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ON THE INFLUENCE OF FUZZY PERCEPTION OF THE TIME PASSAGE SPEED ON THE SOLUTIONS OF OPTIMIZATION PLANNING PROBLEMS

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Annotation. The paper considers an approach to the construction of fuzzy structured numerical sets based on the formation of a fuzzy original of a special type with subsequent replication on the number axis. The constructive principle of fuzzy original formalization is to define a fuzzy triangular number with the appropriate support. A variant of the formation of fuzzy numerical sets, formalizing the "fast" and "slow" passage of time, is proposed. The developed technique allows us to propose a solution to the problem associated with the formalization of the subjective perception of the time reference in processes involving a person, to obtain results and evaluate the impact of fuzzy description and taking into account the dynamics of the time reference on the solution of various optimization problems. A general approach to the formation and solution of fuzzy linear optimization problems is considered, the transition from fuzzy to parametric formulation in the Bellman-Zade form is described. Examples of the use of fuzzy flow of time for different statements of tasks that arise when determining the order of the set of tasks within a given time interval with or without additional restrictions on the execution process are considered. An approach is proposed for the correction of the initial time distribution plans, taking into account different rates of time counting. A mathematical model of the fuzzy traveling salesman problem is formulated as a problem of finding a route to visit a given number of cities without repetitions with a minimum travel time with time parameters specified as right fuzzy numbers, the support value in which depends on various external conditions and factors. The results of calculations of solutions of fuzzy optimization problems are obtained. The influence of the speed of the passage of time on the solution of the considered optimization problems is determined.

Keywords: triangle numbers, time passage formalization, fuzzy optimal tasks.

Introduction

The behavior of people significantly depends on their emotional state. Emotions in human organisms perform the function of a local control criterion, set by some objective function, depending on specific factors influencing emotions. Emotions can “trigger” behavior corresponding to a particular situation, which is not necessarily optimal, but allows avoiding (perhaps with losses) significant consequences of exceeding some important (for example, temporal) resource [1].

On the other hand, it should be noted that the goal of building and developing set-theoretic approaches in the field of modeling human behavior has always been the desire to adapt mathematical models to real life, to be able to organically combine the potential of computational methods with the specifics of human thinking [2]. Such tasks are typical for the implementation of artificial intelligence methods and algorithms, for the development of decision support tools, for solving resource allocation problems taking into account the influence of the human factor, and the like.

As mentioned above, one of the important resources considered in processes involving a person is time. The concept of the magnitude of the time interval is necessary to describe the time intervals, the exact limits of which in this case may not be known until a specific state of the process occurs. As a consequence, each period of time is determined by some interval, the boundaries of which are currently not defined, but can be described approximately taking into account the characteristics of the flow of time. This is possible provided that information about the boundaries of the guaranteed placement of a given interval on the time scale is considered available, and the boundaries of the interval can be specified over time. The measurement of time intervals can also be represented in the form of linguistic terms that determine the speed of the time resource, for example, a quick response, a normal time count or a long wait. Accordingly, when solving problems in which there is a need to implement verbal terms to describe the time reference, one should take into account the uneven flow of time. It is also obvious that the perception of the rate of time reference in various processes involving a person is significantly influenced by emotions.

One of the approaches for formalizing the representation of time intervals that determine different rates of time counting is the construction of time interval boundaries of two types: exact and guaranteed, for which special predicates are used [3]. In [4], an attempt was made to describe the flow of time in the form of fuzzy quantities of a special type. The representation of fuzzy terms is determined using specific membership functions of some fuzzy numbers, which are built on the basis of a body of knowledge obtained from the repository or on the basis of the results of expert information processing.

Purpose of the study

Among the tasks in which, in order to find an optimal or effective solution, it is necessary to take into account the factors influencing the emotional state of a person, and, as a result, the rate of perception of the time reference, it is necessary to single out the tasks of resource allocation, scheduling, planning, etc. In this paper, we propose to consider further detailing the methodology for formalizing the process of time flow based on fuzzy numbers and its application in solving individual fuzzy optimization problems of time distribution, taking into account the uncertainty associated with the influence of non-uniform rates of time counting.

Main materials

1. Formalization of structured fuzzy numbers to describe the timing.

Definition 1[5]. A fuzzy set $\tilde{A}$ in the universal space X is a collection of pairs of the form $\{(x, \mu_{\tilde{A}}(x))\}$, where $x \in X$, $\mu_{\tilde{A}}(x)$: $X \to [0,1]$ is the membership function of the fuzzy set $\tilde{A}$.

The value of the membership function $\mu_{\tilde{A}}(x)$ for an arbitrary element $x \in X$ determines the degree of membership x to the fuzzy set $\tilde{A}$. If a subset of real numbers $X \subseteq R^1$ is considered as a universal space X, the fuzzy set $\tilde{A}$ contains a collection of pairs composed of two scalar values $x \in R^1$ and $\mu_{\tilde{A}}(x)$.

