UDC 519.9

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DEVELOPING A MODEL FOR THE MODULATING MIRROR FIXED ON ACTIVE SUPPORTS: A STOCHASTIC MODEL[1]

Abstract. The paper proposes a stochastic version of the problem of modulating a mirror fixed on active supports. It is assumed that the mirror has several defects of elliptical form with stochastic parameters. The problem is to find the control forces that provide the best approximation of a given shape and phase of the mirror oscillation taking into consideration defects with undefined geometric and mechanical characteristics. It is supposed that the system works inappropriately (i.e., “fails”) if the phase or amplitude deviates from the target more than some specified threshold. To minimize the risk of such deviation, we use Buffered Probability of Exceedance (bPOE) as a measure of risk.

Keywords: risk; CVaR; bPOE; structural reliability; modulating a mirror; amplitude and phase of oscillation; optimization..

INTRODUCTION

Usually, in reliability engineering, risk is quantified by probability of failure, which is the chance that a system fails to carry out its intended task. Although failure probability is very popular, it has undesirable mathematical properties, such as discontinuity of sample distributions. Therefore, it is very difficult to optimize probabilities. To overcome these shortcomings a new alternative risk measure Buffered Probability of Failure (BPF) was developed in [1]. It takes into account the degree of exceeding the failure threshold $h=0$ and is more conservative than the classical probability of failure. Buffered Probability of Exceedance, bPOE, proposed in [2], generalizes BPF in the case where the failure threshold of the system can be any number (not just 0). These risk measures are based on the properties of the CVaR risk measure [3, 4]. bPOE has exceptional mathematical properties (under general conditions bPOE is quasi-convex with respect to the random variable).

In this paper we develop stochastic version of a model, which optimizes parameters of mechanical devices for excitation and formation of wave motion, developed in [5]. Particularly, we consider stochastic model for modulating mirror fixed on active supports taken into consideration multiple defects (inhomogeneities) located on the mirror. It is supposed that a number of defects located on the mirror and their characteristics are uncertain. The problem is to find control forces and their location providing the best approximation of a given shape and phase of the oscillations for the mirror under uncertainty.

1. DETERMINISTIC MODEL

Consider deterministic model, which optimizes parameters of mechanical devices for excitation and formation of wave motion, developed in [5]. Let $I+1$ forces with intensities $F_k = u_k + iv_k$, $k=0,\ldots,I$, be applied to the plate at points $\xi_0(0)$, $\xi_k(r_{\xi_k}, \varphi_{\xi_k})$, $k=0,\ldots,I$. It is supposed that one force, F_0, is always located at the

[1]This paper presents results obtained in the project “Application of Buffered Probability of Exceedance (bPOE) to Structural Reliability Problems” supported by the European Office of Aerospace Research and Development. Grant EOARD #16IOE094.

center of the plate $\xi_0(0)$. The plate contains J defects, located (in polar coordinates) in points (r_{d_j}, φ_{d_j}), $j = 1, \ldots, J$. Denote $r = (r_{d_1}, \ldots, r_{d_J})$, $\varphi = (\varphi_{d_1}, \ldots, \varphi_{d_J})$. Let $\vec{F} = (F_0, \xi_0(0), F_1, \xi_1(r_{\xi_1}, \varphi_{\xi_1}), \ldots, F_I, \xi_I(r_{\xi_I}, \varphi_{\xi_I}))$ be vector of characteristics of forces applied to the plate, $W(r, \varphi) = \operatorname{Re} W(r, \varphi) + i \operatorname{Im} W(r, \varphi)$ be desired mirror oscillation form of the surface of the plate, $w(\vec{F}, r, \varphi)$ be actual mirror oscillation form. Mean-square deviation of $w(\vec{F}, r, \varphi)$ from $W(r, \varphi)$ for the given vector $\vec{F}$ is

$$Q(\vec{F}) = \frac{1}{2\pi} \int_0^1 r dr \int_{-\pi}^{\pi} | w(\vec{F}, r, \varphi) - W(r, \varphi)|^2 d\varphi. \tag{1}$$

