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# ROBUST FOOD–ENERGY–WATER–ENVIRONMENTAL SECURITY MANAGEMENT: STOCHASTIC QUASIGRADIENT PROCEDURE FOR LINKAGE OF DISTRIBUTED OPTIMIZATION MODELS UNDER ASYMMETRIC INFORMATION AND UNCERTAINTY[1]

**Abstract.** The paper presents a consistent algorithm for regional and sectoral distributed models' linkage and optimization under asymmetric information based on iterative stochastic quasigradient (SQG) solution procedure of, in general, non-smooth nondifferentiable optimization. The procedure is used for linking individual sectoral and regional models for integrated and interdependent food–energy–water–environmental security analysis and management.

**Keywords:** decision support, asymmetric information, linkage, SQG solution procedure, non-smooth optimization, subgradient, integrated modeling, food–energy–water–environmental nexus.

## INTRODUCTION

Detailed sectoral and regional models have traditionally been used to anticipate and plan desirable developments of respective sectors and regions. These models operate with a set of feasible decisions and aim to select a solution optimizing a sector- or region-specific objective function, depending on various input scenarios. When interdependencies between sectors and regions are increasing, an independent analysis that does not take the interconnectedness into account can become highly misleading. Hence, the sectoral and regional models must be linked together to produce truly integrated solutions that are optimal for the overall system. Interdependent food–energy–water–environmental (FEWE) security goals

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contribute immensely to signifying the nexus between sectors and regions, notably via common environmental goals, including quotas on total water pollution, GHG emissions, etc.

In this paper, we consider the problem of linking sectoral and/or regional models into an inter-sectoral, inter-regional integrated model under asymmetric information, i.e. without revealing information about corresponding sub-models. The lack of full information on goals, feasible decisions, constraints, and corresponding data sets is typical for modeling international aspects. Accordingly, in this paper we distinguish the term "linkage" under asymmetric information from the term "integrated" modeling based on full information.

In principle, linkage of deterministic models and deterministic approximations of stochastic optimization models into a single model that incorporates all constraints from all models can be treated as a multiple criteria optimization problem under appropriate aggregation principles, including probabilistic principles for problems with uncertainties, e.g. with the welfare weights treated as probabilities. However, in case of asymmetric information when agents are unable to obtain information on each other's models, the linkage, i.e., integration under asymmetric information, is possible by using (Sec. 2) non-smooth optimization methods of a multiagent systems.

Often, it is impossible to consider decisions and constraints of all models simultaneously and solve underlying implicit large-scale optimization models. Instead, we use sequential subgradient optimization methods developed in the form of iterative decomposition for two-stage stochastic optimization models (see discussions and references in [1–4]), which can also be reformulated as large-scale (even infinite dimensional) implicit LP models. These methods have remarkable flexibility enabling optimization of non-smooth systems ([5]; see also Sec. 3). Thus, the approach for models' linkage is based on an iterative stochastic quasigradient (SQG) procedure of, in general, non-smooth nondifferentiable optimization (see also Remark 4, Sec. 4) converging to a socially optimal solution maximizing an implicit nested nondifferentiable social welfare function. The linkage problem can be viewed as a general endogenous reinforced learning problem [6]. The models act as "agents" that communicate with a "central hub" (a regulator) and take decisions in order to maximize the "cumulative reward". The procedure does not require models to exchange full information about their specifications. The "resource quotas" for each sector/region and each resource are recalculated by sectors/regions independently by shifting their current approximation in the direction defined by the corresponding sectoral/regional shadow prices of resources from the primal sectoral optimization problem. In this way, we avoid a "hard linking" of the models in a single code that saves programming time and enables parallel distributed computations of sectoral/regional models instead of a large-scale integrated model, that is, one that addresses the well-known "curse of dimensionality" and challenges large scale data harmonization under asymmetric information. This also preserves the original models in their initial state for other possible linkages. Using detailed sectoral/regional models instead of their aggregated simplified versions allows taking into account critically important local details, which are usually hidden within aggregate data. The linkage procedure has been developed at IIASA (International Institute for applied Systems Analysis) in cooperation with Institutes from NASU (National Academy of Sciences, Ukraine) and applied in various case studies of the NASU-IIASA project on "Integrated robust management of food–energy–water–land use nexus for sustainable development" [7–10] and other projects at IIASA (see e.g. [11]).

The paper is organized as follows. Section 1 discusses the problem of model linkage vs standard integrated modeling. Section 2 presents the original result of this paper — a consistent iterative procedure for model linkage under asymmetric information based on the generalized gradient (sub-gradient) method of non-smooth optimization. Section 3 analyses the properties of the algorithm enabling its convergence. Section 4 and the Annex present an application of the proposed approach in the case study linking energy and agricultural sectoral models for optimal management in water scarce regions, case study in China. The last Section concludes and outlines potential future directions, and in particular, the extension of the linkage approach to stochastic models to manage induced cross-sectoral "hidden" systemic risks.

## 1. LINKING MODELS FOR OPTIMAL RESOURCE DISTRIBUTION

Let us consider $K$ sectors/regions utilizing some common resources. The problem of their linkage can be formulated as follows. Let $x^{(k)}$ be the vector of decision variables in sector/region $k$ and assume that each sector/region aims to choose such $x^{(k)}$ to maximize its objective function (net profits) of the form

$$\langle c^{(k)}, x^{(k)} \rangle \to \max, \tag{1}$$

subject to constraints

$$x^{(k)} \geq 0, \tag{2}$$

$$A^{(k)} x^{(k)} \leq b^{(k)}, \tag{3}$$

$$B^{(k)} x^{(k)} \leq y^{(k)}, \tag{4}$$

where $\langle c^{(k)}, x^{(k)} \rangle$, $k = 1, 2, \ldots, K$, denotes the scalar product of vectors $c^{(k)}$ and $x^{(k)}$, $\langle c^{(k)}, x^{(k)} \rangle = \sum_j c_j^{(k)} x_j^{(k)}$.

