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FAILURE OF CONTROL DEVICES UNDER CONFLICT CONDITIONS

Abstract. The authors consider a nonstationary game problem of control of moving objects in the case of violations in their dynamics caused by a breakdown or failure of the control devices. A game situation is analyzed where the moment of failure of control devices is a priori unknown, and the time required to eliminate it is given. The sufficient conditions for bringing the trajectory of the conflict-controlled process to the terminal set in a certain finite time are established. The results are illustrated using a model example with simple motion.

Keywords: conflict-controlled process, set-valued mapping, resolving function, failure of control devices, stroboscopic strategy, Pontryagin's condition, Aumann's integral.

INTRODUCTION

One of the important areas of artificial intelligence is the methods of decision making and control in various situations [1–4], including motion control in conditions of conflict and uncertainty. Problems from this area are usually called differential or dynamic games, conflict-controlled processes [5–9].

Along with the methods that reveal the structure of the game and are focused on the construction of optimal strategies in the theory of differential games, there are approaches aimed at a guaranteed result that give sufficient conditions for completing the goal without focusing on the issue of optimality. The latter is quite justified from the practical point of view. These approaches include the first direct method of L.S. Pontryagin [6] and the method of resolving functions [8]. Both of them are based on the same principle of constructing the control of the first player using the measurable control theorems [10].

An attractive feature of the method of resolving functions is that it allows effective use of the modern technique of set-valued mappings and their selections [11] in substantiating game constructions and obtaining meaningful results on their basis. The method, in particular, substantiates the rule of parallel pursuit and the method of approach along the ray [12, 13], well known to designers of rocket and space technology.

In this paper, the method of resolving functions is used to solve the linear non-stationary control problem with violations in dynamics arising as a result of a failure of control devices. Previously, such problems were considered in [14, 15]. The essence of the problem is as follows. The conflict-controlled process develops in such a way that at some a priori unknown moment the control devices of the first player fail for the time necessary to eliminate the breakdown and which is known in advance. Then the process continues until the trajectory hits the terminal set. It is necessary to find conditions for the finiteness of time of the trajectory hitting the terminal set and control of the first player providing this result [16–18].

In this work, sufficient conditions are obtained for the solvability of the game problem of approaching the trajectory of a non-stationary conflict-controlled process with a time-varying terminal set in the event of a temporary failure of control devices. An illustrative example is given.

PROBLEM STATEMENT

Let the motion of an object in a finite-dimensional Euclidean space E^n be a non-stationary conflict-controlled process, the evolution of which is given by the equation

$$\dot{z} = C(t)\, z + \alpha_\tau(t)u - v,\ \ z(t_0) = z_0,\ \ t \ge t_0 \ge 0,\ \ u \in U(t),\ \ v \in V(t), \qquad (1)$$

where $C(t)$ is the matrix function of order n, whose elements are measurable and summable on any finite time interval, the admissible control parameters of the opposing sides are measurable selections of the compact-valued measurable mappings $U(t)$ and $V(t)$. Functions $\alpha_\tau(t)$ have forms

$$\alpha_\tau(t) = \begin{cases} 0, & t \in [\tau, \tau + \delta], \\ 1, & t \,\overline{\in}\, [\tau, \tau + \delta], \end{cases}$$

where $\tau, \tau > t_0$, and δ are finite positive numbers, $\tau \in [\tau_1, \tau_2]$, $\tau_1 < \tau_2 < +\infty$.

It is easy to see that

$$\alpha_\tau(t) = h(t) - h(t-\tau) + h(t - \tau - \delta),$$

where

$$h(t-\tau) = \begin{cases} 0, & t < \tau, \\ 1, & t \ge \tau \end{cases}$$

is the well-known Heaviside function [19].

The meaning of the function $\alpha_\tau(t)$ is as follows. At a moment τ, unknown in advance, the only interval of its possible values is known, the failure of the control device occurs, and the coefficient at a control u vanishes. It takes a certain time δ to eliminate the fault and then the process of control in a condition of conflict continues.

In addition to dynamics (1), a cylindrical terminal set is given:

$$M^*(t) = M_0 + M(t), \quad t \in [t_0, +\infty), \qquad (2)$$

where M_0 is a linear subset from E^n, $M(t)$ is a measurable compact-valued mapping and its direct images belong to the orthogonal complement L to M_0 in E^n.