Definition 2 [6]. A fuzzy triangular number $\tilde{b}$ is an ordered triple of numbers $\tilde{b} = \{(a,b,c)\}$, $a \le b \le c$, for which the membership function $\mu_{\tilde{b}}(x)$ is defined in the form:

$$\begin{aligned} &\mu_{\tilde{b}}(x) = \frac{x-a}{b-a}, x \in [a,b]; \\ &\mu_{\tilde{b}}(x) = \frac{c-x}{c-b}, x \in [b,c]; \qquad (1) \\ &\mu_{\tilde{b}}(x) = 0, x \notin [a,c] . \end{aligned}$$

A fuzzy triangular number of the form *(a, b, b)*, called a left fuzzy triangular number [6], is determined by the membership function of the form

$$\begin{aligned} &\mu_{\tilde{A}}(x) = 0, x < a ; \\ &\mu_{\tilde{A}}(x) = \frac{x-a}{b-a}, x \in [a,b]; \qquad (2) \\ &\mu_{\tilde{A}}(x) = 1, x > b , \end{aligned}$$

and the fuzzy triangular number of the form *(b, b, c)*, called the right fuzzy triangular number, is the membership function

$$\begin{aligned} &\mu_{\tilde{A}}(x) = 1, x < b ; \\ &\mu_{\tilde{A}}(x) = \frac{c-x}{c-b}, x \in [b,c]; \qquad (3) \\ &\mu_{\tilde{A}}(x) = 0, x > c . \end{aligned}$$

For a fuzzy number $\tilde{b}$, the support $\operatorname{supp} \tilde{b} = \{x \in X : \mu_{\tilde{b}}(x) > 0\}$ is an interval [6]. In this case, for a fuzzy triangular number $\tilde{b} = (a,b,c)$, the support will be an interval (a,c), for a right fuzzy triangular number - an interval $[b,c)$, for a left fuzzy triangular number - an interval $(a,b]$.

Over fuzzy sets, all operations are defined that are characteristic of the traditional theory of exact sets, but, of course, with their own specifics, given by the very essence of fuzzy sets. Operations of intersection, union, complement and others for fuzzy sets are determined by known relations [5], but cannot fully provide a number of useful properties of fuzzy numbers that generalize and extend operations with ordinary (clear) numbers.

Definition 3. An arbitrary fuzzy triangular number $\tilde{E}(u,v)$, whose support is a numerical interval (u,v), $u<v$, will be called a fuzzy original.

A fuzzy original $\tilde{E}(u,v]$ will be called left if the support of the corresponding fuzzy number is given by an interval $(u,v]$, and a fuzzy original $\tilde{E}[u,v)$will be called right if the support is given by an interval $[u,v)$.

Obviously, an arbitrary right fuzzy triangular number *(b, b, c)* can be considered as a right fuzzy original with the corresponding values $u=b, v=c$, and a left fuzzy triangular number *(a, b, b)* as a left fuzzy original with $u=a, v=b$.

If we assume $u=0, v=1$, then the fuzzy original $\tilde{E}(0,1)=\tilde{E}$ will be called the fuzzy single original, and, accordingly, $\tilde{E}(0,1]$ - the left fuzzy single original, $\tilde{E}[0,1)$ - the right fuzzy single original. The fuzzy original $\tilde{E}(0,v)$, $v>0$, with support $(0,v)$ will be called initial.

The name of the introduced concept is explained by the fact that with the help of a given fuzzy original one can formulate a fuzzy set of a special type with some characteristic properties.

Definition 4. A fuzzy numerical number $\tilde{R}\ (s,t)$ with a support given by a numerical interval (s,t), $s<t$, is a replication of a fuzzy original (or a fuzzy replication) $\tilde{E}(u,v)$, $u<v$, if the condition $|v-u|=|t-s|$ is true.

It should be noted that the membership function defined on the fuzzy original does not necessarily coincide with the replication membership function $\tilde{R}\ (s,t)$. In this case $|t-s|=1$, the set $\tilde{R}\ (s,t)$ is a replication of a fuzzy single original.

Definition 5. A fuzzy number $\tilde{R}=\bigcup_{i=1}^{n}\tilde{R}_i(s_i,t_i)$, where $\tilde{R}_i(s_i,t_i)$, $i=\overline{1,n}$, are replications of a fuzzy original $\tilde{E}(u,v)$, $u<v$, with supports specified by numerical intervals (s_i,t_i), $i=\overline{1,n}$, $\bigcap_{i=1}^{n}(s_i,t_i)=\varnothing$, will be called an n-dimensional replication of a fuzzy original $\tilde{E}(u,v)$.

The multiplicity of replication can be infinite, according to which such a replication will be called the replication of a fuzzy original of infinite multiplicity.

Definition 6. Fuzzy numerical set $\tilde{R}=\bigcup_{i=1}^{n}\tilde{R}_i(s_i,s_{i+1})$, where $\tilde{R}_i(s_i,s_{i+1})$, $i=\overline{1,n}$, is replications of a fuzzy original $\tilde{E}(u,v)$, $u<v$, with supports specified by numerical intervals (s_i,s_{i+1}), $i=\overline{1,n}$, will be called n-dimensional sequential replication of a fuzzy original $\tilde{E}(u,v)$.

Similarly, we can consider the case of replications, for which the values of the membership functions of elements of the fuzzy original are completely preserved.

Definition 7. A fuzzy replication $\tilde{C}\ (s,t)$, $s<t$, of a given original $\tilde{E}(u,v)$, $u<v$, is called a copy if the condition $\mu_{\tilde{C}}(y)=\mu_{\tilde{E}}(x)$ is satisfied for all $x=u+\delta$, $y=s+\delta$, $0\le\delta\le\Delta$, $\Delta=v-u=t-s$.