Here, net unit profits $c^{(k)}$, and matrices $A^{(k)}$ and $B^{(k)}$ define the marginal contribution of each component into the total demand, resource use, and environmental impact, and vectors $b^{(k)}$ and $c^{(k)}$ determine the constraints, which are themselves known only to sector/region $k$. We distinguish between the constraints that are specific to sector/region $k$ expressed by (3) and the constraints that are part of a common inter-sectoral/inter-regional constraint with sectoral/regional quotas $y^{(k)}$ expressed by (4). The sectoral/regional quotas are not fixed, but rather the following joint constraint on the common resources holds:

$$\sum_{k=1}^{K} A^{(k)} y^{(k)} \leq d, \;\; y^{(k)} \geq 0, \tag{5}$$

where matrices $D^{(k)}$ define the marginal contribution of each sectoral/regional quota into a joint constraint described by vector $d \geq 0$ that is known to all sectors/regions.

Thus, each sector/region $k$ maximizes its objective function (1) by choosing $x^{(k)}$ and $y^{(k)}$ from the feasible set defined by (2), (3), and (4).

The joint constraint (5) may be either binding or not. It may happen that for a given $y$ the sectorial/regionally optimal solutions $x_*^{(k)}$ to problem (1)–(4) generate the resource demand $y_*^{(k)} = B^{(k)} x_*^{(k)}$, such that (5) holds. In that case, the sectors/regions are actually not interlinked and decisions that are optimal for each sector are also optimal for the entire system.

In what follows, we are interested in the opposite case, when joint constraints (5) are binding, that is, when for the optimal sectorial/regional solutions the joint constraints (5) do not hold. This means that sectors cannot achieve their optimum independently, and at least one of them has to sacrifice a part of its utility (i.e., accept lower profits or higher costs) in order to satisfy the joint constraints (5).

## 2. A CONSISTENT ITERATIVE PROCEDURE FOR LINKING DISTRIBUTED MODELS UNDER ASYMMETRIC INFORMATION

**2.1. Welfare maximization under asymmetric information.** Truly integrative solutions, by definition, imply cooperation between sectors. From this perspective, the problem of model linkage can essentially be considered a multi-criteria optimization problem under asymmetric information, in which a resource-efficient Pareto solution is to be found. That is, assuming some weights $w_k$, $w_k \geq 0$, $\sum_{k=1}^{K} w_k = 1$, a single welfare function should be maximized producing a Pareto optimal solution:

$$\sum_{k=1}^{K} w_k \langle c^{(k)}, x^{(k)} \rangle \to \max, \tag{6}$$

subject to (2)–(5). By asymmetric information of sectors/regions we mean that a sector/region $k$ does not know $c^{(l)}$, $A^{(l)}$, $B^{(l)}$, $x^{(l)}$ of other sectors/regions $l \neq k$. There is, however, a central hub (regulator) who knows $D^{(k)}$ and $d$. We also assume that there is a network of computers connecting the central computer of each sector/region with the central computer of the hub.

In this section, we suggest a consistent algorithm for iteratively linking sectorial or regional models under asymmetric information, for which convergence is established in Sec. 3. It is important to note that the cyclic coordinate-wise optimization method does not converge if the objective function is non-differentiable continuously (see e.g. [1–4] on SQG methods) hence the direct/naive linkage will not work. A core part of the algorithm is a central hub computer that recalculates the resource quotas $y$ by shifting their current approximation in the direction defined by the corresponding vectors of dual variables (shadow prices of resources) from the primal optimization problems. These quotas are received by sectorial/regional computers enabling parallel computations of solutions and fast adjustments of vector $y$. Ermoliev (1980) [12] initially introduced the idea of this algorithm and current computer capacities enable its implementation to large-scale models used to support decisions.

Consider the main implicit maximization problem. For a given vector $y = (y^{(1)}, \ldots, y^{(K)})$ let us denote the optimal value of function (6) under constraints (2)–(4) by $F(y)$, in other words, in this function $x^{(k)}(y)$ are optimal solutions to (1) under (2)–(4) ignoring joint constraints (5). Therefore,

$$F(y) = \sum_{k=1}^{K} f^{(k)}(y),$$

where $f^{(k)}(y) = w_k (c^{(k)}, x^{(k)}(y))$ are concave non-differentiable (continuously) or non-smooth functions for given weights $w_k$ (Proposition 1, a), b)).

The algorithm defines a rule for adjusting $y$ towards an optimal $y^*$ that maximizes function $F(y)$ under the joint constraints (5) defining the feasible set $Y$.

**2.2. Non-smooth linkage method.** Consider an arbitrary feasible solution $y^s = (y^{s(1)}, \ldots, y^{s(K)})$ for iteration $s = 1, 2, \ldots$ of the algorithm. For given quotas $y^s = (y^{s(1)}, \ldots, y^{s(K)})$, independently and in parallel, computers of sectors/regions solve primal models (1)–(4) and obtain primal solutions $x^{s(k)} = x^{s(k)}(y^s)$ together with the corresponding shadow prices of resources, that is, solutions $(u^{s(k)}, v^{s(k)})$ of the dual problems

$$\langle b^{(k)}, u^{(k)} \rangle + \langle y^{(k)}, v^{(k)} \rangle \to \min, \tag{7}$$

$$A^{(k)} u^{(k)} + B^{(k)} v^{(k)} \ge w_k c^{(k)}, \tag{8}$$

$$u^{(k)} \ge 0,\ v^{(k)} \ge 0, \tag{9}$$

$k = 1, 2, \ldots,$ where vectors $v^{s(k)}$ are the driving force of algorithm (10).

