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INVESTIGATION AND ROBUST SYNTHESIS OF POLYNOMIALS UNDER PERTURBATIONS BASED ON THE ROOT LOCUS PARAMETER DISTRIBUTION DIAGRAM

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ДОСЛІДЖЕННЯ І РОБАСТНИЙ СИНТЕЗ ЗБУРЕНИХ ПОЛІНОМІВ НА ПІДСТАВІ ДІАГРАМИ РОЗПОДІЛУ ФУНКЦІЇ ПАРАМЕТРА КОРЕНЕВОГО ГОДОГРАФА

Investigation of the 4^{th} order dynamic systems characteristic polynomials behavior in conditions of the interval parametric uncertainties is carried out on the basis of root locus portraits. The roots behavior regularities and corresponding diagrams for the root locus parameter distribution along the asymptotic stability bound are specified for the root locus portraits of the systems. On this basis the stability conditions are derived, graphic-analytical method is worked out for calculating intervals of variation for the polynomial family parameters ensuring its robust stability. The discovered regularities of the system root locus portrait behavior allow to extract hurwitz sub-families from the non-hurwitz families of interval polynomials and to determine whether there exists at least one stable polynomial in the unstable polynomial family.

Keywords: polynomial, dynamic system, uncertainty, stability, robustness, root locus portrait, root locus parameter function

На основі використання кореневих портретів проводиться дослідження поведінки характеристичних поліномів динамічних систем четвертого порядку в умовах інтервальної параметричної невизначеності. Для кореневих портретів систем визначаються закономірності поведінки коренів і відповідні діаграми розподілу функції параметра кореневого годографа вздовж межі асимптотичної стійкості. На цій основі формулюються умови стійкості, розробляється графоаналітичний метод для обчислень інтервалів варіації параметрів поліноміального сімейства, що забезпечують його робастну стійкість. Виявлені закономірності поведінки кореневого портрету системи дозволяють виділяти гурвіцеви підсемейства з негурвіцевих сімейств інтервальних поліномів і визначати, чи існує хоча б один стійкий поліном у нестійкому в цілому інтервальному сімействі поліномів.

Ключові слова: поліном, динамічна система, невизначеність, стійкість, робастність, кореневий портрет, функція параметра кореневого годографа

Introduction

The tasks of analysis and synthesis of control processes occurring in dynamic systems of different physical nature, operating in conditions of substantial parametric uncertainty, including the engineering ones, are currently very urgent and challenging as it is emphasized in [1, 2]. One of such problems is a problem of flux control in electric motor vector control systems operating in conditions of uncertainty because the flux control quality affects greatly the electromagnetic torque and speed control quality, and thus the drive power efficiency. Therefore, of great importance are the tasks of stability investigation and parametric synthesis of robust control systems (their characteristic polynomials) for the plants, which parameters are varying within the specified intervals of values.

In the area of investigation and synthesis of dynamic systems characteristic polynomials there exists a lot of approaches and methods. For the first time the necessary and sufficient conditions for systems up to the 3-rd order were formulated by James Maxwell in 1868. Later appeared the stability criteria of Routh – Hurwitz, Mikhajlow, Nyquist, Bode, which made it possible to check stability of the systems of order *n*. The frequency Nyquist criterion was the first one that could be used for synthesis by estimation of the system degree of stability.

Among the modern methods of synthesis [1, 2] together with the frequency ones the root locus and state-space methods could be listed. The main results in the area of the frequency approach to analysis and synthesis of robust dynamic (control) systems are given in [3], where the stability of uncertain polynomials, including interval ones, is also considered.

The methods for analysis and synthesis of polynomial families represent the separate group. The most effective solutions for the task of interval polynomial families investigation within the algebraic approach have been proposed by V.L. Kharitonov [4], where in the general case the task is reduced to consideration of only four specific polynomials of the whole family with constant coefficients. In [3, 5] the frequency criteria of hurwitz robust stability are considered, which allow to define the coefficient perturbation sweep for the nominally stable polynomial and various types of uncertainties. Hurwitz robust stability is also investigated in [6–10].