In deterministic case, for the given form of the surface of the plate $W(r, \varphi)$, it is required to determine optimal characteristics of forces $\vec{F}^*$ from the optimization problem [5]:

$$Q(\vec{F}^*) = \min_{\vec{F}} Q(\vec{F}). \tag{2}$$

2. STOCHASTIC MODEL

Stochastic model for modulating mirror fixed on active supports takes into consideration multiple defects (inhomogeneities) with uncertain characteristics located on the mirror. We suppose that number of defects N_{def} is random, and each defect is characterized by a vector of stochastic parameters (geometric and mechanical characteristics) $\vec{\ell}_{def}$, and a vector of random locations $\vec{\chi}_{def}$. Stochastic vector $\vec{\theta}^T = \{N_{def}, \vec{\ell}_{def}, \chi_{def}\}$ transfers deterministic function of mirror oscillation form $w(\vec{F}, r, \varphi)$ into random function $w(\vec{\theta}, \vec{F}, r, \varphi)$. Therefore, deterministic mean-square deviation $Q(\vec{F})$ in (1) is transformed to the following random function

$$L(\vec{\theta}, \vec{F}) = \frac{1}{2\pi} \int_0^1 r dr \int_{-\pi}^{\pi} | w(\vec{\theta}, \vec{F}, r, \varphi) - W(r, \varphi)|^2 \, d\varphi. \tag{3}$$

Let η be a measure of risk.

We build the following risk functional $H(\vec{F})$ with respect to mean square deviation of the deflection of the plate from the given oscillation form $W(r, \varphi)$:

$$H(\vec{F}) = \eta \left(\int | w(\vec{\theta}, \vec{F}, r, \varphi) - W(r, \varphi)|^2 dr d\varphi \right). \tag{4}$$

where η is a measure of risk, and $\vec{F} = (F_0, \ldots, F_I)$. Information about measure of risk η is given in the next section.

Thus, the control problem is reduced to the problem of minimization of the risk functional (4) on a given domain of control space $\vec{F} \in U$:

$$\min_{\vec{F} \in U} H(\vec{F}). \tag{5}$$

3. RISK MEASURES

Let F_X be the cumulative distribution function (CDF) of a random variable X modeling losses:

$$F_X(x) = P\{X \le x\} \text{ for } x \in (-\infty, \infty).$$

The random variable X can be alternatively described by the quantile function $q_{\alpha}(X)$, which is the inverse of its cumulative distribution:

$$F_X^{-1}(\alpha) = q_{\alpha}(X) = \inf\{x{:}\alpha \le F_X(x)\} \text{ for } \alpha \in (0,1).$$

In financial applications the quantile is called Value-at-Risk (VaR). It is used to estimate tails of distributions.

For a given threshold $x \in (-\infty, \infty)$ the probability of exceedance (POE) with threshold x equals:

$$p_X(x) = P\{X > x\} = 1 - F_X(x).$$

Since POE describes the upper tail of the distribution, it is commonly used in reliability engineering for quantifying of failure probability, which is the chance that a system fails to carry out its intended task. We use POE to quantify the probability that the approximation error would exceed a certain threshold. Both POE and VaR are "optimistic" risk functions which are based on the lower bound of outcomes in the tail. It should be noted that these characteristics do not provide information about large losses, that may occur with low probability. Besides, POE and VaR are difficult to optimize for discrete distributions, because they are non-convex, non-smooth, and they have multiple local extremes.

More attractive properties have Conditional Value-at-Risk (CVaR) and Buffered Probability of Exceedance (bPOE). These measures of risk are conservative counterparts of VaR and POE. By definition, CVaR for a distribution, which has no jump at the point $q_{\alpha}(X)$, is equal to the expected loss exceeding this quantile, see [3, 4]

$$CVaR_{\alpha}(X) = E[X : X \ge q_{\alpha}(X)].$$

In general case, when $F_X(x)$ has jump at the point $q_{\alpha}(X)$, the definition of CVaR is more complicated:

$$CVaR_{\alpha}(X) = \min_u \left(u + \frac{E[X-u]^+}{1-\alpha} \right),$$

where $[\cdot]^+ = \max\{\cdot, 0\}$.

CVaR has a much better mathematical properties compared to VaR. It is convex in random variables. Moreover, it can be optimized with linear programming and convex optimization algorithms.