The next approximation of quotas $y^{s+1} = (y^{s+1(1)}, \ldots, y^{s+1(K)})$ is derived by the computer of the central hub by shifting $y^s$ in the direction of vector $v^s = (v^{s(1)}, \ldots, v^{s(K)})$, that is, optimal dual variables (shadow prices) corresponding to constraints (4). Hence, we have iterative procedure defining in a sense the artificial "intellect" of the designed solution system:

$$y^{s+1} = \pi_Y (y^s + \rho_s v^s),\ s = 1, 2, \ldots, \tag{10}$$

where  is an iteration-dependent multiplier, which is a method's parameter, and $\pi_Y$ is the orthogonal projection operator onto set $Y$ (see also Remark 1, Sec. 2.3). Vector $v^s$ defines sub-gradient of the continuously non-differentiable function $F(x)$. This and the convergence of solutions $y^s$ to an optimal solution of the linkage problem (2)–(6) as $s \to \infty$ is analyzed in Sec. 3. The step-size $\rho_s$ is chosen from rather general and natural requirements: $\rho_s \ge 0$, $\sum_{s=1}^{\infty} \rho_s = \infty$, because generalized gradients are not the increasing directions of functions. Although the standard sub-gradient projection method converges without condition $\sum_{s=1}^{\infty} \rho_s^2 < \infty$, the proposed linkage algorithm for problems under asymmetric information (10) requires this additional condition to enable the convergence of not only function $F(y^s)$, but also solutions $y^s$. This allows us to propose a simple stopping criterion enabling the independent optimization of interdependent sectors by (10).

**2.3. Algorithm and its generalization.** The basic linkage algorithm with more details including stopping criteria is summarized as follows.

**Step 0.** Initialization. Sector $k$, $k = 1, \ldots, K$, chooses initial vectors $y^{0(k)}$ of quotas and submits it to the central computer (hub). The computer projects $y^0 = (y^{0(1)}, \ldots, y^{0(K)})$ onto the set $Y$ defining a first feasible approximation $y^1 = (y^{1(1)}, \ldots, y^{1(K)})$; set $s = 1$.

**Step 1.** Generic step. Suppose by the beginning of iteration $s$ the algorithm arrived at vector $y^s = (y^{s(1)}, \ldots, y^{s(K)})$. Then on iteration $s$ the algorithm proceeds as follows.

**Step 2.** All sectors/regions $k$ receive $y^{s(k)}$ and solve sectorial models (1)–(4) independently. Shadow prices $v^{s(k)}$ of common resources are submitted to the central computer.

**Step 3.** The central computer calculates $y^s + \rho_s v^s$ with a step-size $\rho_s = c_s / s$, where $c_s$ is a scaling parameter, $\underline{c} \le c_s \le \bar{c}$ for some constants $\underline{c}$, $\bar{c}$, which regulate $\rho_s$ so that the product $\rho_s v^s$ corresponds to the scale of $y^s$. Vector $y^s + \rho_s v^s$ is projected onto the set $Y$ and defines $y^{s+1}$. Sectors receive corresponding components of $y^{s+1}$.

**Step 4.** All sectors independently check stopping criteria. Sector $k$ calculates non-negative difference

$$\varepsilon_k(s) = (b^{(k)}, u^{s(k)}(y^s)) + (y^{s(k)}, v^{s(k)}(y^s)) - w_k(c^{(k)}, x^{s(k)}(y^s))$$

and submits values $\varepsilon_k(s)$ to the central computer of the common hub.

If $\sum_k \varepsilon_k(s) \le \varepsilon \ge 0$, where $\varepsilon$ is an admissible accuracy, then stop. Otherwise, continue with increments of 1 and return to step 1.

Section 3 shows that the independent functioning of sectors/regions according to this algorithm is possible without revealing sectorial information due to requirement $\sum_s \rho_s^2 < \infty$.

**Remark 1** (Computing the projection). The orthogonal projection $y^{s+1}$ of vector $\bar{y}^s = y^s + \rho_s v^s$ onto $Y$ is calculated by means of a fast algorithm minimizing the quadratic function $\|\bar{y}^s - y\|^2 = \sum_{k=1}^K \|\bar{y}^{s(k)} - y^{(k)}\|^2$, subject to joint constraints (5). This minimization can be done within a finite number of steps by using LP-transformations of quadratic optimization problems based on optimality equations. Because $y^s \in Y$, then projections $y^{s+1}$ of vector $y^s + \rho_s v^s$, $s = 1, 2, \ldots$, is very fast by minimizing $\| y^s + \rho_s v^s - y\|^2$ due to $\rho_s v^s \to 0$, as vectors $v^s$ are bounded optimal dual solutions (Proposition 1, a) and if $y^s$ is taken as an initial approximation for $y^{s+1}$.

**Remark 2** (Mixed constraints). Joint resource constraints of type (5) can be imposed by an external agency or can be jointly agreed upon by the participating sectors/regions. Sectors/regions may be subsidized or taxed to achieve certain levels of production — say to ensure a secure level of pollution in a common body of water. Such decisions affect not only vectors $x$ of sectorial models (1)–(4), but also vector $y$ of resource constraints (5). In this case, joint sectorial constraints (5) may have the following mixed form involving both $x^{(k)}$ and $y^{(k)}$ under a vector of common resources $\delta$:

$$\sum_{k=1}^{K} M^{(k)} x^{(k)} + \sum_{k=1}^{K} D^{(k)} y^{(k)} \le \delta, \tag{11}$$

where matrices $D^{(k)}$ define the marginal contribution of each $x^{(k)}$ into the constraint $\delta$ and also generate additional asymmetric information regarding common resources $y$.