Of great interest are the problems of ensuring system stability and quality being solved in the modern statements of the problem [2] as tasks of ensuring system robustness, which could be also solved by application of the root locus approach.

Root locus approach to the problem is considered in [11–16]. The task of a stable characteristic polynomial synthesis for the interval dynamic system (IDS) by setting up coefficients of the given (initial) unstable one for the case of location of its root locus initial points (the points where a variable parameter is equal to zero) family within the left half-plane is solved in [15], where the stability is attained via simple setting up the an interval of the free term variation.

In this work the graphic-analytical root locus approaches are described for calculating intervals of uncertainty for coefficients of the given (initial) stable or unstable polynomial with coefficients subject to perturbations, which ensure its robust stability. The proposed methods are based on introduction and application of the notion of the "diagram of the root locus parameter function values distribution along the stability bound" and can be used for both synthesis of interval stable polynomials by setting up (adjusting) the unstable ones and analysis of the polynomial behavior under coefficient perturbations.

The work further develops the results of B.D.O. Anderson [16] and V.L. Kharitonov [4] where they consider the issues of robust interval polynomial families analysis.

The Problem statement

Define a polynomial like

$$g_n(s) = s^n + a_1 s^{n-1} + \dots + a_{n-1}s + a_n, \quad (1)$$

where a_j are given (initial) values of real polynomial coefficients, $j = 1, 2, ..., n$.

In the event of coefficient perturbations, a vector of coefficients of (1), $a = (a_1, \dots, a_{n-1}, a_n)$, belongs to some connected set $A \subset R^n$, $a \in A$; n is a degree of the polynomial (integer value); s is a complex variable, $s = \sigma + i\omega$.

Suppose that coefficients of (1) vary within the following intervals:

$$\underline{a}_j \le a_j \le \overline{a}_j, \ \ j = \overline{1, n}. \quad (2)$$

where $\underline{a}_j$ and $\overline{a}_j$ are minimal and maximal limit values of closed interval (2) of coefficients a_j variation correspondingly. Polynomial (1) can be both, nonhurwitz or hurwitz one.

After substituting $s=\sigma+i\omega$ write the root locus and parameter equations [12] correspondingly:

$$v(\sigma,\omega) = 0, \text{ and} \quad (3)$$

$$a_n = u(\sigma,\omega), \quad (4)$$

where $u(\sigma,\omega)$ and $v(\sigma,\omega)$ are real functions of two independent variables σ and ω.

The root locus method represents a powerful and effective tool for stable and qualitative polynomials synthesis and analysis. Especially in the cases when their parameters are subject for perturbations. Therefore, the task of the work is focused on working out the stability conditions and methods for investigation and parametric synthesis of characteristic polynomials of dynamic systems (in this case of the fourth power) with interval uncertainty (interval dynamic systems (IDS)) based on discovering regularities of the system root locus portraits behavior at the asymptotic stability boundary.

Root locus portrait of an uncertain polynomial

Consider a dynamic system described by the family of interval characteristic polynomials [3, 4, 11, 12] like

$$g_4(s) = s^4 + a_1 s^3 + \dots + a_3 s + a_4, \qquad (5)$$

where s is a complex variable, $s = \sigma + i\omega$. Variation of the dynamic system characteristic polynomial coefficients (parameters) within specific intervals reflexes its physical parameters variation, their undesirable changing relative to the required values, i.e. the system parametric uncertainty. Suppose that coefficients of (5) are real, positive and vary within the intervals (2):

$$\underline{a}_j \le a_j \le \overline{a}_j, \quad j = \overline{0,4}, \quad a_0 = 1, \qquad (6)$$

where $\underline{a}_j$ и $\overline{a}_j$ − correspondingly minimal and maximal values of the closed intervals of coefficient a_j variation.

