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Subjective Entropy Maximum Principle for Preferences Functions of Alternatives Given in the View of Logical Conditions

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Принцип максимума субъективной энтропии для функций предпочтений альтернатив, заданных в виде логических условий

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Принцип максимуму суб'єктивної ентропії для функцій переваг альтернатив, заданих у вигляді логічних умов

In the article the task of finding the optimal combination of objective functions given in the view of a logical conditional system of equations is considered. It is modeled the behavior of an active system controlled by the intellectual (active) element. The principle of the subjective entropy maximum is applied for obtaining the canonical distribution of the individual's preferences as the solution to the problem for the conditional extremum.
Key words: active system, individual's preferences, subjective entropy, multi-alternativeness.

В статье рассматривается задача отыскания оптимальной комбинации целевых функций, заданных в виде логической условной системы уравнений. Моделируется поведение активной системы, управляемой интеллектуальным (активным) элементом. Принцип максимума субъективной энтропии применяется для получения канонического распределения индивидуальных предпочтений в качестве решения задачи на условный экстремум.
Ключевые слова: активная система, индивидуальные предпочтения, субъективная энтропия, многоальтернативность.

У статті розглядається задача відшукання оптимальної комбінації цільових функцій, заданих у вигляді логічної умовної системи рівнянь. Моделюється поведінка активної системи, керованої інтелектуальним (активним) елементом. Принцип максимуму суб'єктивної ентропії застосовується для отримання канонічного розподілу індивідуальних переваг у якості розв'язка задачі на умовний екстремум.
Ключові слова: активна система, індивідуальні переваги, суб'єктивна ентропія, багатоальтернативність.

In subjective analysis we encounter a problem of evaluation of an individual-subject participation in systems and processes of control in the vast majority of scientific researches that consider the functioning and control of the so-called active systems [1-9].

Because an active system includes the active element (a person who is responsible for making managing decisions), therefore, in our opinion the active system has the following peculiarities [4, P. 58, § 3.1]:

1. Being closed, is able to decrease its own entropy
2. Requires the presence (existence) of the active element, the bearer of individual preferences
3. The extremized functional includes the subjective entropy
4. The behavior of the active element is dictated by the postulated variational principle and directed upon the optimization of a certain problem-resource situation with extremizing the entropy
5. Has the ability to aggregate preferences

Thus, an active system is guided by the intellect of its active element and the active element acts on the basis of his own preferences distributed at each moment in time on the achievable for his objectives set of alternatives.

Let us consider theoretically a logical conditional system of, for instance, some three equations [9, P. 20, (1)]:

$$Y_{\max}(x)=\begin{cases} y_1(x), & if \quad y_1(x)>y_2(x)\wedge y_1(x)>y_3(x); \\ y_2(x), & if \quad y_2(x)>y_1(x)\wedge y_2(x)>y_3(x); \\ y_3(x), & otherwise. \end{cases} \tag{1}$$

The concept (1) implies $y_i(x)$ – dependence, for example, of the control of safety of an active system, an effectiveness of the i^{th} operator (alternative) of an engineering system, the i^{th} alternative function of safety, the i^{th} alternative strategy, role of the i^{th} political party (alternative) in sociology/politics, usefulness or utility function of the i^{th} alternative in economics etc. depending upon some parameter of x – for instance, a certain indicator of the mode of operation, parameter that exerts an influence upon the safety of the active system, the value of an income taxation rate, index of policy, profitableness, light and shadow economic proportion and so on and so forth. Here, in the system of equations (1), the intellectual system is seeking after the positive value. Naturally it must be maximal.

Without doubt, if the functions of $y_i(x)$ represent some negative qualities, modeling should be made with respect to the corresponding minimization [9, P. 20, (1)]:

$$Y_{mix}(x)=\begin{cases} y_1(x), & if \quad y_1(x)<y_2(x)\wedge y_1(x)<y_3(x); \\ y_2(x), & if \quad y_2(x)<y_1(x)\wedge y_2(x)<y_3(x); \\ y_3(x), & otherwise. \end{cases} \tag{2}$$

Thus, the systems of (1, 2) look like natural intellect application for decision making, nevertheless they do not represent their system's active element's intentions (individual's preferences), they do not show how much and to which alternative the responsible for making decisions person is inclined to [9, P. 20].