Yet, problem (1)–(4), (11) can be transformed into a problem that has the same structure as (1)–(5) with separate constraints on common resources as follows:

Let us introduce vectors $z^{(k)}$ such that $M^{(k)} x^{(k)} \le z^{(k)}$, $k = 1, \ldots, K$, and re-write (11) as

$$\sum_{k=1}^{K} D^{(k)} y^{(k)} \le \delta - \sum_{k=1}^{K} z^{(k)}, \quad \sum_{k=1}^{K} z^{(k)} \le \delta .$$

After an appropriate re-notation, we indeed arrive at the problem of the form (1)–(5).

### 3. PROPERTIES OF THE ALGORITHM

In this section, we justify the convergence of the proposed linkage method (10). We rely on its connections with the duality theory and the theory of (continuously) non-differentiable optimization.

The following is important for our approach proposition and is derived from the known facts of duality theory. For example, the concavity of $F(y)$ and the importance of non-differentiable optimization follows from Proposition 1 (a). The verification of the stopping criteria in Sec. 2.3. follows from the convergence of $\{y^s\}$ to a point of $Y^*$ due to the theorem of this section.

Consider the sectorial/regional model $k$ defined by equations (1)–(4) for a given feasible $y$ satisfying the constraints of (5). The duality relations are usually established by using the Lagrangian function (see [13, 14]) as follows:

$$L_k(x^{(k)}, y^{(k)}, u, v) = w_k(c^{(k)}, x^{(k)}) + (u^{(k)}, b^{(k)} - A^{(k)}x^{(k)}) + (v^{(k)}, y^{(k)} - B^{(k)}x^{(k)}),$$

where $u$ and $v$ are dual variables.

**Proposition 1** (Duality relations). Assume there exist solutions $x^{(k)}(y)$ of all $K$ sectorial/regional models. Then:

a) Each Lagrangian function $L_k$, $k=1,\ldots,K$, has a saddle point $(x^{(k)}(y), u^{(k)}(y), v^{(k)}(y))$:

$$\min_{u,v\ge 0} L_k(x^{(k)}(y), y, u, v) = L_k(x^{(k)}(y), y, u^{(k)}(y), v^{(k)}(y)) =$$

$$= \max_{x\ge 0} L_k(x, y, u^{(k)}(y), v^{(k)}(y));$$

$$L_k(x^{(k)}(y), y, u^{(k)}(y), v^{(k)}(y)) = w_k(c^{(k)}, x^{(k)}(y)) = f^{(k)}(y), \qquad (12)$$

$$(u^{(k)}(y), b^{(k)} - A^{(k)}x^{(k)}(y)) + (v^{(k)}, y - B^{(k)}x^{(k)}(y)) = 0.$$

Because $L_k(x^{(k)}, y, u^{(k)}(y), v^{(k)}(y))$ for fixed $y$ is jointly concave in $(x^{(k)}, y)$, then after maximizing with respect to $x^{(k)}$, the resulting optimal value (12) $L_k(x^{(k)}(y), y, u^{(k)}(y), v^{(k)}(y)) = f^{(k)}(y)$ remains concave in $y$. Hence, $f^{(k)}(y)$, $F(y) = \sum_{k=1}^{K} f^{(k)}(x^{(k)})$ are concave functions.

The following facts b) and c) justify the stopping criterion of the linkage algorithm (Sec. 2.3.).

b) The dual minimax problem $\min_{u,v\ge 0} \max_{x\ge 0} L_k(x, y, u, v)$ is equivalent to the LP-problem (7)–(9).

The primal LP-model (1)–(4) is equivalent to the maximin problem, that is, maximizing the non-differentiable function in general: $\min_{u,v\ge 0} L_k(x, y, u, v)$.

c) The dual problem has a solution $(u(y), v(y))$ and these solutions satisfy the following equality:

$$f^{(k)}(y) = w_k(c^{(k)}, x(y)) = (b^{(k)}, u(y)) + (y, v(y)).$$

The following fact ([15, 16]) is fundamental for solving the linkage problem through maximizing non-differentiable function $F(y)$ by method (10).

**Proposition 2** (Sub-gradient). Assume there exist solutions $x^{(k)}(y)$ of all $K$ sectorial models. Then for any feasible solution $z$ and $y$,

$$f^{(k)}(y) - f^{(k)}(z) \ge (v^{(k)}(y), y - z),$$

that is, $v^{(k)}(y)$ is a sub-gradient of the concave function $f^{(k)}(y)$.

**Proof.** From Proposition 1 it follows that

$$f^{(k)}(y) - f^{(k)}(z) = (b^{(k)}, u^{(k)}(y)) + (y, v^{(k)}(y)) - (b^{(k)}, u^{(k)}(z)) - (z^{(k)}, v^{(k)}(z)) \ge$$

$$\ge (b^{(k)}, u^{(k)}(y)) + (y, v^{(k)}(y)) - (b^{(k)}, u^{(k)}(y)) - (z^{(k)}, v^{(k)}(y)) = (v^{(k)}(y), y - z).$$

**Corollary 1.** Vector $v(y) = (v^{(1)}(y), \ldots, v^{(K)}(y))$ is a sub-gradient of function $F(y) = \sum_{k=1}^{K} f^{(k)}(y)$, $F_y(y) = v(y)$, that is, $F(y) - F(z) \geq (v(y), y - z)$.

Therefore, the procedure (10) is a specific sub-gradient method for maximizing the (continuously) non-differentiable concave function $F(y)$.

Let us now show that $y^s$ converges to an optimal solution $y^*$, maximizing $F(y)$ subject to joint constraints (5).

**Theorem 1** (Non-monotonic convergence). Assume that

(a) The feasible set $Y$ is bounded;

(b) Step size $\rho_s$ satisfies the conditions:

$$\rho_s \geq 0, \ \sum_{s=1}^{\infty} \rho_s = \infty, \ \sum_{s=1}^{\infty} \rho_s^2 < \infty, \ \text{say} \ \ p_s = 1/s.$$

Then $\lim y^s \in Y^*$ for $s \to \infty$.