Rewrite (5) after substitution of $s = \sigma + i\omega = i\omega$ ($\sigma = 0$):

$$\omega^4 - a_1\omega^3 i - a_2\omega^2 + a_3\omega i + a_4 = 0, \qquad (7)$$

and on the base of (3) and (4) write correspondingly the *root locus equation* [11, 12] at the stability bound:

$$-a_1\omega^3 + a_3\omega = 0 \qquad (8)$$

and the *parameter equation* (*parameter function*) [11, 12] at the stability bound:

$$f(\omega) = -\omega^4 + a_2\omega^2 = a_4, \qquad (9)$$

for the given interval dynamic system (IDS) [11, 12, 15].

The family P of root loci of interval polynomial (1) with coefficients varying within (2) name as *interval polynomial root locus portrait (interval polynomial root locus)* or *interval dynamic system root locus portrait (interval dynamic system root locus).*

The root locus generated while varying free term a_n of (1) name as a *free root locus* [14]. Parameter a_n in this case is named as a *root locus parameter* or a *free parameter*.

Let us together with a_n vary also parameter a_{n-1} of (1). Thus, when varying two parameters, a_n and a_{n-1}, simultaneously we generate a (*free*) *root locus field* F_k (k = 1. 2, ...) in the plane s of system roots, which could also be named a *two-parameter root locus field* or a (*an interval*) *root locus subfamily*. Parameter a_{n-1} used for the field generation is named a *root locus field parameter*.

Evidently, root locus equation (8) represents also the equation of level lines of a *free root locus field* F_k.

Root locus portrait P is then representted by a family of root locus fields,

$$P=\{F_k \mid k=1, 2, \dots\}, \qquad (10)$$

that represents infinite set of root locus fields and, therefore, features their properties.

Crossing region of the root locus portrait

Due to analyticity and continuity properties of functions (8) and (9) it is evident that the cross points of the stability bound (axis $i\omega$) by the branches of root locus family P (10) under condition

$$0 < a_j < +\infty \qquad (11)$$

form on this axis a specific *crossing region*, D_ω^P.

Definition 1. Crossing region D_ω^P of root locus portrait P of an interval dynamic system, described by characteristic polynomial (5), is a region [– ∞, + ∞] on the system asymptotic stability bound i_ω where the given portrait parameter function (9) values family is located provided all coefficients a_j of (5) vary within limits (11).

It is also evident that D_ω^P represents a continuous region. Thus, every field F_k (10) and every branch $b_{k_i}, i = 1,2,\dots$, of the field root loci generate specific sub-regions, correspondingly sub-region $D_{\omega k}^F$ and continuous sub-region $D_{\omega i}^b$, within the above specified crossing region D_ω^P.

Due to the symmetry of the portrait relative to axis σ, hereinafter the only upper complex half-plane s and the upper half-axis $i\omega$ are considered.

Extremum region: root locus parameter function majorant and minorant

Define maximal and minimal values of parameter function (9) within sub-region $D^F_{\omega k} \subset D^P_\omega$. For this purpose it is required to investigate this function for extremum. Evidently, the majorant parameter function (or majorant) is obtained by rewriting (9) as

$$a_{4\max} = -\omega^4 + \bar{a}_2\omega^2 . \qquad (12)$$

Take the first-order derivative of (12) and set it to zero:

$$-4\omega^3 + 2\bar{a}_2\omega = 0 . \qquad (13)$$

After solving (13) obtain three points of extremum for majorant parameter function for the field when $a_2 = \bar{a}_2$:

$$\omega_{e_{\max}} = 0, \qquad a_{4e_{\max}} = 0;$$

$$\omega_{e_{\max}} = \pm\sqrt{\frac{\bar{a}_2}{2}}, \quad a_{4e_{\max}} = \qquad (14)$$

$$= -\omega^4_{e_{\max}} + \bar{a}_2 \cdot \omega^2_{e_{\max}} .$$

Evidently, (12) is a majorant for the whole portrait.

