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DIFFUSION PROCESS WITH EVOLUTION AND ITS PARAMETER ESTIMATION

Abstract. A discrete Markov process in an asymptotic diffusion environment with a uniformly ergodic embedded Markov chain can be approximated by an Ornstein–Uhlenbeck process with evolution. The drift parameter estimation is obtained using the stationarity of the Gaussian limit process.

Keywords: discrete Markov process, diffusion approximation, asymptotic diffusion environment, Ornstein–Uhlenbeck process, phase merging, drift parameter estimation.

We consider a random evolution $\zeta(t)$, $t \geq 0$, that depends on a random environment $Y(t)$, $t \geq 0$, which in turn, is switched by an embedded Markov chain X_k, $k \geq 0$. The connection between the continuous-time $t \geq 0$ and the discrete-time $k \geq 0$ will be explained in the sequel.

The purpose of this work is to prove the convergence (in distribution) of the process $\zeta(t)$, $t \geq 0$, to the Ornstein–Uhlenbeck process under some scaling of the process and its time parameter.

The limit will be considered by a small series parameter $\varepsilon > 0$, $\varepsilon \to 0$.

ASYMPTOTIC DIFFUSION ENVIRONMENT

Consider a discrete Markov process in a semi-Markov asymptotic diffusion environment, determined by a solution of the following scaled difference stochastic equation:

$$\zeta^\varepsilon(t_{n+1}^\varepsilon) = -\varepsilon^2 V(Y_n^\varepsilon)\zeta^\varepsilon(t_n^\varepsilon) + \varepsilon\sigma(Y_n^\varepsilon)\Delta\mu^\varepsilon(t_{n+1}^\varepsilon), \tag{1}$$

where $t_n^\varepsilon := n\varepsilon^2$, hence $t_{n+1}^\varepsilon = t_n^\varepsilon + \varepsilon^2$, $n > 0$, $\varepsilon > 0$, for the process increments $\Delta\zeta^\varepsilon(t_{n+1}^\varepsilon) := \zeta^\varepsilon(t_{n+1}^\varepsilon) - \zeta^\varepsilon(t_n^\varepsilon)$, $n \geq 0$.

The asymptotic diffusion environment Y_n^ε, $n \geq 0$, is also a random evolution process generated by a solution of the following scaled difference evolutionary equation:

$$\Delta Y^\varepsilon(t_{n+1}^\varepsilon) = \varepsilon A_0(Y_n^\varepsilon; X_n^\varepsilon) + \varepsilon^2 A(Y_n^\varepsilon; X_n^\varepsilon), \; n \geq 0, \tag{2}$$

with the embedded Markov chain $X_n^\varepsilon := X(t_n^\varepsilon)$, $n \geq 0$.

The terms $A_0(y;x)$ and $A(y;x)$ are Lipschitz functions, together with the first derivative $A'_{0y}(y;x)$.

Here the predictable evolutional component in (1) is determined by the following conditional expectation [1]:

$$V(Y_n^\varepsilon)\zeta^\varepsilon(t_n^\varepsilon) := E[\Delta\zeta^\varepsilon(t_{n+1}^\varepsilon) \mid Y_n^\varepsilon, \zeta^\varepsilon(t_n^\varepsilon)]\zeta^\varepsilon(t_n^\varepsilon),$$

where it is assumed that the drift regression function $V(z)$ is positive: $V(z) > 0 \; \forall z$.