**Proof.** The property of the projection $\pi_Y(\cdot)$ yields for any optimal $y^* \in Y$:

$$\| y^* - y^{s+1} \|^2 \leq \| y^* - y^s - \rho_s v^s \|^2 \leq \| y^* - y^s \| - 2\rho_s (v^s, y^* - y^s) +$$
$$+ \rho_s^2 \| v^s \|^2 \leq \| y^* - y^s \|^2 + C\rho_s^2,$$

where $0 \leq F(y^*) - F(y) \leq (v^s, y^* - y^s)$, because $v^s$ is a generalized gradient of $F(y)$ at $y = y^s$ (Proposition 2).

Also, $\| v^s \|^2 < C < \infty$, where $C > 0$, is a positive constant because solutions $x^k(y^s)$ of primal and solutions $(u^k(y^s), v^k(y^s))$ of dual sectorial/regional models are bounded, as the feasible set $Y$ is bounded by our assumptions.

The sequence $\{\| y^* - y^s \|\}^2$ satisfying equations $\| y^* - y^{s+1} \|^2 \leq \leq \| y^* - y^s \|^2 + C\rho_s^2$, $\sum_{s=1}^{\infty} \rho_s^2 < \infty$, for all $y^* \in Y^*$ converges for $s \to \infty$ because the sequence $\tau_s = \| y^* - y^s \|^2 + C \sum_{t=s}^{\infty} \rho_t^2$ is monotonic, $\tau_{s+1} \leq \tau_s$ and $\sum_{s=1}^{\infty} \rho_s^2 < \infty$. Therefore, all accumulation points of $\{y^s\}$ are on the sphere of the radius $\lim \| y^* - y^s \|$. Hence, if we now show that one of the limit (accumulation) points of $\{y^s\}$ belongs to $Y^*$, then from this assertion would follow the convergence of $\{y^s\}$ to a point of $Y^*$.

Consider again the inequality

$$\| y^* - y^{s+1} \|^2 \leq \| y^* - y^1 \|^2 - 2 \sum_{t=1}^{s} \rho_t (v^t, y^* - y^t) + C \sum_{t=1}^{s} \rho_t^2.$$

Due to the inequality $F(y^*) - F(y^t) \geq (v^t, y^* - y^t)$ following from the definition of a generalized gradient $v^s = (v^{s(1)}, \ldots, v^{s(K)})$, we have

$$\| y^* - y^{s+1} \|^2 \leq \| y^* - y^1 \|^2 - 2 \sum_{t=1}^{s} \rho_t (F(y^*) - F(y^t)) + C \sum_{t=1}^{s} \rho_t^2.$$

Therefore, $\sum_{t=1}^{\infty} \rho_t (F(y^*) - F(y^t)) < \infty$. Since $\sum_{t=1}^{\infty} \rho_t = \infty$ and $F(y^*) - F(x^t) \geq 0$, then there exists a subsequence $y^{t_s}$ such that $F(y^*) - F(y^{t_s}) \to 0$, for $s \to \infty$. Therefore $\{y^s\}$ converges and the proof is completed.

**Remark 3.** The following sequence of $\rho_s$ for example, satisfies the conditions of the theorem: $\rho_s = \gamma_s / s, 0 \leq \underline{\gamma} \leq \gamma_s \leq \overline{\gamma} < \infty$ for some positive constants $\underline{\gamma}$ and $\overline{\gamma}$.

## 4. LINKING FOR FEWE NEXUS

In this section, we demonstrate the application of the developed iterative linkage procedure for linking agricultural, energy, water, and environmental models. The data are taken from the case study of FEWE nexus in water-scarce Shanxi province in China [11]. In the case study regions, coal and agricultural production are restricted by the availability of natural resources, most notably, water and land. Coal-based industries — mining, washing, chemical production, and power generation — are all extremely water-intensive. What makes the competition for water even worse is a huge mismatch between water resources and coal reserves — 53 % of China's coal reserves are located in water scarce regions, while 30 % are in water stressed regions — and Shanxi is one of them. Water supply to agriculture is also very important as it can significantly improve rural developments and maintain food security by ensuring basic grain sufficiency. Thus, this model addresses the problem of planning sustainable energy and agricultural sectors under water and land scarcity, as well as energy and food security goals in an integrated way. If water and land quotas to sectors are calculated in an independent way, this may lead to the violation of sustainability constraints. The model also addresses linking across production, processing, import, export locations. The model overview can be found in the Annex. Here we present the key results of numerical calculations demonstrating a fast convergence of the approximate solutions $y^s$ to the optimal solutions of the welfare maximization problem.

**Remark 4** (Computational stability). A fast convergence of the linkage algorithm based on the generalized sub-gradient method of non-differentiable optimization is observed and justified theoretically when optimal solutions are points of the non-differentiability. This is due to a fundamental difference between the case of continuously differentiable and non-differentiable functions, because generalized gradients (sub-gradients) do not approach zero at optimal solutions. For example, the minimization of one-dimensional function $F(y) = |y|$ has the solution $y^* = 0$ and sub-gradients equal $+1$ for $y > 0$ and $-1$ for $y < 0$. This type of non-smooth criterion function is used in robust statistics. The robustness of these types of methods with respect to random disturbances is used in stochastic optimization [1–4, 17–25] and references therein).