Rewrite (9) for determination of a minorant parameter function (or a minorant):

$$a_{4\min} = -\omega^4 + \underline{a}_2\omega^2 . \qquad (15)$$

Take the first-order derivative of (15) and set it to zero:

$$-4\omega^3 + 2\underline{a}_2\omega = 0. \qquad (16)$$

After solving (16) obtain three points of extremum (minimum) for minorant parameter function at the stability bound for the field when $a_2 = \underline{a}_2$:

$$\omega_{e_{\min}} = 0, \qquad a_{4e_{\min}} = 0;$$

$$\omega_{e_{\min}} = \pm\sqrt{\frac{\underline{a}_2}{2}}, \quad a_{4e_{\min}} = \qquad (17)$$

$$= -\omega^4_{e_{\min}} + \underline{a}_2 \cdot \omega^2_{e_{\min}} .$$

Evidently (15) is a minorant for the whole portrait.

Definition 2. Extremum region D^e_ω of root locus portrait of an interval dynamic system, described by characteristic polynomial (5), is a region $[0,\ \omega_{e_{\max}}]$ at the system asymptotic stability bound $i\omega$ where the given portrait parameter function (9) extremum values, $a_{4e_{\max}}$ (14) and $a_{4e_{\min}}$ (17), family is located provided all coefficients a_j of (5) vary within limits (11).

A diagram for the parameter function distribution along the stability bound

In Fig. 1 a diagram for the interval root locus portrait parameter function (9) values distribution along the stability bound is represented by majorant (12) and minorant (15) parameter functions. For the purpose of simplification and better understanding the points of extremum in Fig. 1 are connected by strait lines though they represent curves. Thus, region D^P_ω could be divided into three sub-regions (Fig. 1):

– region D_ω^+ where the parameter function increases (*increasing region*);
– region D_ω^- where the parameter function decreases (*decreasing region*);
– region D_ω^c of increasing and decreasing regions combination (*mixed region*).

Consider region Z_ω and interval
$[z', z'']$,
$$[z', z''] \subseteq Z_\omega ,$$
on axis i_ω where the root locus portrait initial points (i.e. the poles, where root locus parameter a_4 is equal to zero) [11, 12, 14] migrate across the stability bound to the right half-plane. Points z', z'' correspond to z_1, z_2 in the diagram.

Within interval $[0, z']$ covering completely region D_ω^+ and partly region D_ω^c,

$$D_\omega^+ \subset [0, z'],\ [0, z'] \cap D_\omega^c ,$$

the stability bound is crossed only by positive branches of the system root locus portrait. Family Z of the root locus portrait initial points here is completely located within the left half-plane L,

$$Z \subset L , \qquad (18)$$

however, positive branches of the portrait are partly located in the right half-plane. Therefore, within polynomial family (5) the unstable polynomials probably could be found, but there also exist specific intervals (6) ensuring stability of the whole family. Interval [0, z'] is thus named the *system stability region*.

Within interval [z', z''] partly belonging to region D_ω^c and partly – to region D_ω^-,