Definition 8. Fuzzy replication $\tilde{C}=\bigcup_{i=1}^{n}\tilde{C}_i\ (s_i,t_i)$, where $\tilde{C}_i(s_i,t_i)$, $i=\overline{1,n}$, are copies of fuzzy original $\tilde{E}(u,v)$, $u<v$, with supports given by numerical intervals (s_i,t_i), $i=\overline{1,n}$, $\bigcap_{i=1}^{n}(s_i,t_i)=\varnothing$, will be called an n-dimensional copy of the fuzzy original $\tilde{E}(u,v)$.

Definition 9. Fuzzy replication

$\tilde{C} = \bigcup_{i=1}^{n} \tilde{C}_i(s_i, s_{i+1})$, where $\tilde{C}_i(s_i, s_{i+1})$, $i = \overline{1,n}$, is copies of the fuzzy original $\tilde{E}(u,v)$, $u < v$, with supports specified by numerical intervals (s_i, s_{i+1}), $i = \overline{1,n}$, will be called an n-dimensional sequential copy of the fuzzy original $\tilde{E}(u,v)$.

As in the case of replications of infinite multiplicity, copies will be called copies of the fuzzy original of infinite multiplicity. It is also clear that the definitions of right and left replications and copies of the fuzzy original will be similar.

We use formalized fuzzy structured numerical sets to describe the uncertainty in the measurement of the time count. The duration of time intervals, presented in linguistic form, is a reflection of the subject's assessment of the time interval, taking into account the emotional impact of specific conditions associated with the mode of observing time. It is known that the lack of time that occurs in situations with prompt decision-making, speed testing, competitions with a time limit, leads to "acceleration" of the flow of time. Accordingly, in the presence of an excess of time and the absence of "consumed" processes (under the conditions of waiting for events, prolonged inactivity, etc.), the passage of time slows down. In other words, the measurement of the time reference in different situations is determined by a subjective assessment, which can be described by a fuzzy triangular value.

Suppose that the measurement of time occurs through intervals of the same duration (such intervals can be considered any unit of time, for example, 1 second, 1 day or 1 year, depending on the pace of the process). To monitor the fuzzy time count, we will intuitively estimate the interval remaining until the end of each time interval. In this case, the “fast” flow of a unit of time can be given by a right fuzzy triangular number with a support whose length is less than the duration of the time interval, and “slow” - by a fuzzy number of the same type with a support whose length is greater than the duration of the interval. Obviously, if time flows in a natural way, then the value of the support of this fuzzy number coincides in length with the value of a unit time interval.

Thus, having a sample of the time interval measurement reference in the form of the right initial original $\tilde{E}[0,v)$, it is possible to determine a fuzzy n-dimensional sequential copy based on it, which will describe the changes in time over a given time interval.

Let $\tilde{E}^Q[0,v_Q)$ be a right fuzzy original in the form of a triangular number with a linear descending membership function that defines a “fast” unit time interval ($v_Q < 1$), and $\tilde{E}^D[0,v_D)$ - a similar fuzzy original that defines a “slow” unit interval ($v_D > 1$). Let's define $\tilde{C}^Q = \bigcup_{i_Q=1}^{n_Q} \tilde{C}^Q_{i_Q}[s_{i_Q}, s_{i_Q+1})$ - a fuzzy n_Q-fold sequential copy of the original $\tilde{E}^Q[0,v_Q)$, which describes a "fast" change in time over a period $[s_1, s_{n_Q+1})$, and $\tilde{C}^D = \bigcup_{i_D=1}^{n_D} \tilde{C}^D_{i_D}[s_{n_Q+i_D}, s_{n_Q+i_D+1})$ - a fuzzy n_D -fold sequential copy of the original $\tilde{E}^D[0,v_D)$, which determines a "slow" change in time over an interval $[s_{n_Q+1}, s_{n_Q+n_D+1})$. Here $\tilde{C}^Q_{i_Q}[s_{i_Q}, s_{i_Q+1})$, $i_Q = \overline{1,n_Q}$, $\tilde{C}^D_{i_D}[s_{n_Q+i_D}, s_{n_Q+i_D+1})$, $i_D = \overline{1,n_D}$, are copies of right fuzzy originals $\tilde{E}^Q[0,v_Q)$ and $\tilde{E}^D[0,v_D)$, respectively, with supports given numerical intervals of given length v_Q and v_D .

Then a fuzzy numerical set of the form $\tilde{C}^Q \cup \tilde{C}^D$ will describe the countdown of time on the interval $[s_1, s_{n_Q+n_D+1}]$. This countdown consists of two phases: the fast time change phase on $[s_1, s_{n_Q+1}]$ and the slow time change phase on $[s_{n_Q+1}, s_{n_Q+n_D+1}]$. It is obvious that by changing the nature of the time counting on the intervals and combining the obtained fuzzy copies, it is possible to build a fuzzy number that reflects the fuzzy perception of the flow of time on an arbitrary time interval. In addition, the resulting fuzzy numerical set can be supplemented with a copy of a single original $\tilde{E}$ that determines the natural time reference.

It should be noted that "fast" or "slow" timing is typical not only for human perception, but also for various processes in technical systems. The speed of operation of a technical device is often determined by the frequency (number per unit of time) of clock pulses applied to the input. Increasing or decreasing the frequency within the allowable interval allows you to do the same job faster or slower. In other words, the clock frequency actually determines the "fast" and "slow" passage of time. Thus, this once again confirms that a fuzzy estimate of the rate of change of the time count can be obtained on the basis of the basic fuzzy original describing the verbal term "remainder until the end of a unit of time".