The goals of the players are the opposite. The first (u) strives to bring the trajectory of the process (1) to the terminal set (2) in the shortest time, despite the fault, and second (v) — to maximally postpone the moment of the trajectory $z(t)$ hitting the set $M^*(t)$ or to avoid the meeting altogether, using, perhaps, the moment τ.

Let us focus on the sufficient conditions for the first player to win in the game (1), (2) under various-kind information availability.

If the game (1), (2) runs on the interval $[t_0, T]$, Krasovskii' strategy [7] of the first player assigns to choose control at the moment t, $t \in [t_0, T]$, in the form of the measurable function

$$u(t) = u(t_0, z_0, v_t(\cdot)),\ u(t) \in U(t),$$

$v_t(\cdot) = v(s) : v(s) \in V(s), s \in [t_0, t]$, i.e. based on control prehistory of the second player. In the same game, Hayek's stroboscopic strategy [9] assigns to the first player [7]

$$u(t) = u(t_0, z_0, v(t)), \quad u(t) \in U(t),$$

in addition, with measurable control $v(t)$; $v(t) \in V(t)$, the function $u(t)$ must be measurable.

SCHEME OF THE METHOD

Let us denote by π the orthoprojector, acting from R^n to L, and by $\Phi(t, s)$ — the fundamental matrix of homogeneous system (1). Consider the set-valued mappings

$$W(t, s, v) = \pi\Phi(t, s)U(s) - \pi\Phi(t, s)v, \ \ v \in V(s),$$

$$W(t, s) = \pi\Phi(t, s)U(s) \overset{*}{-} \pi\Phi(t, s)V(s), \ \ s \in [t_0, t],$$

$\overset{*}{-}$ is the operation of geometric subtraction by Minkowski [20, 21].

Pontryagin's condition. Set-valued mapping $W(t, s)$ has non-empty direct images for s, $t_0 \le s \le t < +\infty$.

As this takes place, the set-valued mapping $W(t, s)$ is measurable in s and closed-valued [21–23]. Therefore [10], there exists a measurable in s the selection $\gamma(t, s)$, $\gamma(t, s) \in W(t, s)$, $t \ge s \ge t_0$, which is measurable in s function at each finite t. We fix it and set

$$\xi(t) = \xi(t_0, z_0, t, \gamma(t, \cdot)) = \pi\Phi(t, t_0)\, z_0 + \int_{t_0}^{t} \gamma(t, s) ds.$$

Let us denote

$$\Theta_* = \Theta(\tau, t) = \min\{\Theta\colon \Theta \in (\tau + \delta, t), \ \int_{\tau}^{\Theta} \gamma(t, s) ds \in \\ \in \int_{\tau+\delta}^{\Theta} W(t, s) ds \overset{*}{-} \int_{\tau}^{\tau+\delta} \pi\Phi(t, s)V(s) ds \ne \varnothing\}. \quad (3)$$

Condition 1. There is a moment, t, $t > t_0$ such that function $\Theta(\tau, t)$ takes finite value at each $\tau \in [\tau_1, \tau_2]$.

For such t, given Pontryagin's condition, we consider the set-valued mapping:

$$R(t, \tau, s, v) = \{\rho \ge 0\colon [\pi\Phi(t, s)U(s) - \pi\Phi(t, s)v - \\ -\gamma(t, s)] \cap \rho[M(t) - \xi(t)] \ne \varnothing\}, \quad (4)$$

$\tau \in [\tau_1, \tau_2]$, $s \,\overline{\in}\, [\tau, \Theta_*]$, $t \ge s \ge t_0$, $v \in V(s)$.

Its support function in direction +1 is called resolving function [22, 23]:

$$\rho(t, \tau, s, v) = \sup\{\rho\colon \rho \in R(t, \tau, s, v)\}.$$

For $s \in [\tau, \Theta_*]$ we set $\rho(t, \tau, s, v) \equiv 0$. In other words, at the interval of failure of control devices $[\tau, \tau + \delta]$ and at the interval of elimination of consequences of an accident $(\tau + \delta, \Theta_*]$ the resolving function takes zero values.