$$[z',z'']\cap D_\omega^{\ c},\ [z',z'']\cap D_\omega^{\ -},$$

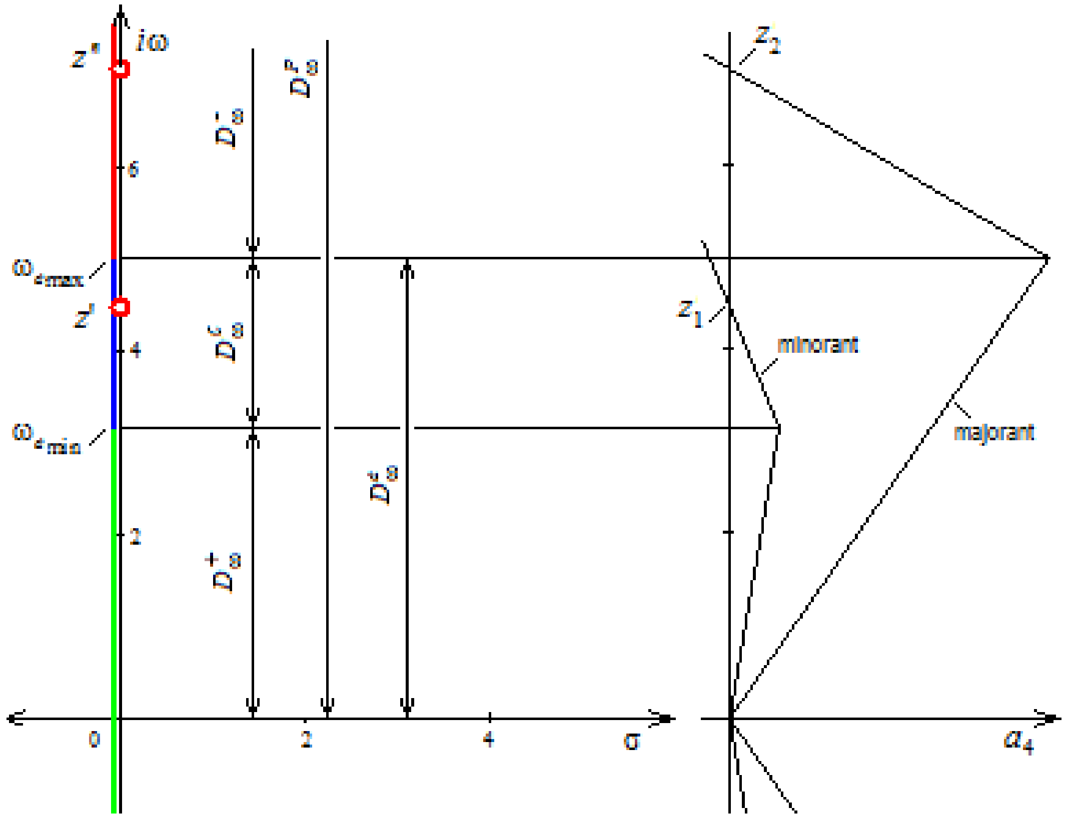

Fig. 1. A diagram for distribution of the interval system root locus portrait parameter function along the asymptotic stability bound

the stability bound is crossed by both positive and negative branches of the system root locus portrait.

Some initial points and, therefore, some positive branches of the root locus have migrated to the right half-plane. Thus, in this case family (5) comprises both stable and unstable polynomials. Interval [z', z"] is thus called the *system instability region*.

Within interval [z", ∞], completely belonging to D_ω^-,

$$[z'',\infty]\subset D_\omega^{\ -}, \qquad (19)$$

the stability bound is crossed by only negative branches of the root locus family. The whole family Z and, therefore, the corresponding positive root locus branches have migrated into the right half-pane,

$$Z\subset R, \qquad (20)$$

and in this case the system root locus family doesn't contain a single positive branch (or a portion of a positive branch), and, therefore, family (5) doesn't comprise a single stable polynomial. This region is called the *instability region*.

Real crossing region

Define the limits of crossing region D_ω^R where the stability bound is crossed by branches of the root locus portrait. For this purpose consider equation (8) and define values of its roots. For the root locus portrait (when $\omega>0$)

$$\omega_{max} = \sqrt{\frac{\overline{a}_3}{\underline{a}_1}}, \; \omega_{min} = \sqrt{\frac{\underline{a}_3}{\overline{a}_1}}, \qquad (21)$$

where ω_{max} and ω_{min} are limits of the *real crossing region*.

Definition 3 Real crossing region D_ω^R of the given interval dynamic system root locus portrait, described by characteristic equation (5), is a region $[\omega_{min}, \omega_{max}]$ (21), located at the system asymptotic stability bound $i\omega$ where this bound is crossed by the root locus portrait branches.

$$[\omega_{min}, \omega_{max}] \subseteq D_\omega^R. \qquad (22)$$

Graphic-analytical stability conditions for interval polynomials

These conditions depend on how does the real crossing region D_ω^R is located relative to the increasing region D_ω^+, decreasing region D_ω^- and mixed region D_ω^c (see sections 3-6). Define three possible options of this location and the stability conditions corresponding to each one of them.