2. Fuzzy optimization problems and methods for their solution.

Let us consider the problem of formalizing the influence of the subjective perception of the time reference in processes involving a person using the example of mathematical programming problems with parametric uncertainty.

Recall a typical formulation of a linear programming problem. Suppose that within the framework of a certain production process, it is planned to release various products in the amount of $x_1,...,x_n$. Denote by c_j the expected profit per unit of sold products of the type $j, j=\overline{1,n}$. For the production of any products are used the resources $b_1,...,b_m$ - the production capacities of the company, and the unit costs of the *i*-th resource $i=\overline{1,m}$, in the production of a unit of output of type *j*, $j=\overline{1,n}$ are a_{ij} units. It is necessary to find such a rational plan for the production of products of each type that provides the maximum profit for the manufacturer. The mathematical model of this problem with fixed known values of the parameters c_j, a_{ij}, $j=\overline{1,n}$, $i=\overline{1,m}$, is similar to the standard linear programming problem of the form [7]:

$$\max \sum_{j=1}^{n} c_j x_j \tag{4}$$

with constraints

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i , \quad i=\overline{1,m}, \tag{5}$$

$$x \ge 0; \; x \in R^n .$$

If the set of feasible solutions defined by the system of constraints is complete or enumerated, then we obtain a class of discrete linear programming problems [7], which includes a Boolean programming problem of the form (4), (5) with additional constraints on variables in the form $x_j = \{0,1\}$, $j=\overline{1,n}$.

One of the most well-known problems of Boolean programming is the knapsack problem, the content of which is the optimal placement of a set of a given number of objects in a knapsack of a given capacity, provided that two parameters are given for each of them - volume and value.

Suppose that V is the volume of the knapsack, each of the n items has a volume v_j and value c_j, $j=\overline{1,n}$, and all characteristics are given by positive numbers. One of the deliverables of the knapsack problem is to determine the set of items placed in the knapsack, the total value of which is maximum, taking into account the constraint on the size of the knapsack [7], which is written as:

$$\max \sum_{j=1}^{n} c_j x_j \tag{6}$$

$$\sum_{j=1}^{n} v_j x_j \le V , \tag{7}$$

where $x_j = \{0,1\}$, $j=\overline{1,n}$.

In practice, the parameters of optimization problems are often not known, or only intervals of possible values can be determined for them. Problems of this type are called problems with multivalued coefficients. In this case, the specification and refinement of the parameters of mathematical models is possible based on the description of the coefficients in the form of fuzzy numbers. Additional information is introduced into the model by means of membership functions of fuzzy values. These functions can be considered as a way for an expert to approximate an informal representation of the real value of a particular parameter. The value of membership functions is the weight coefficients that experts assign to different possible values of each parameter of the problem.

Let us consider the formalization of such a refinement using the example of setting a general fuzzy mathematical programming problem [8], similar to a similar standard linear programming problem of the form (4), (5). In this case, it is considered that a linear objective function of the form

$$\sum_{j=1}^{n} \tilde{c}_j x_j \to \max, \tag{8}$$

in which the values of the coefficients $\tilde{c}_j$ are presented in the form of fuzzy numbers and the constraint is given

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \le \tilde{b}_i, \quad i = \overline{1,m}, \quad x_j \ge 0, \; j = \overline{1,n},$$

where the values of the coefficients $\tilde{a}_{ij}$, $\tilde{b}_i$ are also described in the form of the corresponding fuzzy numbers. It is necessary to make a rational choice of a solution $x \in R^n$, that, in a certain sense, maximizes the given fuzzy linear form (8).

The general formulation of the fuzzy optimization problem makes it possible to determine its individual variants, which are characterized by the presence of uncertainty only in part of the parameters. In the case of a fuzzy problem of linear mathematical programming with fuzzy technological coefficients, we obtain the problem of optimizing the objective function (7) under the condition

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \le b_i, \; i = \overline{1,m}, \tag{9}$$

$$x \ge 0; \; x \in R^n.$$

Here $\tilde{a}_{ij}, i = \overline{1,m}, j = \overline{1,n}$, are technological coefficients of the problem, given in the form of right fuzzy numbers $(a_{ij}, a_{ij}, a_{ij} + d_{ij}), i = \overline{1,m}, j = \overline{1,n}$, with membership functions:

$$\mu_{a_{ij}}(s) = \begin{cases} 1, & s = a_{ij}, \\ (a_{ij} + d_{ij} - s)/d_{ij}, & a_{ij} < s < a_{ij} + d_{ij}, \\ 0, & s \ge a_{ij} + d_{ij} \; ma \; s < a_{ij}, \end{cases}$$

$$i = \overline{1,m} \;\; j = \overline{1,n}, \tag{10}$$

where $d_{ij} > 0$, $i = \overline{1,m}$ $j = \overline{1,n}$.