The martingale difference $\Delta\mu^{\varepsilon}(t^{\varepsilon}_{n+1})$, $n \ge 1$, generated by the process $\Delta\zeta^{\varepsilon}(t^{\varepsilon}_{n+1})$, $n \ge 1$, is determined by the following conditional second moment:

$$-\varepsilon^2\sigma^2(Y^{\varepsilon}_n) := E[(\Delta\,\zeta^{\varepsilon}(t^{\varepsilon}_{n+1}) + \varepsilon^2 V(Y^{\varepsilon}_n)\zeta^{\varepsilon}(t^{\varepsilon}_n))^2 | Y^{\varepsilon}_n].$$

The embedded Markov chain $X^{\varepsilon}_n := X(t^{\varepsilon}_n)$, $t^{\varepsilon}_n := n\varepsilon^2$, $n \ge 0$, is supposed to be homogeneous ergodic Markov chain with transition probabilities $P(x, B)$, $x \in E$, $B \in \mathcal{E}$, having a stationary distribution $\rho(B)$, $B \in \mathcal{E}$, which satisfies the condition $\rho(B) = \int_E \rho(dx)P(x, B)$; $\rho(E) = 1$.

The stochastic difference equations (1), (2) generate a discrete stochastic basis [2, Ch. 1] with filtration $\mathrm{F}_m(\zeta^{\varepsilon}, Y^{\varepsilon}) = \sigma\{\zeta^{\varepsilon}(t^{\varepsilon}_n), Y^{\varepsilon}(t^{\varepsilon}_n), n \le m\}$, $m \ge 0$.

Now we consider three components $(\zeta^{\varepsilon}(t^{\varepsilon}_n), Y^{\varepsilon}(t^{\varepsilon}_n), X^{\varepsilon}_n)$, $n \ge 0$, as piecewise constant functions with continuous time:

$$\left.\begin{aligned} &\zeta^{\varepsilon}(t) = \zeta^{\varepsilon}(t^{\varepsilon}_n) \\ &Y^{\varepsilon}(t) = Y^{\varepsilon}(t^{\varepsilon}_n) \\ &X^{\varepsilon}_t = X^{\varepsilon}_n \end{aligned}\right\} \text{ for } n\varepsilon^2 < t \le (n+1)\varepsilon^2.$$

In what follows, a solution of equations (1), (2) is given by martingale characterization [3, Section 4.4] of three-component Markov process $(\zeta^{\varepsilon}(t), Y^{\varepsilon}(t), X_t)$, $t \ge 0$:

$$M^{\varepsilon}(t) = \varphi(\zeta^{\varepsilon}(t), Y^{\varepsilon}(t), X_t) - \varphi(\zeta^{\varepsilon}(0), Y^{\varepsilon}(0), X_0) -$$
$$-\int_0^{\varepsilon^2[t/\varepsilon^2]} L^{\varepsilon}\varphi(\zeta^{\varepsilon}(s), Y^{\varepsilon}(s), X^{\varepsilon}_s)ds,$$

and the generator of three-component Markov process $(\zeta^{\varepsilon}(t), Y^{\varepsilon}(t), X_t)$, $t \ge 0$, is represented as follows [4, Ch. 5]:

$$L^{\varepsilon}\varphi(c, y, x) := \varepsilon^2 E[\varphi(c + \Delta\zeta^{\varepsilon}(t^{\varepsilon}_{n+1}, Y^{\varepsilon}(t^{\varepsilon}_{n+1}), X^{\varepsilon}_{n+1}))|$$
$$\zeta^{\varepsilon}(t^{\varepsilon}_n) = c, Y^{\varepsilon}(t^{\varepsilon}_n) = y, X^{\varepsilon}_n = x].$$

APPROXIMATION OF A DISCRETE MARKOV PROCESS IN A SYMPTOTIC DIFFUSION ENVIRONMENT

Let the singular term $A_0(y; x)$ satisfies the balance condition

$$\int_E \rho(dx)A_0(y; x) \equiv 0. \tag{3}$$

The approximation of a discrete Markov process in asymptotic diffusion environment gives the following theorem.

Theorem 1. Let the Markov chain X_n, $n \ge 0$, be uniformly ergodic with the stationary distribution $\rho(B)$, $B \in \mathcal{E}$.