Under Pontryagin's condition, the set-valued mapping $R(t, \tau, s, v)$ is well defined and has non-empty closed direct images for all admissible values of arguments.

If for some $t > t_0$ $\xi(t) \in M(t)$, then from (4) it follows that $R(t, \tau, s, v) = [0, +\infty)$ and, consequently, $\rho(t, \tau, s, v) = +\infty$ at all $v \in V(s)$, $s \in [t_0, t]$. From the theorem on inverse image and the characterization theorem [10], it follows that the set-valued mapping $R(t, \tau, s, v)$ is jointly $L \times B$ — measurable in (s, v), $v \in V(s)$, $s \in [t_0, t]$. What is more, the function $\rho(t, \tau, s, v)$ is $L \times B$-measurable in the same parameters, under the theorems on support function [10], and therefore is superposition measurable [21].

Let us denote by Ω_v the set of all measurable selections of mapping $V(t)$ and consider the set

$$T(z_0) = \left\{ t > t_0\colon \inf_{\tau \in [\tau_1, \tau_2]} \inf_{v(\cdot) \in \Omega_v} \int_{t_0}^{t} \rho(t, \tau, s, v(s)) d\tau \ge 1 \right\}. \quad (5)$$

If for some t, $t > t_0$, $\rho(t, \tau, s, v) \equiv +\infty$, the value of integral in (5) is set equal $+\infty$. Then the corresponding inequality is readily satisfied and $t \in T(z_0)$. In the case when the inequality in (5) does not hold at all $t > t_0$, we set $T(z_0) = \varnothing$.

MAIN RESULTS

The following statement is true.

Theorem 1. Let for a given conflict-controlled process (1), (2) Pontryagin's condition be fulfilled and the mapping $M(t)$, $t \geq t_0$, be convex-valued. Assume that for a given initial state (t_0, z_0) there is a measurable in s the selection $\gamma(t, s)$, $t_0 \leq s \leq t < +\infty$, such that $T(z_0) \neq \varnothing$, and the moment T, $T \in T(z_0)$, satisfies Condition 1.

Then the trajectory of the process (1) can be brought to the terminal set (2) at the moment T with the help of appropriate quasi-strategy, despite the failure of control devices at any moment of interval $[\tau_1, \tau_2]$ and the time δ to eliminate the breakdown.

Proof. Let $v(t)$, $v(t) \in V(t)$, $t \geq t_0$, be an arbitrary admissible control of the second player and τ be a moment of failure of control devices, which is a priori unknown to the first player. Let us define the control of the first player on the intervals $[t_0, \tau]$, $(\tau + \delta, \Theta_*]$, and $(\Theta_*, T]$, where $\Theta_* = \Theta(\tau, T)$. Since τ is the moment of the accident, and the time δ is allowed for its elimination, there is no control of the first player in the interval $[\tau, \tau + \delta]$, and only the second player acts on the system (1) in his interests. This negative influence is eliminated on the interval $(\tau + \delta, \Theta_*]$ using the inclusion in (3).

Thus, the accumulation of the resolving function occurs only on the intervals $[t_0, \tau)$ and $(\Theta_*, T]$. To implement this process, consider the test function

$$f(t, v(\cdot)) = 1 - \int_{t_0}^{t} \rho(T, \tau, s, v(s))ds.$$

It is absolutely continuous in t, does not increase, what is more, $f(t_0, v(\cdot)) = 1$. From the definition of the moment T and the well-known theorem of analysis it follows that there is such a moment t_*, $t_* \leq T$, that $f(t_*, v(\cdot)) = 0$. The moment of switching t_* depends on the control prehistory $v_{t_*}(\cdot)$ and the moment of failure of control devices τ. There are two possible options: before the moment of switching t_*: there was no failure of control devices, i.e., $t_* \in [t_0, \tau)$, and the moment τ has already come in the past, and it is known, i.e. $t_* \in (\Theta_*, T]$. Note that the moment t_* depends on the control prehistory of the second player $v_{t_*}(\cdot)$.

In each of these cases, the point t_* separates the set $[t_0, \tau) \cup (\Theta_*, T]$ into the passive and active parts: on the active section the resolving function is accumulated, and on the passive one it is equal to zero.