To find solutions to the fuzzy problem (4), (9)-(10), it is necessary to carry out its defuzzification. To do this, we calculate the optimal values of the levels of the objective function (1) Z_l and Z_u by solving two linear programming problems [7]

$$Z_l = \max_x \sum_{j=1}^{n} c_j x_j,$$

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i, \; i = \overline{1,m} \; x \ge 0; \; x \in R^n,$$

and

$$Z_u = \max_x \sum_{j=1}^{n} c_j x_j,$$

$$\sum_{j=1}^{n} (a_{ij} + d_{ij}) x_j \le b_i, \; i = \overline{1,m} \; x \ge 0; \; x \in R^n.$$

Assuming that both problems have a solution, we obtain that the optimal value of the objective function (4) will lie between the values Z_l and Z_u and when placing technological coefficients between the values a_{ij} and $a_{ij} + d_{ij}$, $i = \overline{1,m}$, $j = \overline{1,n}$.

Let $L = \min(Z_l, Z_u)$, $U = \max(Z_l, Z_u)$. The calculated values L and U will be, respectively, the lower and upper limits of the optimal values of the objective function.

The fuzzy value of optimal values (let us denote it as $\tilde{G}$), given in space R^n, is described by the membership function of the form

$$\mu_{\tilde{G}}(x) = \begin{cases} 1, & \sum_{j=1}^{n} c_j x_j \le L, \\ (U - \sum_{j=1}^{n} c_j x_j)/(U - L), & L < \sum_{j=1}^{n} c_j x_j < U, \\ 0, & \sum_{j=1}^{n} c_j x_j \ge U, \end{cases} \tag{11}$$

Fuzzy constraints ($\tilde{F}_i$, $i = \overline{1,m}$) from (9) are determined by the membership functions

$$\mu_{\tilde{F}_i}(x) = \begin{cases} 1, & \sum_{j=1}^{n} a_{ij} x_j \le b_i, \\ \dfrac{\sum_{j=1}^{n}(a_{ij}+d_{ij})x_j - y}{\sum_{j=1}^{n} d_{ij} x_j}, & for\, y = \sum_{j=1}^{n}(a_{ij}+\alpha_{ij})x_j \le b_i, \\ & 0 \le \alpha_{ij} \le d_{ij}, \exists k\ 0 < \alpha_{ik} < d_{ik}, \\ 0, & \sum_{j=1}^{n}(a_{ij}+d_{ij})x_j \le b_i, \end{cases}$$

$$d_{ij} > 0,\ i = \overline{1,m},\ j = \overline{1,n}. \quad (12)$$

Using the definition of a fuzzy solution as understood by Belman-Zade [6], we rewrite the fuzzy linear programming problem (4), (9)-(10) in the form of a crisp optimization problem

$$\max_x \lambda$$
$$\begin{matrix} \mu_{\tilde{G}}(x) \ge \lambda, \\ \mu_{\tilde{F}_i}(x) \ge \lambda,\ i = \overline{1,n}, \end{matrix} \quad x \ge 0. \quad (13)$$

Substituting (11) and (12) into (13), we write the final form of the problem

$$\max_x \lambda$$
$$\begin{aligned} &\sum_{j=1}^{n} c_j x_j + \lambda(U-L) \le U, \\ &\sum_{j=1}^{n} (a_{ij} + \lambda d_{ij}) x_j \le b_i,\ i = \overline{1,m}, \\ &x \ge 0,\ 0 \le \lambda \le 1. \end{aligned} \quad (14)$$

It is necessary to designate that the subdivision of the problem (14) is to take care of the production λx_j, $j = \overline{1,n},$, and that is not ocular. In this way, in order to solve the problem (14), it is necessary to develop methods that are recognized for the search for the solution of non-broad optimization problems.

3. The fuzzy problem about the knapsack as a way to find the optimal distribution of the clock resource with the fuzzy terms of the implementation (implementation).

One of the most important optimization tasks, which allows fuzzy formulation on the basis of fuzzy technological coefficients, is the task of distributing the hourly resource, which can be formalized in a look at the tasks about the knapsack.

Let's assume that it is necessary to complete N tasks Z_i, $i = \overline{1,N}$, each of which are characterized by a time of performance T_i, $i = \overline{1,N}$. It is possible that for switching from the one task to another, a necessary time interval (time of changeover) π_{ij}, $i = \overline{1,N}$, $j = \overline{1,N}$, $i \ne j$. It is important that the performance of tasks is carried out by one performer, and the time for performance of all tasks is surrounded by a certain value $\overline{T}$, $\sum_{i=1}^{N} T_i \ge \overline{T}$.

Let the variance of the skin problem be assessed according to the number of balls B_i, $i = \overline{1,N}$. For the purpose of working to achieve the highest possible score for the mind, so that the intervals of an hour for reworking the job are not insured, we can formulate the task of the most efficient hourly resource for looking at the task of a knapsack

$$W = \sum_{i=1}^{N} B_i x_i \to \max, \quad (15)$$

$$\sum_{i=1}^{N} T_i x_i \le \overline{T}, \quad (16)$$

$$x_i \in \{0,1\},\ i = \overline{1,N}.$$

Here $x_i \in \{0,1\}$, $i = \overline{1,N}$, - boolean variables whose single values correspond to the performance of the task, and zero - its non-performance.

For the formalization of the fuzzy problem, we will take into account the time resource, that the terms of completion tasks are only approximate. Let's write down the parameters of the hour, which is necessary for the work to be carried out at the sight of fuzzy triangle numbers $\tilde{T}_i = (T_i, T_i, T_i + \Delta_i)$, $i = \overline{1,N}$, де $\Delta_i \ge 0$, $i = \overline{1,N}$, - allowable delays in work carried out, depending on the conditions of the task.