The finite-dimensional distributions of the discrete Markov process (1), together with asymptotical diffusion $Y^{\varepsilon}(t)$, $t \ge 0$, converge, as $\varepsilon \to 0$, to a diffusion Ornstein–Uhlenbeck process with evolution:

$$(\zeta^{\varepsilon}(t), Y^{\varepsilon}(t)) \xrightarrow{D} (\zeta^0(t), Y^0(t)),\ \varepsilon \to 0,\ 0 \le t \le T.$$

The limit two-component diffusion process with evolution $(\zeta^0(t), Y^0(t))$, $t \ge 0$, is set by the generator

$$\mathfrak{L}^0(y)\varphi(c, y) = -V(y)c\varphi'_c(c, y) + \frac{1}{2}\sigma^2(y)\varphi''_c(c, y) + \\ + \hat{A}(y)\varphi'_y(c, y) + \frac{1}{2}\hat{B}^2(y)\varphi''_y(c, y), \ \varphi \in C_0^2(R, \mathfrak{B}), \tag{4}$$

where by definition

$$\hat{A}(y) := \int_E \rho(dx)[A(y;x) + A_1(y;x)], \tag{5}$$

$$A_1(y;x) := A_0(y;x)PR_0A'_{0y}(y;x), \tag{6}$$

$$\hat{B}^2(y) = \int_E \rho(dx)B(y;x), \\ B(y;x) = A_0(y;x)P[R_0 + \frac{1}{2}\mathbb{I}]A_0(y;x). \tag{7}$$

Here $\mathbb{I}$ is the standard identity matrix, P is the transition operator of the Markov chain X_t, $t \ge 0$, and the potential kernel R_0 is defined as in [3, Section 5.2]:

$$R_0 = (\mathbb{Q} + \Pi)^{-1} - \Pi, \ \mathbb{Q} := P - \mathbb{I}, \ \Pi\varphi(x) := \int_E \rho(dx)\varphi(x). \tag{8}$$

Remark 1. The limit two-component diffusion process $(\zeta^0(t), Y^0(t))$, $t \ge 0$, set by the generator (4)–(8), has a stochastic representation by the stochastic differential equation

$$d\zeta^0(t) = -V(Y^0(t))\zeta^0(t)dt + \sigma(Y^0(t))dW(t), \\ dY^0(t) = \hat{A}(Y^0(t))dt + \hat{B}(Y^0(t))dW_0(t).$$

Consequently, the parameters of the limit diffusion $\zeta^0(t)$, $t \ge 0$, depend on the diffusion process $Y^0(t)$, $t \ge 0$.

Proof of Theorem 1. The basic idea is that any Markov process is determined by its generator on the class of real-valued test functions, defined on the set of values of Markov process [5].

First of all, the extended three-component Markov chain is used

$$(\zeta^\varepsilon(t_n), Y^\varepsilon(t_n), X^\varepsilon(t_n) = X_n), \ t_n = n\varepsilon^2, \ \varepsilon > 0, \tag{9}$$

with operator characterization in the following form.

Lemma 1. The extended Markov chain is determined by the generator

$$\mathbb{L}^\varepsilon(x)\varphi(c, y, x) = \varepsilon^{-2}[\Gamma^\varepsilon(y)A^\varepsilon(x)\mathbb{P} - \mathbb{I}]\varphi(c, y, x), \tag{10}$$

where the transition operators are defined as follows:

$$\Gamma^\varepsilon(y)\varphi(c) := E[\varphi(c + \Delta\zeta^\varepsilon(t; y)) | \zeta^\varepsilon(t) = c, Y^\varepsilon(t) = y, X^\varepsilon(t) = x], \\ \mathbb{A}^\varepsilon(x)\varphi(y) := E[\varphi(y + \Delta Y^\varepsilon(t; x)) | Y^\varepsilon(t; x) = y, X^\varepsilon(t) = x], \\ \mathbb{P}\varphi(x) := \int_E P(x, dz)\varphi(z). \tag{11}$$

The assertion of Lemma 1 follows from the next argumentation. The extended three-component Markov chain (11), under the additional condition $Y^{\varepsilon}(t) = y$, $X^{\varepsilon}(t) = x$, has independent components. So its transition probabilities are given by the product of transition probabilities of each component.