Let $t_* \in [t_0, \tau)$, and $\xi(t_0, z_0, T, \gamma(T, \cdot) \mathrel{\overline{\in}} M(T)$. Consider the set-valued mapping

$$U(s, v) = \{u \in U(s): \pi\Phi(T, s)(u - v) - \gamma(T, s) \in \rho(T, \tau, s, v)[M(T) - \xi(T)]\}. \quad (6)$$

It is a $L \times B$-measurable and closed-valued mapping [22, 23]. Therefore, by the theorem on measurable choice [10], it has $L \times B$-measurable selection $u(s, v)$, which is a super-position measurable function [21].

Let us put control of the first player on the active section equal

$$u(s) = u(s, v(s)), \ s \in [t_0, t_*), \quad (7)$$

For $s \in [t_*, \tau) \cup (\Theta_*, T]$, we set in the relationship (6) $\rho(T, \tau, s, v) = 0$ and choose $u_0(s, v)$ in the form of $L \times B$-measurable selection of the obtained set-valued mapping

$$U_0(s,v)=\{u\in U(s):\pi\Phi(T,s)(u-v)-\gamma(T,s)=0\}. \tag{8}$$

Then the control of the first player is

$$u_0(s)=u_0(s,v(s)),\ s\in[t_*,\tau)\cup(\Theta_*,T]. \tag{9}$$

Let $t_*\in(\Theta_*,T]$ and $\xi(T)\,\overline{\in}\,M(T)$. Then the moment τ is known and on the active section $[t_0,\tau)\cup(\Theta_*,t_*)$ we choose control of the first player analogously to the case $t_*\in[t_0,\tau)$, that is with an account of the relationships (6), (7).

In the passive section $[t_*,T]$, when $\xi(T)\,\overline{\in}\,M(T)$, control of the first player is determined by expressions (8), (9).

If $\xi(T)\in M(T)$, then the control of the first player on the entire set $[t_0,\tau)\cup\cup(\Theta_*,T]$ is given in the form (9), $u_0(s,v)$ is the superposition measurable selection of the mapping $U_0(s,v)$.

Next, we indicate the control of the first player on the section $(\tau+\delta,\Theta_*]$. From the inclusion in (3), for,$t=T$ by definition of the resolving function, we obtain

$$\int_{\tau}^{\Theta_*}\gamma(T,s)ds+\int_{\tau}^{\tau+\delta}\pi\Phi(T,s)V(s)ds\subset\int_{\tau+\delta}^{\Theta_*}W(T,s)ds. \tag{10}$$

Since the control of the first player on the interval $[\tau,\tau+\delta]$ becomes known at the moment $\tau+\delta$, then becomes also known the value

$$\int_{\tau}^{\tau+\delta}[\pi\Phi(T,s)v(s)+\gamma(T,s)]ds=\omega.$$

Then from the inclusion (10) we have

$$\omega\in\int_{\tau+\delta}^{\Theta_*}[W(T,s)-\gamma(T,s)]ds.$$

By definition of Aumann's integral [24], there is a measurable selection $\omega(s)$ of the set-valued mapping $W(T,s)-\gamma(T,s)$, such that

$$\omega=\int_{\tau+\delta}^{\Theta_*}\omega(s)ds.$$

We introduce the set-valued mapping

$$U_\delta(s,v)=\{u\in U(s):\pi\Phi(T,s)(u-v)-\gamma(T,s)-\omega(s)=0,\ v\in V(s)\}, \tag{11}$$

$$s\in[\tau+\delta,\Theta_*].$$

It is $L\times B$-measurable and closed-valued and, therefore, there exists $L\times B$ measurable selection $u_\delta(s,v)$, which determines the measurable control of the first player:

$$u_\delta(s)=u_\delta(s,v(s)),\ s\in[\tau+\delta,\ \Theta_*], \tag{12}$$

under the superposition measurability of the function $u_\delta(s,v)$.

Let us show that the control laws (6)–(9), (11), (12) of the first player ensure the bringing of the trajectory of (1) to the terminal set at the moment T for any counteractions of the adversary.