Similarly, the fuzzy problem of the most efficient choice of the hourly resource can be written down in a seemingly fuzzy task about a knapsack with an objective function

(15) and formally written down as an exchange

$$\sum_{i=1}^{N} \tilde{T}_i x_i \le \bar{T} , \qquad (17)$$

$$x_i \in \{0,1\} ,\ i = \overline{1,N} ,$$

with fuzzy defined terms, the calculation of fuzzy numbers (interpretation of unevenness (17) transfers, so that the addition of that pairing of fuzzy numbers is carried out according to the given rules for fuzzy quantities [5]).

The problem is formulated as a problem of mathematical programming with fuzzy technological coefficients. The best sum score is assigned to the value $W^o = W(x^0, \lambda^0)$ of goal function (15) on the optimal fuzzy solution (x^0, λ^0) of the Bellman-Zadeh optimization problem (14).

If you want to pay an hour for a changeover when switching from one job to another, the order of work becomes important. It is necessary to bring to the point of necessity the development of combinatorial optimization methods. The solution to the problem of optimal timing $\bar{T}$ will be the permutation of task numbers $p = (p_1,..., p_m)$, $m \le N$, $p_i \in \overline{1,N}$, $i = \overline{1,m}$, as it determines the sequence of the task to work and as a solution to the problem of optimization, similar to (15), (17):

$$W = \sum_{i=1}^{m} B_{p_i} y_{p_i} \to \max , \qquad (18)$$

$$\sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \ne i}}^{N} (\tilde{T}_i + \pi_{ij}) x_{ij} \le \bar{T} , \qquad (19)$$

where $x_{ij} \in \{0,1\}$ - variables characterizing the order of work execution (with a single value, work i is performed immediately before work j, in other cases - 0), $i, j = \overline{1,N}, i \ne j$,

$$y_i = \begin{cases} 1, if\ x_{ij} = 1, \\ 0, otherwise, \end{cases} ,\ i = \overline{1,m} .$$

Examples of the above formulations are the usual situations of students' knowledge assessment based on the results of test tasks in different disciplines on module tests or final semester tests and exams, during which it is necessary to complete a number of tasks with an established time schedule for each of them and for the total number of tasks. in general.

It is also clear that the formulation of the task of distributing a time resource with fuzzy deadlines is associated not only with the impossibility of estimating the time of work, but also with an attempt to take into account the influence of subjective factors of perception of the time count on the speed of task completion.

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Assuming further that the state of the executor (subject or object) can affect the performance of tasks of a certain set of works (for example, the timely completion of individual tasks has a positive effect, and failure to complete or delay the completion of work slows down the solution of the entire set of tasks), optimization problems arise for the effective use of time in the form $\bar{T}$ (15), (16); (15), (17) and (18), (19) taking into account the fuzzy timing of the deadlines for the implementation of individual tasks.

Let us consider that the “fast” passage of time unit on the i-th interval, $i = \overline{1,N}$, is described by the right fuzzy triangle number $\tilde{E}^Q[0, v_i)$, $i = \overline{1,N}$, with the support, the length of some less for the trivality of the single hour interval ($v_i < 1$), and the “slow” passage – by the fuzzy triangle number $\tilde{E}^D[0, v_i)$, $i = \overline{1,N}$, with the support, the length of what more for the triviality of a unit interval ($v_i > 1$). The aging of the dermal task is carried out in a special mode, which is not changed by the length of the hourly interval of the specific task.

Let the solution of the problem of optimal use of time (15)-(16); (15), (17) or (15)-(19) be obtained by permuting from task numbers $p = (p_1,..., p_m)$, $m \le N$, $p_i \in \overline{1,N}$, $i = \overline{1,m}$, the elements of which specify the sequence of work execution. Then the values

T_{p_i}, $p_i \in \overline{1,N}$, $i=\overline{1,m}$, determine the nominal (planned) execution time of each task, and the values $T^r_{p_i} = T_{p_i} * v_{p_i}$, $p_i \in \overline{1,N}$, $i=\overline{1,m}$, at the same time will set the time actually spent on the execution of the current task, taking into account its “fast” or “slow” execution.

For such minds, it was understood that the result of the "quick" task completion is the rest of the time, which can be further used to solve other tasks, nd the "slow" execution, on the contrary, leads to a shortage of time, the coverage of which must be carried out at the expense of time resources allocated for the execution of further tasks. Such a correction of the execution time can lead to a change in the solutions obtained in solving optimization problems: a task that is not included in the original plan may be attached to the list, or there will not be enough time from the available actual resource $\overline{T}$ to complete the entire plan obtained.

To simulate and demonstrate the influence of time dynamics on the solution of the problem of optimal use of time $\overline{T}$, a number of numerical experiments were carried out. Formalization of the fuzzy assessment was carried out according to a simplified scheme. The result of the execution or failure of the current task was modeled randomly. At the same time, the fulfillment of the task “slowed down” the rate of decrease in the hour, and the failure to fulfill it, on the contrary, “accelerated” it. The change in speed during the calculation process was fixed as a modification of the time $\overline{T}$ allocated for the execution of all tasks: the value increased in proportion to the time of the current task completed (the effect of slowing down the rate of time flow) and decreased - if the current task was not completed (the effect of acceleration).

The results of numerical distributions of the time resource are obtained when performing a set of tasks for the case of fuzzy terms for their implementation [9].