The conceptual framework of subjective analysis [1-9] allows creating a model of an artificial intellect of active systems with the use of the postulated functional in the view of [2, P. 119, (3.38)], [9, P. 20, (2)]:

$$\Phi_{\pi}=-\sum_{i=1}^{N}\pi_i\ln\pi_i-\beta\sum_{i=1}^{N}\pi_i F_i+\gamma\left[\sum_{i=1}^{N}\pi_i-1\right], \tag{3}$$

where π_i – function of the individual's subjective preferences of the i^{th} achievable alternative; N – number of the achievable alternatives; β – structural parameter; F_i – fun-

ction, related to the i^{th} achievable alternative; γ – structural parameter. The structural parameters β and γ can be considered in different situations as Lagrange coefficients, weight coefficients or endogenous parameters that represent some certain properties of the individual's psych.

Let us consider, as an example, the problem setting with the logical conditions systems of (1) and (2). Hence, $N = 3$. Assume, the functions of $y_i(x)$ are

$$y_1(x) = a(x-d_1)^k + b(x-d_1)^{k-1} + c(x-d_1)^{k-2},$$

$$y_2(x) = -a(x-d_2)^k - b(x-d_2)^{k-1} - c(x-d_2)^{k-2} + z_2, \quad y_3(x) = nxe^{nx} + z_3. \qquad (4)$$

where a, b, c, d_1, d_2, k, z_2, z_3, n – coefficients and parameters of the models.

The necessary conditions for the extremum of functional (3) to exist, in the form of the system of equations $\frac{\partial \Phi_\pi}{\partial \pi_i} = 0$, yield the canonical distributions of the individual's preferences, likewise in [2, P. 115-135], [5, P. 58, (4)], that is for the systems (1, 2), where $N = 3$, $F_i = y_i$, it will be, [9, P. 21, (3)]

$$\pi_j = \frac{e^{-\beta y_j}}{\sum_{i=1}^{3} e^{-\beta y_i}}. \qquad (5)$$

For the system (1) the optimal combination of the preferences

$$\pi_{\max}(x) = \begin{cases} \pi_1(x), & if \ \ \pi_1(x) > \pi_2(x) \wedge \pi_1(x) > \pi_3(x); \\ \pi_2(x), & if \ \ \pi_2(x) > \pi_1(x) \wedge \pi_2(x) > \pi_3(x); \\ \pi_3(x), & otherwise\ . \end{cases} \qquad (6)$$

Analogously for (2)

$$\pi_{\min}(x) = \begin{cases} \pi_1(x), & if \ \ \pi_1(x) < \pi_2(x) \wedge \pi_1(x) < \pi_3(x); \\ \pi_2(x), & if \ \ \pi_2(x) < \pi_1(x) \wedge \pi_2(x) < \pi_3(x); \\ \pi_3(x), & otherwise\ . \end{cases} \qquad (7)$$

Let us introduce a *function of the intellectual diapason* in the view of

$$\pi_\Sigma(x) = \pi_{\max}(x) + \pi_{\min}(x) \qquad (8)$$

The function (8) is not normalized itself, though being formed as a summation of the optimized combinations of the normalized functions of preferences comprised by the systems of (6, 7), has the possible values diapason for its changes from "0" up to "1".

For the special data given: $a = 0.5$; $b = -0.632$; $c = 9.487$; $d_1 = 5$; $d_2 = 0$; $k = 2$; $z_2 = 70$; $z_3 = 30$; $n = -0.4$; $\beta = -0.063$ the results of modeling by the procedure of (1-8) are illustrated in fig. 1 [9, P. 21, fig. 1].

From the diagrams plotted in fig. 1 in the corresponding scales it is visible that the optimal combinations of functions $y_i(x)$ correspond the optimal combinations of the preferences functions $\pi_i(x)$. The corner points of the changes of $y_i(x)$ have the same abscissas as for the $\pi_i(x)$. The function of the intellectual diapason on one hand has its own minima corner points with the same abscissas as for the maximal combination of the objective functions of $y_i(x)$ and corresponding individual's preferences $\pi_i(x)$. On the other hand the

function of the intellectual diapason has its own maxima corner points with the same abscissas as for the minimal combination of the individual's preferences and the objective functions of $y_i(x)$.