Table 1 presents the comparison of the optimal utility values between separately optimized models and overall welfare optimization. When sectorial models are solved separately, no joint constraints (5) are imposed, which allows the agriculture and coal sectors to gain about 15 % and 36 % of their utility, respectively. However, joint systemic constraints (5) are violated. In practice, this can lead to a shortage of water for one of the sectors and to a systemic failure. Under joint constraints, i.e. when models are hard-linked and/or when they are linked via a central "hub" thus ensuring systemic security, the total gains of feasible optimal solutions, that is, the net profits of sectors, can be lower. Numerical calculations by method (10) allow for fast identification of the most critical parts of the optimal solutions responsible for systemic stability and efficiency. The calculations also easily illustrate the value of mathematical models vs. simple calculations of "intuitively evident" direct net profits that ignore indirect systemic gains and losses, which may dramatically affect final conclusions and policies. For example, the lack of a bridge or a sectorial link connecting otherwise disconnected sectors/regions can cause losses that are incomparable with the direct cost of the bridge. This kind of systemic interdependencies are addressed by the proposed approach.

Figure 1 presents the values of the overall welfare function in each iteration of the algorithm for three different initial approximations, i.e. initial allocation of water and land quotas between two sectors. On the vertical axis are the total net profits (in bln. CNY) while method iterations appear on the horizontal axis. The three curves

**T a b l e 1.** Exact and iterative comparison of the optimal utility values between separately optimized models and overall welfare optimization

| Model | **Net profit, Total, bln. CNY** | **Net profit, Agriculture, bln. CNY** | **Net profit, Coal, bln. CNY** |
|---|---|---|---|
| Two separate sectorial optimizations (no joint constraints) | 254.2 | 17.7 | 236.5 |
| Welfare optimizing (hard-linked) | 176.2 | 14.1 | 162.1 |
| Linked via a central “hub” | 177.3 | 13.9 | 163.4 |

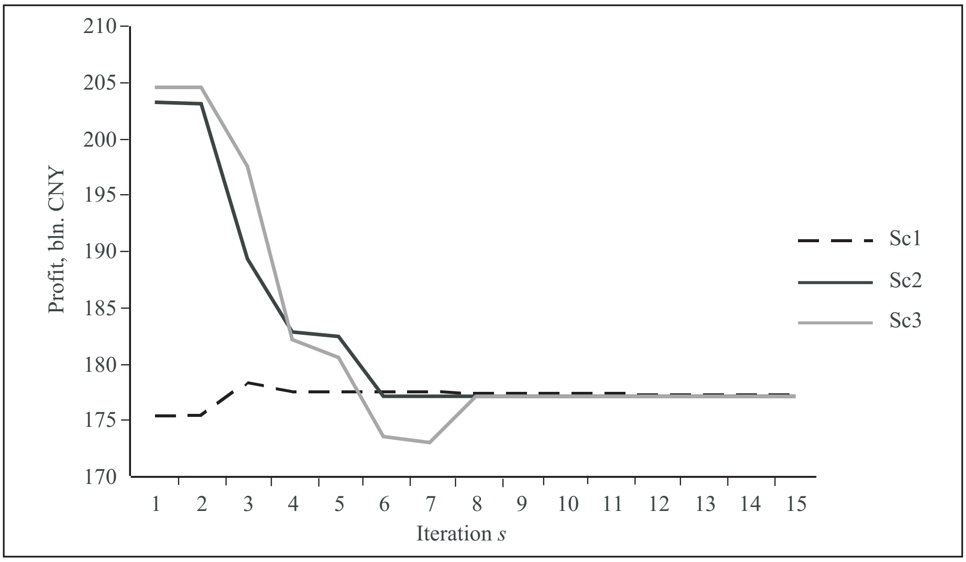


*Fig. 1.* Convergence (in terms of the utility value) of the iterative procedure to the exact solution of the overall welfare maximization problem

(Sc1, Sc2, Sc3) correspond to different initial conditions. In all three cases the iterative process converges rather quickly, that is, in the sixth iteration the optimal value is practically reached. After the tenth iteration, the accuracy becomes 0.6 %.

An essential factor affecting the convergence speed is of course the choice of the step-size $\rho_s$. A general rule is that the scale of the product $\rho_s v^s$ must correspond to the scale of the solutions $y^s$.

## CONCLUSIONS

This paper presented two novel results. First, it is the algorithm — or a model-based negotiation process — that allows the hub (central authority, principal agent), who pursues the maximization of public welfare, to distribute limiting resources between several actors without knowing the details of the internal structure of these actors and requiring only the shadow prices for the individual resource bounds. More specifically, this method links different linear optimization models into one system model without re-coding the sub-models into a single integrated model. Second, it is the illustrative demonstration of the algorithm for a case study in China.

While in this paper, we meant linking regional and/or sectorial models when referring to model linkage, more generally, linking models may refer to different local-global scales. Therefore, the linkage problem can also be formulated much more generally in terms of sub-models and integrated models and the approach presented in this paper can still be applicable. Remark 2 illustrates this in more detail.

The linkage of models is, in a sense, opposite to decomposition methods (e.g., [26, 27]). While in the decomposition we split an existing integrated optimization model into a number of smaller sub-models, in the linkage we obtain an integrated model of the system by linking existing explicitly unknown sub-models. Remark 2, Sec. 2.3, also demonstrates that the proposed methodology has a fundamentally new type of flexibility enabling the simultaneous use of linkage and decomposition procedures, in other words, endogenously disaggregating models to make their further integration more efficient.

The proposed computational algorithm is based on sub-gradient methods invented for the optimization of non-smooth systems, which may be subject to shocks and discontinuities. Therefore, these methods will be naturally developed further for linking stochastic models with known marginal distributions of sectorial uncertainties, into cross-sectorial integrated models with joint distributions of collective systemic risks induced by sectorial uncertainties and decisions maximizing a stochastic version of the function (6).