A. Real crossing region belongs to the region D_ω^+,

$$D_\omega^R \subset D_\omega^+. \qquad (23)$$

This case of D_ω^R location takes place when:

$$\omega_{max} < \omega_{e_{min}}.$$

Based on the commonly known asymptotic properties of root loci [2], the following statement could be formulated.

Statement 1. If condition (23) holds, the family Z of initial points of the dynamic system root locus portrait, described by polynomial (5), is located in the left half-plane L,

$$Z \subset L. \qquad (24)$$

Consider a set S of intervals s_i of the system root locus portrait P branches:

$$S = \{s_i = [0, a_4(\omega_i)], \; i = 1,2,...\}. \qquad (25)$$

where $a_4(\omega_i)$ are values of parameter function (9) at points of axis $i\omega$ having coordinates ω_i; $S \subset P$ and $S \subset L$ (see (21)). On the base of (23) and (24) write the following expression:

$$\bigcap_{i=1}^{\infty} s_i = \inf S = [0, \underline{a}_4(\omega_{min})], \qquad (26)$$

where $\underline{a}_4(\omega_{min})$ is the minimal value of function (9) at a point having coordinate ω_{min} (21). Therefore,

$$\forall\, a_4 \in [\underline{a}_4, \overline{a}_4] \; [a_4 \in [0, \underline{a}_4(\omega_{min})] \rightarrow a_4 \in S \,\&\, P \subset L], \qquad (27)$$

$$\forall\, a_4 \in [\underline{a}_4, \overline{a}_4] \; [a_4 \notin [0, \underline{a}_4(\omega_{min})] \rightarrow a_4 \notin S \,\&\, P \not\subset L]. \qquad (28)$$

Based on (25) and (28) formulate the following statement.

Statement 2. The asymptotic stability of the dynamic system, described by interval characteristic polynomial family (5) and satisfying expression (23), is ensured when the following condition holds:

$$\overline{a}_4 < \underline{a}_4(\omega_{min}). \qquad (29)$$

Definition 4. Dominating polynomials in polynomial family (5) are specific polynomials with constant coefficients, which stability guarantees stability of the whole family.

From condition (29) follows that the system asymptotic stability in this case is defined by the value of $\underline{a}_4(\omega_{min})$. It means that in this case the *dominating* one is a polynomial of (5) defined on the branch passing through the point having a coordinate ω_{min} corresponding to the parameter function value $\underline{a}_4(\omega_{min})$. In its turn, a point having coordinate ω_{min} is defined by expression (21), i.e. by coefficients $\overline{a}_1, \underline{a}_3$ of (5); the parameter minimal value, $\underline{a}_4(\omega_{min})$ at this point is defined by (15), i.e. by the coefficient $\underline{a}_2$. Based on Statement 2 and the above conclusions, formulate the following stability condition.

Stability condition 1. If the interval dynamic system root locus portrait P (10) satisfies expression (23), the system asymptotic stability is ensured when polynomial

$$s^4 + \overline{a}_1 s^3 + \underline{a}_2 s^2 + \underline{a}_3 s + \overline{a}_4 = 0 \qquad (30)$$

of family (5) is stable:

Stability condition 1 is applied for verifying the polynomial (system) stability and Statement 2 - for the system parametric synthesis.

B. Real crossing region belongs to the region D_ω^-,

$$D_\omega^R \subset D_\omega^- . \quad (31)$$

Such case of D_ω^R location takes place when:

$$\omega_{\min} \geq \omega_{e_{\max}} .$$

Based on conclusions made in sections 3-6, see case (19) and expression (20), and the well known asymptotic properties of root loci [2, 11, 12], the following statement could be formulated.

Statement 3. If condition (31) holds for interval dynamic system root locus portrait *P*, the whole family *Z* of its initial points satisfies expression (20), and hence, the system is unstable.