Risk measure bPOE equals to one minus the inverse of the superquantile [2]:

$$bPOE_x(X) = \begin{cases} 0 & \text{if } x \ge \sup X; \\ 1 - CVaR^{-1}(x; X) & \text{if } EX < x < \sup X; \\ 1 & \text{otherwise.} \end{cases}$$

Risk measure bPOE describes the chance that the average of the data points in the upper tail of the distribution will equate to a specified threshold, while POE only describes the likelihood that some specified threshold will be exceeded [6]. Threshold used in the calculation of the POE is set by user, while threshold used in the calculation of the bPOE is strictly specified to be the averaged of the data points in the tail. Risk measure bPOE is an upper bound for POE because it includes all outcomes exceeding the threshold, as well as some outcomes below the threshold. The outcomes below the threshold form the so called buffer [7]. Results of optimization problems, where bPOE and CVaR are used only in constraints, are equivalent. However, the problems of minimizing bPOE and CVaR are quite different [8]. Risk measure bPOE was used in many optimization problems, for example, in [6–13].

4. NUMERICAL APPROACH FOR SOLVING THE PROBLEM

We solve problem (1) in three steps:

Step 1. For given form of vibration, wave number, and number of forces, solve deterministic optimization problem without defects, and obtain optimal characteristics of forces (application points, real and imaginary parts of their amplitudes).

Step 2. Perturb the optimal deterministic solution by adding a random number of defects with random parameters. We simulate locations, geometric and mechanical characteristics of defects as independent random values, which are uniformly distributed within the given limits. As a result we obtain a sample of deviations of the mirror deflection (and the phase deflection) from the given oscillation form.

Step 3. Minimize bPOE as a measure of risk [2].

To demonstrate this approach, consider the following examples.

Example 1.

Parameters: wave number is 1.23; number of forces is 5.

Step 1. Deterministic problem.

Statement of Problem 1.

For given form of the plate surface $W(r,\varphi) = \operatorname{Re} W(r,\varphi) + i \operatorname{Im} W(r,\varphi)$ and these parameters we determined the optimal characteristics of forces, which minimize the value of the deterministic function (2):

$$\min_{\vec F} Q(\vec F) \qquad (6)$$

We determined optimal forces $\vec F^* = (u_0^* + iw_0^*, \ldots, u_4^* + iw_4^*)$. Real parts of $\vec F^*$ are presented in Table 1, and imaginary parts are presented in Table 2. $F_0^* = u_0^* + iw_0^*$ is the value of the force applied at the center of the plate ($R_0 = 0$). The other forces form two groups with two forces, having a common radius and a centrally symmetrical arrangement:

$$R_1 = R_2 = 0.443,\ \Phi_1 = 0,\ \Phi_2 = \pi,$$

$$R_3 = R_4 = 0.443,\ \Phi_3 = \pi/2,\ \Phi_4 = 3\pi/2.$$

Optimal value of mean square deviation of the deflection of the plate (and the deflection phase) from the given oscillation form in deterministic case is $Q(\vec F^*) = 1.826\text{E}{-3}$.

Step 2. Perturbation of the optimal deterministic solution of problem (6).

We generated $N_{Sample} = 200$ independent random realizations of defects $\vec\theta_1, \ldots, \vec\theta_{N_{Sample}}$. Defects where modeled by small inhomogeneities with changed elastic characteristics. Using iterative technique, developed in [5], we modeled finite-size defects in a Kirchhoff plate by point quadrupoles. Then we perturbed the optimal deterministic solution of optimization problem (6).

