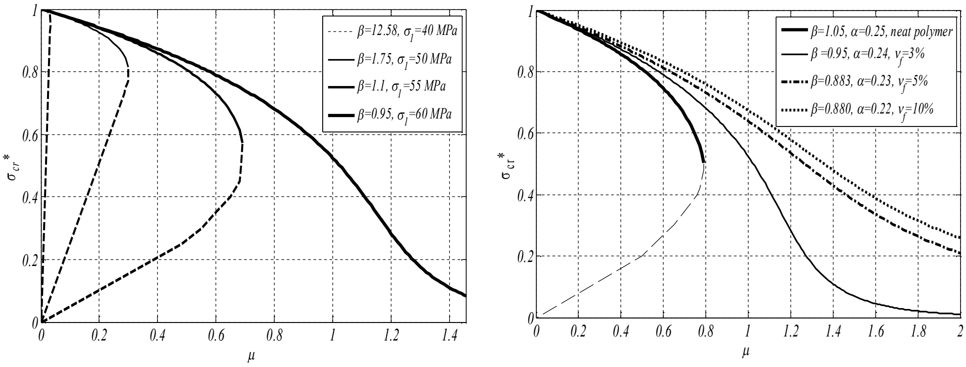


a *b*

Fig. 1

Influence of volume fraction of CNTs nanofibers on thermal stability of the rod under $\sigma_1 = 75MPa$, is illustrated in Fig 1, (*b*). It is clearly observed that embedding axially aligned CNTs nanofibers in polymer matrix improves thermal stability of the rod. The normalized loading parameter, μ, decreases with increasing volume fraction of the filament, so that the loss creep compliance modulus, J'', decreases and the heat transfer coefficient of rod improves.

Variation of the normalized Eulerian critical stress, σ^*_{cr}, with the load parameter, μ, for $V_f = 3\%$ under different amplitudes of the loading are presented in Fig 2, (*a*). As seen in this figure, the mechanical instability occurs under harmonic loading with $\sigma_1 = 40$ and $50\ MPa$ for $\mu > \mu^*$, while there exists no mechanical instability for $\sigma_1 = 60\ MPa$. It means that the rod is always stable for all values of loading parameter and the dissipative heating does not have any effect on the mechanical stability of the considered nanocomposite rod. This behavior has great conformity with the results reported in [3]. For this particular case, it can be said that J'' increases with increasing temperature because the coefficient β is less than one (β=0.95). As a result, one can conclude that if the coefficient β is less than one then mechanical stability is occurred for arbitrary value of the loading parameter. The effect of volume fraction on normalized critical stress for the constant amplitude of the harmonic loading, $\sigma_1 = 60\ MPa$, is demonstrated in Fig. 2, (*b*). This figure shows that with increasing of volume fraction of CNT nanofibers, the critical stress increases while the coefficients β and α decrease. It can be clearly viewed that embedding CNT nanofibers with $V_f = 3, 5$ and $10\ \%$ in polymeric matrix provides mechanical stability for all values of loading parameter. The values of β are less than one while for neat polymer it is equal to 1.05 providing the occurrence of thermal instability.



a *b*

Fig. 2

Conclusion. In this work, the influence of volume fraction on thermal and mechanical stability was studied. The results show that filament volume fraction of nanofibers (up to 10%) affects significantly the temperature evolution and mechanical stability due to dissipative heating of nanocomposite rod under combined steady compressive and harmonic loading. The approach presented in this investigation can be useful to understanding and appropriate description of the cyclic behavior of nanocomposite structures and further investigations in the area of the description of fatigue mechanisms with regard to vibro-heating effect caused by the mechanical energy dissipation.

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Теплова нестійкість та прогноз руйнування конструкційного елемента з трансверсально ізотропного нанокомпозитного матеріалу

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Статтю присвячено прогнозу руйнування нанокомпозитного елемента конструкції внаслідок теплової нестійкості при циклічному навантаженні. Дослідження ґрунтується на використанні моногармонічного наближення і застосуванні концепції комплексних модулів. Температурно- та амплітудназалежні комплексні модулі використані для вивчення впливу дисипативного розігріву на механічну стійкість полімерного нанокомпозитного стержня, що перебуває під дією комбінованого статичного і моногармонічного навантаження. Досліджено вплив амплітуди навантаження і об'ємного вмісту нановолокон на теплову стійкість полімерного нанокомпозитного стержня.

Тепловая неустиойчивость и прогноз разрушения конструкционного элемента из трансверсально изотропного нанокомпозитного матеріала

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Статья посвящена прогнозу разрушения нанокомпозитного элемента конструкции вследствие тепловой неустойчивости при цикличном нагружении. Исследование основывается на использовании моногармонического приближения и применении концепции комплексных модулей. Температурно- и амплитуднозависимые комплексные модули использованы для изучения влияния диссипативного разогрева на механическую устойчивость полимерного нанокомпозитного стержня, находящегося под воздействием комбинированного статического и моногармонического нагружения. Исследовано влияние амплитуды нагружения и объемного содержания нановолокон на тепловую устойчивость полимерного нанокомпозитного стержня.

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