It is worth noting that we can also carry out the linkage of dynamic systems using the same equations (1)–(5) with vectors $x^{(k)} = (x^{(k)}(1), \dots, x^{(k)}(T))$, $y^{(k)} = (y^{(k)}(1), \dots, y^{(k)}(T))$ characterizing decisions and quotas of sectors $k = 1, \dots, N$ at time $t = 1, \dots, T$. Additional complications arise in the situation when vectors $x^{(k)}(t)$ have two components $x^{(k)}(t) = (z^{(k)}(t), u^{(k)}(t))$ representing the state variables $z^{(k)}(t)$ and the control variables $u^{(k)}(t)$. In this case, the saddle points in duality relations (Proposition 1) may have the form of discrete (in time) Pontryaging Maximum Principle. This enables the decomposition of the dynamic optimization model over interval $[1, T]$ into independent sub-problems for each $t = 1, \dots, T$, which can be solved by the proposed algorithm as indicated in Remark 2.

Another fundamentally important possible extension of the presented method is the case of stochastic sectoral/regional models in which the distribution of uncertainties are shaped by the decisions of various agents. The mitigation of floods by new land use decisions, for example, affect flood scenarios. As a rule, this makes it impossible to separate scenario generations and optimization procedures. This calls for linking both simulation and optimization procedures in a similar manner to algorithm (10), thus combining simulations of scenarios with optimization steps.

## ANNEX: MODEL OVERVIEW

In the model, variables $x_{ijmt}$ denote the amount of coal (in tons) of type *i* (brown, anthracite, etc.) produced in location *j*, transported to location *m*, and utilized (converted) by technology *t*. Variables $y_{kjm}$ denote the amount of crop (in tons) of type *k* produced in location *j* and exported to location *m*. Index *k* is used to represent a particular type of crop (such as corn, wheat, soybean, etc.).

**Energy, water, and agricultural nexus.** In the integrated energy-agricultural model, a social planner chooses how much coal *i* to exctract in location *j*, transport to location *m*, and convert by technology *t*. In addition, decisions are made on how much agricultural commodities *k* to produce in location *l* so that the total costs from the coal and agricultural production, transportation, and conversion is minimized, thus fulfilling constraints on natural resources, environmnetal pollution, and end-product demand. This is represented by:

$$\min_{x,y} \sum_{i,j,k,m,t} [c_{ij}^{CP} x_{ijmt} + c_{ijm}^{CT} x_{ijmt} + c_{ijt}^{CC} x_{ilmt} + c_{kj}^{AP} y_{kjm} + c_{kjl}^{AT} y_{kjm}], \qquad (13)$$

where $c_{ij}^{CP}$ stands for the production cost of a unit (i.e., ton) of coal of type $i$ in location $j$, $c_{ijm}^{CT}$ stands for the transportation cost of a unit of coal $i$ from location $j$ to location $m$, $c_{ijt}^{CC}$ stands for the conversion costs of a unit of coal $i$ by technology $t$ in location $j$, $c_{kj}^{AP}$ denotes costs associated with production of a unit agricultural commodity $k$ in location $j$, and $c_{kjm}^{AT}$ stands for the transportation cost of a unit of the agricultural commodity $k$ from location $j$ to location $m$.

**Energy and food security interdependencies.** We impose a constraint that defines the minimum required level of production of each agricultural commodity $k$ in each location $j$ as follows

$$\sum_j y_{kjm} \geq D_{km}^A, \tag{14}$$

where the right-hand side $D_{km}^A$ stands for the demand for agricultural commodity $k$ in location $m$. $D_{km}^A$ can be measured in terms of the minimum amount of daily calories required per capita suggested by the World Health Organization (WHO) accounting for size, age, sex, physical activity, climate, and other factors.

Demand for useful or final energy converted from coal, for example, electricity, which accounts for more than half of total coal conversion in China, heat, coke, gas, and oil. Thus, we introduce the constraint on the energy produced from coal as follows:

$$\sum_{m,t} \alpha_{ijt}^d x_{ijmt} \geq D_j^d, \tag{15}$$

where $\alpha_{jt}^d$ denotes the conversion efficiency of coal type $i$ in location $j$ by technology $t$, the end-product of type $d$, and $D_j^d$ stands for the demand for $d$.

**Natural resource constraints.** Another important driver and the main limitation of both energy and agricultural sectoral growth are natural (land and water) constraints. Energy and food security targets have to be jointly fulfilled, which is why they compete for resources. Land constraints prescribe that the total land used for agriculture, the land that subsides due to coal mining, and the land occupied by coal waste deposits cannot exceed the total available land $L_j$ in each location $j$. Thus, the constraint is formulated as follows:

$$\sum_{k,m} l_{kj} y_{kjm} + \sum_{i,m,t} x_{ijmt}(1-r_{ij})\Delta l_j l_{ij} + g\sum_{i,m,t} x_{ijmt} \leq L_j, \tag{16}$$

where $l_{kj}$ stands for the area of farmland required for the production of a unit of crop $k$ in location $j$, $l_{ij}$ stands for the area of land that subsides as a result of the mining of a unit of coal of type $i$ in location $j$, $\Delta l_j$ represents the fraction of the farmland overlapped with the coal filed in the location $j$, and $r_{ij}$ stands for the land reclamation rate (or efficiency rate) for coal $i$ in location $j$.

As water plays a key role in coal production and is simultaneously essential for agriculture, we impose a constraint on the total water required for coal production, processing, and conversion and for crop irrigation purposes in each location $j$:

$$\sum_{i,m,t} w_{ij}^P x_{imlt} + \sum_{i,m,t} w_{ij}^d x_{ijmt} + \sum_{k,m} w_{kj}^c y_{kmj} \leq W_j, \tag{17}$$

where $w_{ij}^P$ defines the amount of water required to produce a unit of coal $i$ in location $j$, $w_{ij}^d$ is the amount of water required to convert a unit of coal $i$ in location $j$, $w_{km}^{c}$ is the amount of water required to irrigate a unit of crop $k$ in location $j$, and $W_j$ defines water availability in location $j$. In the following numerical experiments, we do not account for environmental constraints such as, for example, $SO_2$ and $CO_2$ emissions targets.