Table 1. Real parts of optimal forces u_j^*, $j = 0, \ldots, 4$

u_0^*	u_1^*	u_2^*	u_3^*	u_4^*
-8.49E-03	-1.74E+00	1.67E+00	-3.80E-02	-3.80E-02

Table 2. Imaginary parts of optimal forces w_j^*, $j = 0, \ldots, 4$

w_0^*	w_1^*	w_2^*	w_3^*	w_4^*
3.55E-15	-1.71E-08	1.71E-08	-1.71E+00	1.71E+00

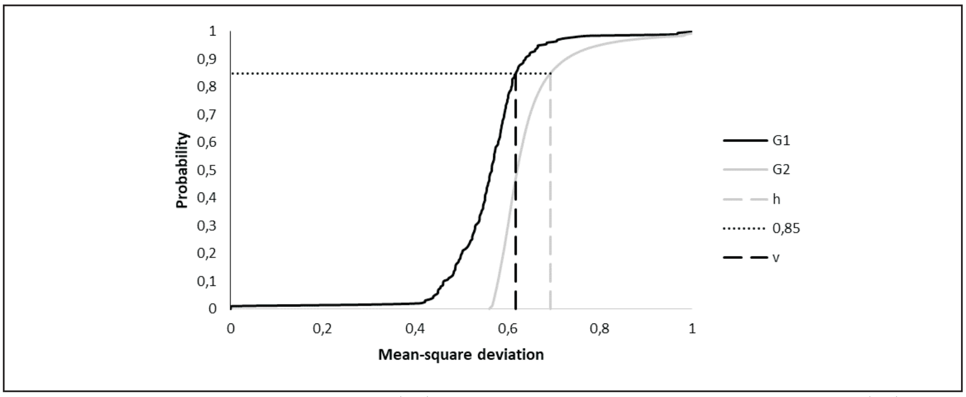

Fig. 1. Distribution G1 of random value $L(\vec{\theta}, \vec{F}^*)$ vs distribution G2 of random value $CVaR(L(\vec{\theta}, \vec{F}^*))$ before optimization. Wave number is 1.23, number of forces is 5

As a result, we obtained N_{Sample} values $L_1(\vec{\theta}_1, \vec{F}^*), \ldots, L_{N_{Sample}}(\vec{\theta}_{N_{Sample}}, \vec{F}^*)$, which we consider as independent realizations of the random value $L(\vec{\theta}, \vec{F}^*)$ in (3). Using these values, we determined $CVaR_{\alpha_i}(L(\vec{\theta}, \vec{F}^*))$, where $\alpha_i = \dfrac{i}{N_{Sample}}$, $i = 1, \ldots, N_{Sample}$. Let $CVaR(L(\vec{\theta}, \vec{F}^*))$ denote discrete random value taking equally probable values $CVaR_{\alpha_i}(L(\vec{\theta}, \vec{F}^*))$, $i = 1, \ldots, N_{Sample}$.

Cumulative distribution functions of the random value $L(\vec{\theta}, \vec{F}^*)$, denoted by G1, and random value $CVaR(L(\vec{\theta}, \vec{F}^*))$, denoted by G2, are presented in Fig. 1.

The black solid line in Fig. 1 corresponds to the distribution of random value $L(\vec{\theta}, \vec{F}^*)$, and the gray solid line corresponds to distribution of random value $CVaR(L(\vec{\theta}, \vec{F}^*))$. The black dotted line indicates 0.85-quantile of the distribution function of the random value $L(\vec{\theta}, \vec{F}^*)$. It corresponds to the threshold $\upsilon = 0.6193943$. The gray dotted line indicates 0.85-quantile of the distribution function $CVaR(L(\vec{\theta}, \vec{F}^*))$. It corresponds to the threshold $h = 0.693582$. The threshold υ was chosen such that $E(L(\vec{\theta}, \vec{F}^*) | L(\vec{\theta}, \vec{F}^*) \geq \upsilon) = CVaR_{0.85}(L(\vec{\theta}, \vec{F}^*)) = h.$ Interception between the CDF G1 and the threshold υ and interception between the CDF G2 and the threshold h are located on the black dotted line, which corresponds to 0.85 level of probability. Thus, before optimization, the value of POE, corresponding to the threshold υ, and the value of bPOE, corresponding to the threshold h, are the same and are equal to 0.15 (1–0.85).

After perturbation of the optimal deterministic solution, the value of mean square deviation of the deflection of the plate $Q(\vec{F}^*)$ from the given oscillation form increased from 1.826E-3 to 0.15.

Step 3. Minimize bPOE as a measure of risk (4) in (5).

Statement of problem 2.

For threshold $h = 0.693582$ determine the optimal characteristics of forces, which minimize bPOE.

$$\min_{\vec{F} \in U} \; bPOE_h(L(\vec{\theta}, \vec{F})). \tag{7}$$

To solve this optimization problem we used AORDA PSG package [14]

Optimal solution of stochastic optimization problem 2.