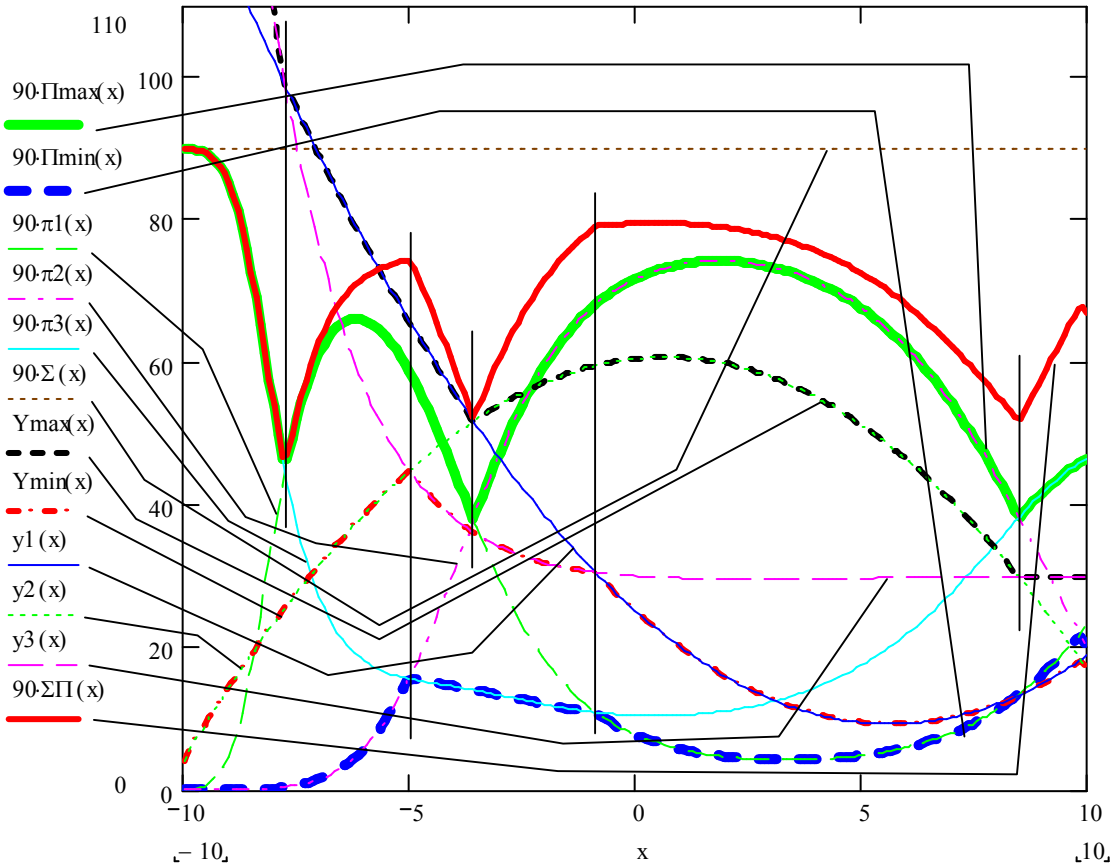

Figure 1 – Objective functions, preferences, their optimal combinations, and function of intellectual diapason

The considered problem means the application of the subjective entropy extremization principle being developed in subjective analysis [1-9]. For the given problem setting the entropy of preferences is

$$H_{\pi}(x) = -\sum_{i=1}^{N=3} \pi_i(x) \ln \pi_i(x); \tag{9}$$

the entropy of optimal combinations of preferences –

$$H_{\pi_{opt}}(x) = -\left(\pi_{\max}(x) \ln \pi_{\max}(x) + \pi_{\min}(x) \ln \pi_{\min}(x)\right); \tag{10}$$

the entropy of normalized optimal combinations of preferences –

$$\pi_{\max\_n}(x) = \frac{\pi_{\max}(x)}{\pi_{\Sigma}(x)}; \qquad \pi_{\min\_n}(x) = \frac{\pi_{\min}(x)}{\pi_{\Sigma}(x)}; \tag{11}$$

will be –

$$H_{\pi_{opt\_n}}(x) = -\left(\pi_{\max\_n}(x) \ln \pi_{\max\_n}(x) + \pi_{\min\_n}(x) \ln \pi_{\min\_n}(x)\right). \tag{12}$$

The entropies calculated by formula (9-12) are presented in fig. 2.

The entropy paradigm illustrated with the corresponding diagrams plotted in fig. 2 makes it noticeable that the measures of uncertainty are appropriate for the estimation of

the decision making thresholds. Nevertheless, the comparison of the entropies with the function of the intellectual diapason and the optimal combinations of preferences functions testifies that the corner points of the function of the intellectual diapason, thus, the optimal transitions from one objective function to other, do not necessarily coincide with the extremums of the traditional entropy (9) and the entropy of the traditional form (9) has no corner points as it is the smooth (everywhere differentiated curve).