**Individual sectoral models.** In the individual sectoral models, that is, the coal industry and the agricultural sector, the individual goal functions are as follows

$$\min_{x,y} \sum_{i,j,k,m,t} [c_{ij}^{CP} x_{ijmt} + c_{ijm}^{CT} x_{ijmt} + c_{ijt}^{CC} x_{ilmt}] \qquad (18)$$

and

$$\min_{x,y} \sum_{i,j,k,m,t} [c_{kj}^{AP} y_{kjm} + c_{kjl}^{AT} y_{kjm}] \qquad (19)$$

for the coal and agricultural sectors, respectively. Individual sectoral land constraints

$$\sum_{i,m,t} x_{ijmt} (1-r_{ij}) \Delta l_j l_{ij} + g \sum_{i,m,t} x_{ijmt} \le L_j^C \qquad (20)$$

and

$$\sum_{k,m} l_{kj} y_{kjm} \le L_j^A \qquad (21)$$

incorporate exogenous assumptions on land allocated for coal and crop production $L_j^C$ and $L_j^A$ in locations $j$, for the coal and agricultural sectors respectively.

Water quotas allocated to the sectors limit the choice of coal and crop technologies

$$\sum_{i,m,t} w_{ij}^{P} x_{imlt} + \sum_{i,m,t} w_{ij}^{d} x_{ijmt} \le W_j^C, \qquad (22)$$

and

$$\sum_{k,m} w_{kj}^{c} y_{kmj} \le W_j^A, \qquad (23)$$

where $W_j^C$ and $W_j^A$ define water provided to the coal and agricultural sectors in location $j$.

As discussed in Sec. 1, the independent allocation of $L_j^A$, $L_j^C$ and $W_j^A$, $W_j^C$ can lead to failures.

**Model linkage.** To link the energy ((18), (20), (22)) and the agricultural ((19), (21), (23)) models in such a way that joint constraints (16) and (17) are fulfilled, we implement the approach (1)–(11), where the procedure (10) sequentially adjusts the right hand-side of the resource constraints towards maximization of the aggregate welfare function

$$F(y) = \sum_{i,j,k,m,t} [c_{ij}^{CP} x_{ijmt} + c_{ijm}^{CT} x_{ijmt} + c_{ijt}^{CC} x_{ilmt}] + \sum_{i,j,k,m,t} [c_{kj}^{AP} y_{kjm} + c_{kjl}^{AT} y_{kjm}], \qquad (24)$$

where $x_{ijmt}$ and $y_{kjm}$ depend on the resource constraints.

The linkage of the models is accomplished as follows:

At the initial step 0, individual sectoral models are solved using arbitrary assumptions on resource distribution $L_j^C(0)$, $L_j^A(0)$ and $W_j^C(0)$, $W_j^A(0)$ between the sectors. The solutions of the individual models depend on the resource constraints $x_{ijmt}(L_j^C(s-1), L_j^A(s-1), W_j^C(s-1), W_j^A(s-1))$ and $y_{kjm}((L_j^C(s-1), L_j^A(s-1), W_j^C(s-1), W_j^A(s-1))$.

The resource distribution at step $s$ is adjusted according to (10) using marginal values (dual variables) of the constraints

$$\sum_{i,m,t} x_{ijmt}(1-r_{ij})\Delta l_j l_{ij} + g \sum_{i,m,t} x_{ijmt} \leq L_j^C(s-1), \tag{25}$$

$$\sum_{k,m} l_{kj}\, y_{kjm} \leq L_j^A(s-1), \tag{26}$$

$$\sum_{i,m,t} w_{ij}^P x_{imlt} + \sum_{i,m,t} w_{ij}^d x_{ijmt} \leq W_j^C(s-1), \tag{27}$$

$$\sum_{k,m} w_{kj}^c\, y_{kmj} \leq W_j^A(s-1). \tag{28}$$

It is important to mention that the proposed approach allows us to link the models under asymmetric information on remote computers.

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**РОБАСТНЕ УПРАВЛІННЯ ДЛЯ БЕЗПЕКИ У ВЗАЄМОЗАЛЕЖНІЙ СИСТЕМІ ПРОДОВОЛЬСТВО–ЕНЕРГІЯ–ВОДА–ДОВКІЛЛЯ: ПРОЦЕДУРА СТОХАСТИЧНИХ КВАЗІГРАДІЄНТІВ ДЛЯ ЗВ'ЯЗУВАННЯ РОЗПОДІЛЕНИХ ОПТИМІЗАЦІЙНИХ МОДЕЛЕЙ В УМОВАХ АСИМЕТРИЧНОЇ ІНФОРМАЦІЇ ТА НЕВИЗНАЧЕНОСТІ**

**Анотація.** Запропоновано послідовний алгоритм для зв'язування децентралізованих розподілених оптимізаційних регіональних і секторальних моделей в умовах асиметричної інформації та невизначеності на основі ітеративних процедур стохастичних квазіградієнтів, розроблених для негладкої та недиференційовної оптимізації. Розроблену процедуру використовують для об'єднання індивідуальних регіональних і секторальних моделей для інтегрованого взаємозалежного аналізу та управління безпекою в системі продовольство–енергія–вода–довкілля.

**Ключові слова:** підтримка під час прийняття рішень, асиметрична інформація, зв'язування, процедури стохастичних квазіградієнтів, негладка оптимізація, субградієнт, інтегроване моделювання, управління взаємозалежностями в системі продовольство–енергія–вода–довкілля.

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