Table 3. Real parts of optimal forces u_j^{**}, $j = 1, \ldots, 5$

u_1^{**}	u_2^{**}	u_3^{**}	u_4^{**}	u_5^{**}
-6.40E-03	-1.74E+00	1.67E+00	-3.59E-02	6.31E-02

Table 4. Imaginary parts of optimal forces w_j^{**}, $j = 1, \ldots, 5$

w_1^{**}	w_2^{**}	w_3^{**}	w_4^{**}	w_5^{**}
1.01E-01	2.09E-03	2.09E-03	-1.61E+00	1.81E+00

Let $\vec{F}^{**} = (u_1^{**} + iw_1^{**}, \ldots, u_5^{**} + iw_5^{**})$ denote optimal solution of the problem (7) with $h = 0.693582$.

Real parts of $\vec{F}^*$ are presented in Table 3, and imaginary parts are presented in Table 4.

Similar to Step 2, we used optimal solution $\vec{F}^{**}$ of the problem (7) to determine independent realizations $L_1(\vec{\theta}_1, \vec{F}^{**}), \ldots, L_{N_{Sample}}(\vec{\theta}_{N_{Sample}}, \vec{F}^{**})$ of new discrete random value $L(\vec{\theta}, \vec{F}^{**})$ and independent realizations $CVaR_{\alpha_i}(L(\vec{\theta}, \vec{F}^{**}))$, $i = 1, \ldots, N_{Sample}$, of new discrete random value $CVaR(L(\vec{\theta}, \vec{F}^{**}))$. Tails of cumulative distribution functions of the random value $L(\vec{\theta}, \vec{F}^{**})$, denoted by G1_1, and the random value $CVaR(L(\vec{\theta}, \vec{F}^{**}))$, denoted by G2_1, are presented in Fig.2 together with tails of CDFs G1 and G2.

Characters G1, G2, h, and v in the legend of Fig. 2 have the same meaning as in the legend of Fig. 1. The black solid line, denoted by G1_1, corresponds to distribution of random value $L(\vec{\theta}, \vec{F}^{**})$, obtained after bPOE minimization. The gray solid line, denoted by G2_1, corresponds to the distribution of the random value $CVaR(L(\vec{\theta}, \vec{F}^{**}))$, obtained after bPOE minimization. The black dotted line segment “delta_POE” demonstrates how much the value of POE, corresponding to the threshold v, diminished after bPOE minimization. The line segment “delta_bPOE” demonstrates how much the value of bPOE, corresponding to the threshold h, diminished after bPOE minimization.

In the considered case $\text{delta\_ bPOE} = 0.07 - 0.15 = -0.08$, and $\text{delta\_ POE} = 0.04 - 0.15 = -0.11$. This example demonstrates that minimization of bPOE reduces not only bPOE (by 53.33%), but also POE (by 73.33%).

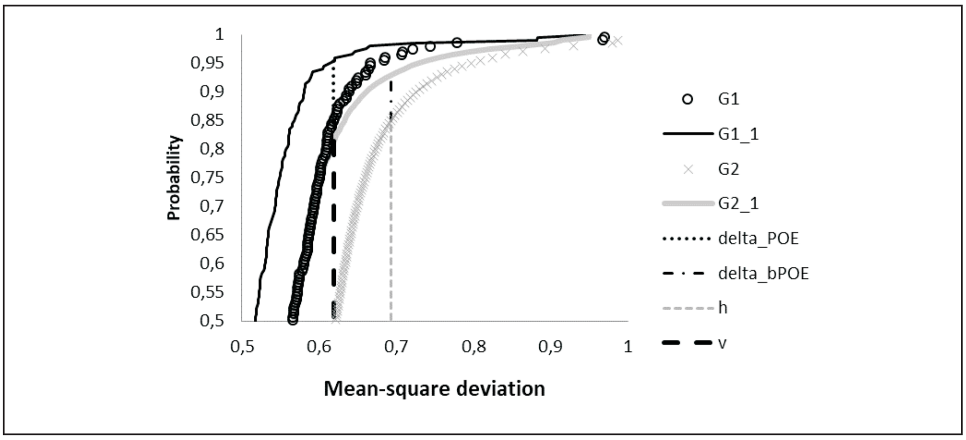

Fig. 2. Comparison of cumulative distribution functions before and after minimization of bPOE with threshold h is 0.693582. Wave number is 1.23, number of forces is 5