C. Real crossing region completely or partially belongs to the mixed region D_ω^c

$$D_\omega^R \subset D_\omega^c \vee D_\omega^R \cap D_\omega^c . \quad (32)$$

This case of D_ω^R location takes place when conditions

$$\omega_{\max} < \omega_{e_{\min}} ,$$

$$\omega_{\min} \geq \omega_{e_{\max}} .$$

are not satisfied.

In this case the specific portion, P^+, of the root locus portrait *P* crosses the stability bound within the parameter function increasing regions and the specific one, P^-, – within the decreasing regions:

$$P = P^+ + P^-, \quad (33)$$

The increasing part of (33), when $P^- = \varnothing$, was considered in section A. Therefore, in this section consider only the decreasing part, P^-. Consider first the family *Z* of the root locus portrait P^- initial points. Based on the previous conclusions and the well known peculiarities of the root loci asymptotic properties [2, 11, 12], the following statement could be formulated.

Statement 4. If condition (32) holds, family *Z* of initial points of the dynamic system root locus portrait, described by characteristic polynomial (5), can be located in both left half-plane *L* and right half-plane *R,* i.e. the following options of *Z* location may take place:

$$Z \subset L , \quad (34)$$

$$Z \subset (L + R) , \quad (35)$$

$$Z \subset R . \quad (36)$$

Evidently, options (35) and (36) take place when

$$D_\omega^R \subset D_\omega^- \quad (37)$$

$$\text{or } D_\omega^R \cap D_\omega^- . \quad (38)$$

As options (35) – (38) a priori indicate instability of the system in whole, consider below the only option (34) of the system poles location,

$$\omega_{\max} < \omega(z') , \quad (39)$$

where $\omega(z')$ is the coordinate ω at point z' (Fig. 1).

Therefore, define a set *S* of intervals s_i of the system root locus portrait P^- branches by expression (25). Then, based on (32) and (34) write for P^-:

$$\bigcap_{i=1}^{\infty} s_i = \inf S = [0, \underline{a}_4(\omega_{\max})], \quad (40)$$

where $\underline{a}_4(\omega_{\max})$ is the minimal value of function (9) at point with coordinate $\omega_{\max}$ (21). Therefore,

$$\forall\, a_4 \in [\underline{a}_4, \overline{a}_4]\ [a_4 \in [0, \underline{a}_4(\omega_{\max})] \rightarrow a_4 \in S \,\&\, P^- \subset L], \quad (41)$$

$$\forall\, a_4 \in [\underline{a}_4, \overline{a}_4]\ [a_4 \notin [0, \underline{a}_4(\omega_{\max})] \rightarrow a_4 \notin S \,\&\, P^- \not\subset L]. \quad (42)$$

Based on (29), (32) – (42) formulate the following statement for the whole portrait *P*.

Statement 5. The asymptotic stability of the dynamic system, described by interval characteristic polynomial family (5) and satisfying expression (32), is ensured when the following condition holds:

$$\overline{a}_4 < \min\{\underline{a}_4(\omega_{\min}), \underline{a}_4(\omega_{\max})\}. \quad (43)$$

From condition (43) follows that system asymptotic stability for the part P^- of portrait (33), provided that condition (34) holds, is defined by the value of $\underline{a}_4(\omega_{\max})$. It means that in this case (i.e. for part of the portrait P^-) the dominating one is a single polynomial of (5) defined at a point having coordinate $\omega_{\max}$ corresponding to the parameter function value

$\underline{a}_4(\omega_{max})$. In its turn, a point having coordinate ω_{max} is defined by expressions (21), i.e. by coefficients $\underline{a}_1, \overline{a}_3$ of (5); the parameter minimal value, $\underline{a}_4(\omega_{max})$ at this point is defined by (15), i.e. by coefficient $\underline{\boldsymbol{a}}_{\mathbf{2}}$. Therefore, for checking stability of P^- (33) it is enough to check the only one following polynomial of (5):

$$s^4 + \underline{a}_1 s^3 + \underline{a}_2 s^2 + \overline{a}_3 s + \overline{a}_4 = 0. \quad (44)$$

Taking into account that in this case the portrait is compound (33), the stability should be checked by checking two polynomials, (30) and (44) and, thus, formulate the following condition of stability.