An essential step in the proof of Theorem 1 is realized in the next lemma.

Lemma 2. The generator (10), (11) of three-component Markov chain (9) on the class of real-valued test functions $\varphi(c, y, x)$, having bound derivatives up to the third order inclusively, admits an asymptotic representation

$$\mathbb{L}^{\varepsilon}(x)\varphi(c, y, x) = \tag{12}$$
$$= [\varepsilon^{-2}\mathbb{Q} + \varepsilon^{-1}\mathbb{A}_0(x)\,\mathbb{P} + \mathbb{A}(x)\,\mathbb{P} + \mathbb{L}^0(y)P + \mathbb{R}_{\varepsilon}(y;x)]\varphi(c, y, x);$$

$$\mathbb{A}_0(x)\varphi(y) = A_0(y;x)\varphi'(y),\ \mathbb{A}(x)\varphi(y) = A(y;x)\varphi'(y); \tag{13}$$
$$\mathbb{L}^0(y)\,\varphi(c) = -V(y)c\varphi'(c) + \frac{1}{2}\sigma^2(y)\varphi''(c).$$

The residual term is expressed as:

$$\mathbb{R}_{\varepsilon}(y;x)\varphi(c, y, x) \to 0,\ \ \varepsilon \to 0,\ \ \varphi \in C^3(R^2).$$

Here one intends the uniform convergence for all the arguments.

Proof of Lemma 2. We use transformation of generator (12) by the formula

$$\mathbb{L}^{\varepsilon}(x)\,\varphi(c, y, x) =$$
$$= \varepsilon^{-2}[\mathbb{Q} + (A^{\varepsilon}(x) - \mathbb{L})\,\mathbb{P} + (\Gamma^{\varepsilon}(y) - \mathbb{I})\,\mathbb{P} + \mathbb{R}_{\varepsilon}(y;x)]\varphi(c, y, x). \tag{14}$$

The residual term has the following form:

$$\mathbb{R}_{\varepsilon}(y;x)\varphi(c, y, x) = (\Gamma^{\varepsilon}(y) - \mathbb{I})(A^{\varepsilon}(x) - \mathbb{I})\,\mathbb{P}\,\varphi(c, y, x).$$

Then we calculate

$$\varepsilon^{-2}[\Gamma^{\varepsilon}(y) - \mathbb{I}]\varphi(c) = \varepsilon^{-2}\{E[\varphi(c + \Delta\zeta^{\varepsilon}(t; y))\,|\,\zeta^{\varepsilon}(t; y) = c] - \varphi(c)\} =$$
$$= [\,\mathbb{L}^0(y) + \mathbb{B}_{\varepsilon}(y; c)]\varphi(c);$$

The next term in (17) has the next representation:

$$\varepsilon^{-2}[A^{\varepsilon}(x) - \mathbb{I}]\varphi(y) = \varepsilon^{-2}\{E[\varphi(y + \Delta Y^{\varepsilon}(t; y))\,|\,Y^{\varepsilon}(t; y) = y] - \varphi(y)\} =$$
$$= \varepsilon^{-2}\left[E[\Delta Y^{\varepsilon}(t; y)]\varphi'(y) + \frac{1}{2}E[\Delta Y^{\varepsilon}(t; y)]^2\varphi''(y) + \varepsilon^2\mathbb{R}_{\varepsilon}(x)\varphi(y)\right] =$$
$$= [\varepsilon^{-1}\mathbb{A}_0(y;x) + \mathbb{A}_0(y;x)]\varphi'(y) + \frac{1}{2}\mathbb{A}_0^2(y;x)\varphi''(y) + \mathbb{R}_{\varepsilon}(x)\varphi(y),$$