On the contrary, the curves of the entropies (10, 12) indeed have those corner points at the same with the function of the intellectual diapason abscissas; therefore they have the diagnostic value for the optimal transitions.

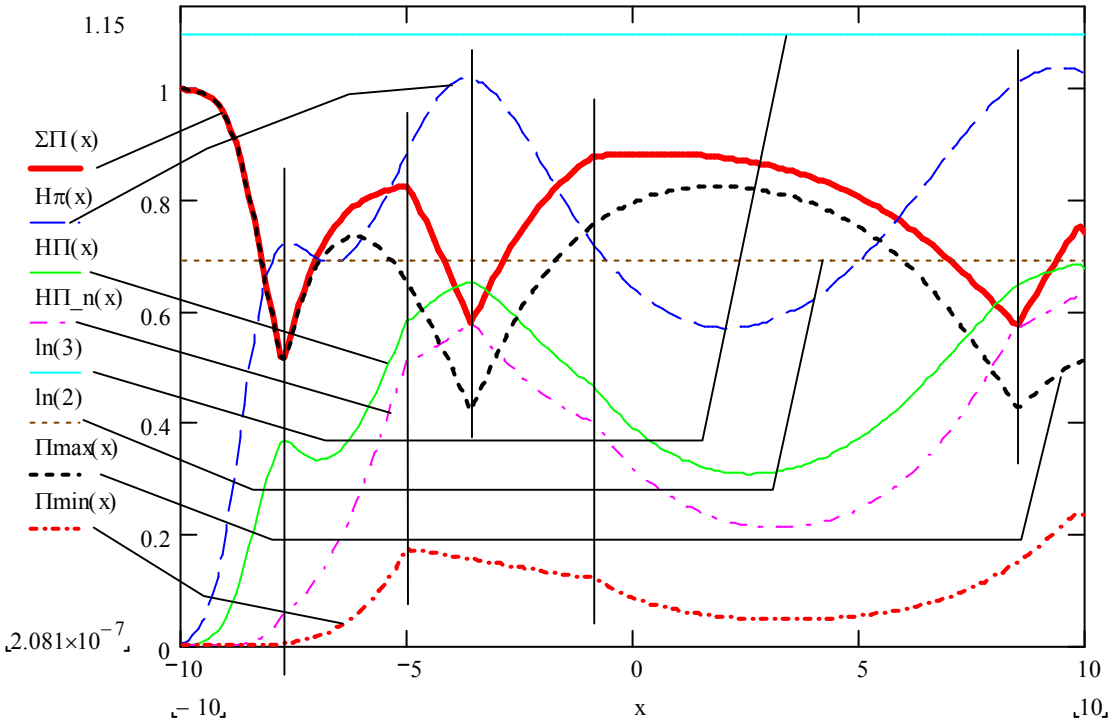

Figure 2 – Entropies of preferences and function of intellectual diapason

For the estimation of the conflictability of the making managing decisions and control process, as well as conflictability of the individual's preferences distributions it is applicable the pseudo entropy hybrid function proposed in [6], [7].

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RESUME

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Subjective Entropy Maximum Principle for Preferences Functions of Alternatives Given in the View of Logical Conditions

In the given article the models for evaluation of an individual-subject participation in systems and processes of control in the problems, and the vast majority of scientific researches, that consider the functioning and control of the so-called active systems are developed.

The models are developed on the basis of the subjective analysis theory elaborated in monographs [1], [4] with the application of the principle of the subjective entropy of individual preferences extremization. The concept of an active system is interpreted in terms of monograph [4]. The active system is guided by the intellect of its active element and the active element acts on the basis of his own preferences distributed at each moment in time on the achievable for his objectives set of alternatives.

Postulated in subjective analysis functional allows obtaining canonical distributions of individual's preferences on the basis of the systems of the necessary conditions for extremums to exist equations solutions. The corner points of the optimal combinations of the preferences represent the optimal combinations of the objective functions.

The developed in the article entropies and function of intellectual diapason allow estimating the potential for control in situations of multi-alternativeness and conflicts. The conflictability of individual preferences is estimated with the application of the pseudo entropy hybrid function proposed in works [6], [7].

Realization of such approach creates a theoretical background for solutions of the conceptual problems of the artificial intellect.

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