Stability condition 2. If the interval dynamic system root locus portrait P (33), describing the family of characteristic polynomials (5), satisfies expression (32), the system asymptotic stability is ensured when polynomials

$$s^4 + \overline{a}_1 s^3 + \underline{a}_2 s^2 + \underline{a}_3 s + \overline{a}_4 = 0, \quad (45)$$

$$s^4 + \underline{a}_1 s^3 + \underline{a}_2 s^2 + \overline{a}_3 s + \overline{a}_4 = 0 \quad (46)$$

of family (5) are both stable.

However, the above results show that for case (32) for checking system asymptotic stability one can use only a single equation of (5) with constant coefficients. What equation should be chosen, depends of condition (43) checking results. If it shows that $\min\{\underline{a}_4(\omega_{min}), \underline{a}_4(\omega_{max})\} = \underline{a}_4(\omega_{min})$, then equation (45) should be used for stability check. If it is shown that $\min\{\underline{a}_4(\omega_{min}), \underline{a}_4(\omega_{max})\} = \underline{a}_4(\omega_{max})$, then the stability is checked according to (46).

For determination of coefficients of (5), ensuring satisfaction of (34) and (39), formulas (9) and (21) are used. Then, coefficients a_1 and a_3 should satisfy relationship:

$$\sqrt{\frac{\overline{a}_3}{\underline{a}_1}} < \omega(z'), \quad \overline{a}_3 < \underline{a}_1 \omega^2(z') \quad (47)$$

Stability conditions 1 and 2 are applied for checking the system stability; expressions (29), (43) and (47) could be applied for the system parametric synthesis.

Conclusions

Investigation of the fourth power dynamic system characteristic polynomial behavior in conditions of the interval parameter variations has been carried out on the basis of root locus portraits and introduction of a notion of the "diagram of the root locus parameter function values distribution along the stability bound". Behavior regularities for the root locus portraits of interval polynomials at the stability bound have been formulated. On this basis the stability conditions have been derived, graphic-analytical method has been worked out for calculating intervals of parameter variation ensuring the system robust stability. In continuation to the results of B.D.O. Anderson [16] and V.L. Kharitonov [4] in this work it is proved that for the 4th power interval systems families asymptotic stability analysis it is enough to use the only one polynomial of the family. Moreover, the discovered regularities of the root locus portrait behavior allow to extract stable subfamilies from the unstable families of interval polynomials.

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РЕЗЮМЕ

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Дослідження і робастний синтез збурених поліномів на підставі діаграми розподілу функції параметра кореневого годографа

У роботі розглянуті питання аналізу і синтезу характеристичних поліномів динамічних систем четвертого порядку, дослідження їхньої поведінки в умовах інтервальної параметричної невизначеності на підставі використання кореневих портретів. Метод кореневого годографа є потужним та ефективним інструментом для синтезу і аналізу робастно стійких поліномів відповідно до умов до їхньої якості. У результаті його застосування для кореневих портретів систем визначені закономірності поведінки та відповідні діаграми розподілу функції параметра кореневого годографа вздовж межі асимптотичної стійкості.

На цій основі отримані умови стійкості, розроблений графоаналітичний метод для обчислень інтервалів варіації параметрів поліноміального сімейства, що забезпечують його робастну стійкість.

На розвиток результатів, що отримані B.D.O. Anderson і В.Л. Хариноновим, у даній статті доказано, що для аналізу асимптотичної стійкості інтервальних поліноміальних сімейств четвертого порядку достатньо перевірити тільки один поліном сімейства.

Закономірності поведінки кореневого портрету, що виявлені, дозволяють виділяти гурвіцеви підсімейства з негурвіцевих сімейств інтервальних поліномів і визначати, чи існує хоча б один стійкий поліном у нестійкому в цілому поліноміальному сімействі.

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