gives the asymptotic expansion in Lemma 2:

$$\mathbb{L}^{\varepsilon}(x)\,\varphi(c, y, x) = [\varepsilon^{-2}\mathbb{Q} + \varepsilon^{-1}\mathbb{A}_0(x)\,\mathbb{P} + \mathbb{A}(x)\,\mathbb{P} +$$
$$+ \frac{1}{2}(\mathbb{A}_0(x)\,\mathbb{P})^2 + \mathbb{L}^0(y)\,]\,\varphi(c, y, x) + \mathbb{R}_{\varepsilon}(y;x)\varphi(c, y, x).$$

Next, we use the solution of singular perturbation problem for the truncated operator [3].

Lemma 3. The solution of the singular perturbation problem for the truncated operator is realized on perturbed test functions:

$$\mathbb{L}_0^{\varepsilon}(x)\varphi^{\varepsilon}(c, y, x) = [\varepsilon^{-2}\mathbb{Q} + \varepsilon^{-1}\mathbb{A}_0(x)\mathbb{P} + A(x)\mathbb{P} +$$

$$+\frac{1}{2}(\mathbb{A}_0(x)\mathbb{P})^2 + \mathbb{L}^0(y)][\varphi(c, y) + \varepsilon\varphi_1(c, y, x) + \varepsilon^2\varphi_2(c, y, x)] =$$

$$= \mathfrak{L}^0(y)\varphi(c, y) + \mathbb{R}_{\varepsilon}(x)\varphi(c, y). \quad (15)$$

The averaging parameters are determined by the formulas (4)–(8).

The limit operator is calculated by the formula

$$\mathfrak{L}^0(y)\varphi(c, y) = \mathbb{L}^0(y)\varphi(c, y) + \hat{A}(y)\varphi'_y(c, y) + \frac{1}{2}\hat{B}^2(y)\varphi''_y(c, y). \quad (16)$$

Proof of Lemma 3. To solve the singular perturbation problem for the truncated operator, consider the asymptotic representation by the powers of ε:

$$\mathbb{L}_0^{\varepsilon}(x)\,\varphi^{\varepsilon}(c, y, x) = \varepsilon^{-2}\mathbb{Q}\varphi(c, y) + \varepsilon^{-1}[\mathbb{Q}\varphi_1(c, y, x) +$$

$$+\mathbb{A}_0(x)\varphi(c, y)] + [\mathbb{Q}\varphi_2(c, y, x) + \mathbb{A}_0(x)\,\mathbb{P}\,\varphi_1(c, y, x) +$$

$$+[\mathbb{A}(x) + \frac{1}{2}(\mathbb{A}_0(x)\mathbb{Q})^2 + \mathbb{L}^0(y)]\varphi(c, y)] + \mathbb{R}_{\varepsilon}(x)\varphi(c, y).$$

Obviously that $\mathbb{Q}\varphi(c, y) = 0$.

The balance condition (3) is then used. The solution of the equation

$$\mathbb{Q}\varphi_1(c, y, x) + \mathbb{A}_0(x)\varphi(c, y) = 0$$

is given by the formula [4, Section 5.4]:

$$\varphi_1(c, y, x) = \mathbb{R}_0\mathbb{A}_0(x)\varphi(c, y).$$

Lemma 3 implies the following equation

$$\mathbb{Q}\varphi_2(c, y, x) + [\mathbb{B}(x) + \mathbb{A}(x) + \mathbb{L}^0(y)]\varphi(c, y) = \mathrm{L}^0(y)\varphi(c, y). \quad (17)$$

Here, by definition

$$\mathbb{B}(x) := \mathbb{A}_0(x)\,\mathbb{P}\,R_0\mathbb{A}_0(x) + \frac{1}{2}(\mathbb{A}_0(x)\,\mathbb{P})^2.$$