Table 5. Results obtained after bPOE optimization

Confidence level	Wave number	Number of forces	Threshold h	Threshold v	delta bPOE	delta POE
0.85	1.23	5	0.693582	0.6193943	-53.33%	-73.33%
0.9	1.23	5	0.726248	0.6393482	-48.47%	-64.82%
0.85	5.91	7	0.893926	0.4931682	-42.30%	-35.40%
0.9	5.91	7	1.057949	0.6432257	-43.28%	-50.68%
0.95	1.23	5	0.7977	0.6664913	-41.46%	-60.50%
0.99	1.23	5	0.987562	0.9695511	-100.00%	-100.00%
0.8	5.91	7	0.785515	0.4161413	-40.08%	-28.05%
0.95	5.91	7	1.381567	0.6470629	-41.93%	-10.23%
0.99	5.91	7	2.312896	1.452711	-14.75%	-54.65%
0.85	3.23	7	0.452861	0.1950224	-4.33%	-83.13%
0.99	3.23	7	1.540914	1.263241	-100%	-75.49%
spnum0.9	1.23	7	1.272189	1.02435	-16.01%	-15.99%
0.95	1.23	7	1.445576	1.150952	-100%	-94.28%
0.99	1.23	7	1.886962	1.502909	-21.36%	-17.36%

5. RESULTS OF CALCULATIONS

Using approach described in Sec. 4, we solved the problem (7) with the following parameters: confidence level: 0.8, 0.85, 0.9, 0.95, 0.99; wave number: 1.23, 3.23, 5.91, number of forces: 5, 7. The results obtained after bPOE optimization are summarized in Table 5.

CONCLUSIONS

We have demonstrated how bPOE can be used to structural reliability problems. As an example, we considered a problem of modulating a mirror fixed on active supports. We assumed that the mirror has several defects (inhomogeneities) with uncertain characteristics and developed appropriate stochastic models. We performed computer simulations to find control forces, which provide the best approximation of a given shape and phase of the mirror oscillation taking into consideration structural inhomogeneities with undefined geometric and mechanical characteristics. Each run of simulation consists of three steps.

At the first step, for given form of vibration, wave number, and number of forces, we solved deterministic optimization problem without defects, and obtained optimal characteristics of forces (application points, real and imaginary parts of their amplitudes).

At the second step, we perturbed the optimal deterministic solution by adding defects. We generated a random number of defects with random parameters. We simulated locations, geometric and mechanical characteristics of defects as independent random values, which are uniformly distributed within the given limits. As a result, we obtained a sample of deviations of the mirror deflection (and the phase deflection) from the given oscillation form.

At the third step we minimized risk of high deviation of the deflection of the plate (and the deflection phase) from the given oscillation form using bPOE and CVaR measures of risk and found optimal forced. Our numerical experiments demonstrated that minimization of bPOE result to minimization of probability of exceedance (POE). Taking into consideration that optimization of POE is a very had problem, optimization of bPOE is a good alternative to optimization of POE in practical problems of structural reliability problems.

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РОЗРОБЛЕННЯ МОДЕЛІ МОДУЛЮВАЛЬНОГО ДЗЕРКАЛА, ЗАКРІПЛЕНОГО НА АКТИВНИХ ОПОРАХ: СТОХАСТИЧНА МОДЕЛЬ

Анотація. Запропоновано стохастичну версію моделі модулювального дзеркала, закріпленого на активних опорах за припущення, що на поверхні дзеркала можуть бути дефекти з випадковими параметрами. Задача полягає в пошуку таких сил керування, які б забезпечили найкраще наближення заданої форми та фази коливань однорідного дзеркала, а також враховували дефекти з випадковими геометричними та механічними характеристиками. Зроблено припущення, що система працює неадекватно (тобто «відмовляє»), якщо фаза або амплітуда відхиляються від заданих значень на величину, більшу за певний заданий поріг. Під час мінімізації ризику цього відхилення використано bPOE (міра ризику).

Ключові слова: ризик, CVaR, bPOE, структурна надійність, модуляція дзеркала, амплітуда і фаза коливань, оптимізація.

Надійшла до редакції 16.06.2022