4. Statement of the traveling salesman problem (TSP, Traveling Salesman Problem).

Let's assume that he is a representative of a trading company, who wants to propagate or reconsider the presence of his company's goods at the points of sale of the actual stock of a particular region. The route of yoga traffic is to blame for the revenge of the points, which are registered at the head, moreover, it is necessary to see the leather at the points no more than once. It’s more expensive to pick up a lot of time, so it’s logical that a representative wants to lay down his route in such a rank, so that he’ll stand, like it’s necessary to repair it, it will be minimal (as a criterion, you can also look at the sign of a path of minimal mileage or by passing expenses). The search for such a route is the main task in solving the traveling traveling problem [7].

The Traveler Problem is a combinatorial problem, for which one can use methods of mathematical programming. To reduce the problem to a general form, we number the cities by numbers *(1, 2, 3, ..., n)*, and describe the traveling salesman's route by a cyclic permutation of numbers, moreover all , $t = (j_1, j_2, ..., j_n, j_1)$, are different numbers. The number repeated from the beginning to the end indicates that the permutation is cyclic.

The set of cities can be considered as the vertices of some graph with given distances (or travel time) between all pairs of vertices c_{ij} that form the matrix $C=(c_{ij})$, $i,j=\overline{1,n}$. We assume that this matrix is symmetric. The formal problem then is to find the shortest route (in time or length) that goes through each city and ends at the starting point. In this formulation, the problem is called the closed traveling salesman problem (TSP), which is a well-known mathematical integer programming problem.

The mathematical statement of the TSP problem.

Let $I=\{1,...,n\}$ be the set of vertex indices of the problem graph. The objective function is the total time of the route, including all the vertices of the task graph. The parameters of the problem are the elements of the matrix $C=(c_{ij})$, $i, j \in I$.

The variables of the problem are the elements of the binary matrix of transitions between the vertices $X = \{x_{ij}\}$, $i, j \in I$, which are equal to 1 if there is an edge(v_i, v_j) in the constructed route for the task, 0 otherwise [10]. The optimal route is the shortest route in terms of time:

$$E = \sum_{i \in I} \sum_{j \in I, j \neq i} c_{ij} x_{ij} \to \min \qquad (20)$$

with constraints

$$\sum_{j \in I, j \neq i} x_{ij} = 1,\ i \in I,$$

$$\sum_{i \in I, j \neq i} x_{ij} = 1,\ j \in I, \qquad (21)$$

$$v_i - v_j + n x_{ij} \leq n - 1,\ 1 \leq i \neq j \leq n.$$

The last inequality ensures the connectivity of the vertex traversal route; it cannot consist of two or more unconnected parts.

Algorithms that allow solving the problem of finding the optimal route can be divided into exact and heuristic. For small problems (for example, for the purpose of primary design of a small transport network), it is advisable to use exact algorithms, since their implementation requires high computing power, which cannot be said about real problems: for them, as a rule, it is necessary to use heuristic algorithms. One of these methods is the simulated annealing method [11].

Mathematical model of fuzzy problem TSP.

We will assume that the movement along each of the stages of the path under normal conditions can be carried out in time $b_{ij} \geq 0$, $i, j = \overline{1, n}$. Suppose that each stage of the travel path can be divided into separate sections (for example, ascent, descent, road repair section, etc.), the number of which is denoted by $d_{ij} \geq 0$, $i, j = \overline{1, n}$. It is known that, under normal conditions, movements in these areas are carried out in time $b_{ijk} \geq 0$, $k = \overline{1, d_{ij}}$, $i, j = \overline{1, n}$.

Taking into account information about the state of the paths between individual points of the transport network at the beginning of solving the traveling salesman problem, we estimate the travel time for all sections, which can be less (the section is overcome faster) $\underline{b_{ijk}} \geq 0$, $k = \overline{1, d_{ij}}$, $i, j = \overline{1, n}$, or more (delays are possible) $\overline{b_{ijk}} \geq 0$, $k = \overline{1, d_{ij}}$, $i, j = \overline{1, n}$, of usual time.

It is clear that the shortest time $\underline{b_{ijk}} \geq 0$, $k = \overline{1, d_{ij}}$, $i, j = \overline{1, n}$, must be limited by the rules and possibilities of movement, and the longest time $\overline{b_{ijk}} \geq 0$, $k = \overline{1, d_{ij}}$, $i, j = \overline{1, n}$, is considered to be finite (i.e., the option of a complete stop is not considered).

Under such conditions, the time of movement along individual sections of the path can be described by fuzzy triangular numbers $\tilde{b}_{ijk} = (\underline{b_{ijk}}, b_{ijk}, \overline{b_{ijk}})$, $k = \overline{1, d_{ij}}$, $i, j = \overline{1, n}$, and the parameters of the problem specified by the elements of the matrix $C=(\tilde{c}_{ij})$, $i, j = \overline{1, n}$, are fuzzy triangular numbers $\tilde{c}_{ij} = \tilde{b}_{ij} = (\underline{b_{ij}}, b_{ij}, \overline{b_{ij}})$, $b_{ij} = \sum_{k=1}^{d_{ij}} b_{ijk}$, $\underline{b_{ij}} = \sum_{k=1}^{d_{ij}} \underline{b_{ijk}}$, $\overline{b_{ij}} = \sum_{k=1}^{d_{ij}} \overline{b_{ijk}}$, $i, j = \overline{1, n}$, with linear membership functions of the form (1).