The limit operator is calculated using the balance condition

$$\mathfrak{L}^0(y)\varphi(c, y) = \Pi[\mathbb{B}(x) + \mathbb{A}(x)]\Pi + \mathbb{L}^0(y)\varphi(c, y). \quad (18)$$

Recall the projector's operation:

$$\Pi\mathbb{B}(x)\,\Pi = \int_E \rho(dx)\,B(y,x), \quad B(y,x) = A_0(y,x)\,\mathbb{P}\,\mathbb{R}_0 A_0(y,x) + \frac{1}{2}(A_0(y,x))^2.$$

Taking into account the definition of evolutionary operators (13), the limit generator is determined by formula (16).

The limit operator (18) provides a solution of equation (17), which is a function of $\varphi_2(c, y, x)$. The existence of perturbing functions $\varphi_i(\cdot)$, $i = 1,2$, ensures the asymptotical representation (15). That completes the proof of Theorem 1.

The volatility is generated by introduction of a random environment $Y^0(t)$, $t \geq 0$, into the diffusion parameter of the Ornstein–Uhlenbeck diffusion process $\zeta^0(t)$, $t \geq 0$.

PARAMETER ESTIMATION OF THE LIMIT PROCESS

The limit Ornstein–Uhlenbeck diffusion process parameters estimation is substantiated in this section without the assumption of volatility, which greatly changes the kind of estimates. The stationarity of the Gaussian statistical experiment is essentially used [6].

It is known [7], that a diffusion type processes are given by stochastic differential

$$d\xi_t = \alpha_t(\xi_t)dt + dW_t. \tag{19}$$

The predictable component satisfies the conditions

$$P\left(\int_0^T \alpha_t^2(\xi_t)dt < \infty\right) = 1, \ T < \infty, \tag{20}$$

$$P\left(\int_0^\infty \alpha_t^2(\xi_t)dt = \infty\right) = 1,$$

which ensures the absolute continuity of the measure $\mu_\xi(B) := P\{\omega : \xi \in B\}$ and the measure $\mu_W(B) := P\{\omega : W \in B\}$ for all $B \in \mathcal{B}_T = \sigma(\xi_t : 0 \le t \le T)$.

The Radon–Nicodemus derivative specifies the density of the measure

$$\zeta_T(\xi) := \frac{d\mu_\xi}{d\mu_W}(\xi, T), \tag{21}$$

which for processes of diffusion type (19) has the following representation.

Theorem 2 [7]. The measure density (21) for processes of diffusion type (19) with additional conditions (20) is given by exponential martingale

$$\frac{d\mu_\xi}{d\mu_W}(\xi_t, T) = \exp\left[\int_0^T \alpha_t(\xi_t)d\xi_t - \frac{1}{2}\int_0^T \alpha_t^2(\xi_t)dt\right]. \tag{22}$$

In particular, the exponential martingale (21) is determined by a solution of the stochastic Doléans–Dade equation [2]

$$d\zeta_T(\xi) = \zeta_T(\xi)\alpha_T(\xi)d\xi_T, \ \zeta_0(\xi) = 1, \tag{23}$$

or in equivalent form:

$$\zeta_T(\xi) - 1 = \zeta_T(\xi)\alpha_T(\xi)d\xi_T.$$

The relationship of the density (22) with the stochastic Doléans–Dade equation (23) can be explained, using the Itô formula for exponential function $\varphi(\xi) = \exp[\eta_T(\xi) - \frac{1}{2}\langle\eta(\xi)\rangle_T]$, with $\eta_T(\xi) := \int_0^T \alpha_t(\xi)dt$, $\langle\eta(\xi)\rangle_T := \int_0^T \alpha_t^2(\xi)dt$, namely (see [2]):

$$d\varphi(\xi_T) = \varphi'(\xi_T)[d\eta_T(\xi) - \frac{1}{2}d\langle\eta(\xi)\rangle_T] + \frac{1}{2}\varphi''(\xi_T)d\langle\eta(\xi)\rangle_T.$$

Taking into account equality $\varphi(\xi_T) = \varphi'(\xi_T) = \varphi''(\xi_T)$ for exponential function $\varphi(\xi)$, we have a stochastic Doléans–Dade differential equation for exponential martingale (22). According to the results of the previous section, the limit diffusion process for normalized discrete Markov processes is the Ornstein–Uhlenbeck process with a linear predictable component

$$d\alpha_t = -V_0\alpha_t dt + \sigma dW_t, \ 0 \le t \le T,$$

Without limiting of generality, let us put $\sigma = 1$.

The maximum likelihood method for estimating the parameter V_0 of a diffusion process with a stochastic differential (19) ($\sigma = 1$) is realized for the logarithm of the measure density (22):

$$L(V,T) := \ln \zeta_T(\xi) = -V\int_0^T \alpha_t dt - \frac{V^2}{2}\int_0^T \alpha_t^2 dt.$$

Therefore the equation for estimating the maximum likelihood method is

$$\max_{0 \le V \le 2} \partial L(V,T)/\partial V = -\int_0^T \alpha_t dt - V_T \int_0^T \alpha_t^2 dt = 0,$$

and the estimate of the maximum likelihood method has the following form:

$$V_T = -\int_0^T \alpha_t d\alpha_t \Big/ \int_0^T \alpha_t^2 dt.$$

The least square method estimation of parameter V_0 of diffusion process with stochastic differential (19) ($\sigma = 1$) is implemented using equality

$$\int_0^T \alpha_t d\alpha_t = -V_0 \int_0^T \alpha_t^2 dt + \int_0^T \alpha_t dW_t.$$

So we have a relationship

$$V_0 - V_T = \int_0^T \alpha_t dW_t \Big/ \int_0^T \alpha_t^2 dt.$$

The estimation of the least squares method has a representation

$$V_T^0 = \int_0^T \alpha_t d\beta_t \Big/ \int_0^T \alpha_t^2 dt.$$

Corollary 1. The estimates of maximum likelihood and least squares coincide: $V_T = V_T^0$.

Corollary 2. Estimation by the least squares method, and hence estimation by the method of maximum likelihood of the parameter V_0 are strongly consistent:

$$P1 \lim_{T \to \infty} V_T^0 = V_0. \tag{24}$$

Remark 2. In the presence of volatility (see [8]), the maximum likelihood estimate and the least squares estimate are different, but the property of strong consistency (24) is retained.

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ДИФУЗІЙНИЙ ПРОЦЕС З ЕВОЛЮЦІЄЮ ТА ОЦІНЮВАННЯ ЙОГО ПАРАМЕТРА

Анотація. Показано, що дискретний марковський процес в асимптотичному дифузійному середовищі з рівномірним ергодичним вкладеним ланцюгом Маркова може бути наближений процесом Орнштейна–Уленбека з еволюцією. Оцінку параметра дрейфу отримано з використанням стаціонарності гаусівського граничного процесу.

Ключові слова: дискретний марковський процес, дифузійна апроксимація, асимптотичне дифузійне середовище, процес Орнштейна–Уленбека, фазове укрупнення, оцінка параметра зсуву.

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ДИФФУЗИОННЫЙ ПРОЦЕСС С ЭВОЛЮЦИЕЙ И ОЦЕНКА ЕГО ПАРАМЕТРА

Аннотация. Показано, что дискретный марковский процесс в асимптотической диффузионной среде с равномерной эргодической вложенной цепью Маркова может быть приближен процессом Орнштейна–Уленбека с эволюцией. Оценка параметра дрейфа получена с использованием стационарности гауссовского предельного процесса.

Ключевые слова: дискретный марковский процесс, диффузионая аппроксимация, асимптотическая диффузионная среда, процесс Орнштейна–Уленбека, фазовое укрупнение, оценка параметра сдвига.

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