The objective function of the traveling salesman problem takes the form

$$E = \sum_{i \in I} \sum_{j \in I, j \neq i} \tilde{c}_{ij} x_{ij} \to \min, \qquad (22)$$

and the problem, together with constraints (21), is a fuzzy optimization problem [8], which can be solved using the annealing simulation method.

To implement the technique for solving the fuzzy traveling salesman problem, we use an approach based on the use of level sets [8]. To do this, we are looking for two separate solutions that correspond to the smallest and largest values of the sets of the level of fuzzy parameters specified by the matrix C. The reliability level of the fuzzy solution is determined by the value that specifies the assessment of the level of influence of various factors on the duration of the route sections. This approach allows for any fuzzy numbers, $\tilde{c}_{ij}$, $i, j = \overline{1, n}$, to construct slices that are given in the form of time intervals necessary to overcome each of the stages of the traveling salesman's route.

Applying the simulated annealing method for the extreme values of these intervals, we solve the traveling salesman problems in the best and worst case of travel time along the route. Under such circumstances, we obtain a time interval that gives an estimate of the values of the criterion function (22) for the selected parameter . The minimum value will determine the time under ideal traffic conditions, the maximum - taking into account the greatest influence of factors that worsen the duration of the passage of route sections.

Different results of calculations obtained by solving the traveling salesman problem for different values of the parameter are obtained [12].

Conclusions and discussions

The article presents a constructive method for constructing a fuzzy set that describes changes in the speed (rate) of the passage of time. To formalize the fuzzy estimate of the time reference, the concept of triangular fuzzy numbers with a linear membership function is used. The approach is logically and mathematically justified. Optimization problems are formulated and investigated that operate with a time resource, the solution of which depends on taking into account the speed of counting time. The application of this approach allows us to solve practical problems of time resource allocation, taking into account the impact of the performance or non-performance of individual tasks on the search for effective solutions to problems.

The annealing simulation method is considered for solving the fuzzy traveling salesman problem, which is formulated as the problem of finding a route to visit a given number of cities without repetitions with a minimum travel time. The content of the annealing simulation method is presented, the method formalization algorithm is described. A fuzzy traveling salesman problem is formulated, in which the time parameters of movement between cities are given in the form of right fuzzy numbers, the carrier value in which depends on various external conditions and factors. The results of calculations for solving the traveling salesman problem in clear and fuzzy forms with different parameters of slices of fuzzy numbers are presented.

Further study of the topic under consideration is connected with the analysis of fuzzy optimization problems, the area of feasible solutions in which is described by a fuzzy set of indices, with the development of the heuristics of the greedy scheme for the approximate solution of problems of efficient use of time resources and with the study of the possibility of generalizing the description of fuzzy time counting using other formalization methods.

References

1. Golitsyn G.A., Petrov V.M. (1991) *Informatsiya - po¬vedeniye – tvorchestvo*. M.: Nauka.
2. Golitsyn G.A., Petrov V.M. (1995) *Information and Creation: Integrating the «Two Cultures»*. Basel; Boston; Berlin: Birkhauser Verlag.
3. Razin B.V., Tuzovskiy A.F. (2013). Predstavleniye znaniy o vremeni s uchotom neopredelonnosti v ontologiyakh Semantic Web. *Upravleniye, vychislitel'naya tekhnika i informatika. Doklady TUSURa.*, №2 (28). – 157-162.
4. Ivohin E.V, Makhno M.F. (2017) On an approach to construction of structured fuzzy sets and their application for description of fuzzy time response. *Journal of Automation and Information Science,* V.49. – Iss.10. – 55-63.
DOI 10.1615/JAutomatInfScien.v49.i10.60
5. Orlovskiy S.A. (1981) *Problemy prinyatiya resheniya pri nechetkoy iskhodnoy informatsii*. M.: Nauka.
6. Bablu Jana, Tapan Kumar Roy. (2005) Multi-Objective Fuzzy Linear Programming and Its Application in Transportation Model. *Tamsui Oxford Journal of Mathematical Sciences*, V.21. – N.2. – 243-268.
7. Zaychenko Yu.P. (2006) *Doslidzhennya operatsiy*. K.: «Slovo».
8. Zimmermman, H.J. (1985) Application of Fuzzy Set Theory To Mathematical Programming. *Information Sciences*, 36. – 25-58.
9. Ivohin E.V. (2021) Formalization of Influence Processes of Fuzzy Time Flow on the Solution to Time Resource Distribution Problems. *Cybernetics and Systems Analysis*, 57. – Iss. 3. – 363-373.
DOI: 10.1007/s10559-021-00361-x
10. Rai S., Ettam R.K. (2013) Simulation-based optimization using simulated annealing for optimal equipment selection within print production environments. *Winter Simulations Conference (WSC)*. - 1097-1108.
11. Kostenko O.M. (2016) Syntez optymal′nykh kombinatornykh planiv bahatofaktornoho eksperymentu. *Visnyk Poltavs'koyi derzhavnoyi ahrarnoyi akademiyi*, №1-2. – 62-71.
12. Havrylenko V.V., Ivokhina K.Ye., Rudoman N.V. (2022) Pro zastosuvannya metodu imitatsiyi vidpalu dlya rozv'yazannya nechitkoyi zadachi komivoyazhera. *Systemy upravlinnya, navihatsiyi ta zv'yazku*, №3. – 77-82.

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