Using the Cauchy formula to represent the solution to system (1) and dividing an integral part into special time sections, depending on the switching time, we obtain for the case $t_* \in [t_0, \tau)$ $\xi(T) \overline{\in} M(T)$,

$$\pi z(T) = \pi\Phi(T, t_0)\, z_0 + \int_{t_0}^{t_*} \pi\Phi(T, s)(u(s) - v(s))ds + \int_{t_*}^{\tau} \pi\Phi(T, s)(u(s) - v(s))ds -$$

$$- \int_{\tau}^{\tau+\delta} \pi\Phi(T, s)v(s)ds + \int_{\tau+\delta}^{\Theta_*} \pi\Phi(T, s)(u(s) - v(s))ds + \int_{\Theta_*}^{T} \pi\Phi(T, s)(u(s) - v(s))ds,$$

and for the case $t_* \in (\Theta_*, T]$, $\xi(T) \overline{\in} M(T)$,

$$\pi z(T) = \pi\Phi(T, t_0)\, z_0 + \int_{t_0}^{\tau} \pi\Phi(T, s)(u(s) - v(s))ds - \int_{\tau}^{\tau+\delta} \pi\Phi(T, s)v(s)ds +$$

$$+ \int_{\tau+\delta}^{t_*} \pi\Phi(T, s)(u(s) - v(s))ds + \int_{t_*}^{T} \pi\Phi(T, s)(u(s) - v(s))ds.$$

Taking into account the previously determined control laws on the active and the passive sections, and on the section of eliminating the consequences of the accident, we obtain an inclusion for the projection of the trajectory onto the subspace L at the moment T:

$$\pi z(T) \in \xi(T)[1 - \int_{t_0}^{T} \rho(T, \tau, s, v(s))ds] + \int_{t_0}^{T} \rho(T, \tau, s, v(s)) M(T)ds. \qquad (13)$$

Since $\int_{t_0}^{T} \rho(T, \tau, s, v(s))ds = 1$, by the construction of the resolving function for any measurable selections $v(s), v(s) \in V(s)$, and the set $M(T)$ is convex, it follows from (13) that $\pi z(T) \in M(T)$ and, hence, $z(T) \in M^*(T)$.

If, otherwise, $\xi(T) \in M(T)$, then

$$\pi\Phi(T, t_0)\, z_0 \in M(T) - \int_{t_0}^{T} \gamma(T, s)ds \subset M(T) - \int_{t_0}^{T} W(T, s)ds.$$

Let us choose control of the first player on the entire interval $[t_0, T]$, using a superposition measurable selection $u_0(s, v)$ of the set-valued mapping $U_0(s, v)$, in the form of counter-control $u_0(s) = u_0(s, v(s))$. Substituting it into the Cauchy formula, we obtain $\pi z(T) \in M(T)$.

Further, we will reason in a slightly different way. Assuming Pontryagin's condition is satisfied, as before, $\gamma(t, s)$ is the selection of the set-valued mapping $W(t, s)$, $t_0 \le s \le t < +\infty$, we consider the set-valued mapping

$$M(t, \tau) = \begin{cases} M(t) \overset{*}{-} \int_{\tau}^{\tau+\delta} [\pi\Phi(t, s)V(s) - \gamma(t, s)]ds, & t > \tau + \delta > t_0, \\ M(t), & t_0 \le t \le \tau + \delta. \end{cases} \qquad (14)$$

Condition 2. The set-valued mapping $M(t, \tau)$ has non-empty direct images for all $t, \tau, t_0 < \tau, \quad \tau + \delta < t < +\infty$.

Let us set

$$R^*(t, \tau, s, v) =$$

$$= \{\rho \ge 0: [\pi\Phi(t, s)U(s) - \pi\Phi(t, s)v - \gamma(t, s)] \cap \rho[M(t, \tau) - \xi(t)] \ne \varnothing, \qquad (15)$$

$$t \ge s \ge t_0, \; s \overline{\in} [\tau, \tau + \delta], \; v \in V(s).$$

We define the resolving function in the following way:

$$\rho^*(t, \tau, s, v) = \begin{cases} \sup(\rho: \rho \in R^*(t, \tau, s, v), & s \overline{\in} [\tau, \tau+\delta], \\ 0, & s \in [\tau, \tau+\delta]. \end{cases}$$

Denote

$$T^*(z_0) = \left\{ t \ge t_0 : \inf_{\tau \in [\tau_1, \tau_2]} \inf_{v(\cdot) \in \Omega_v} \int_{t_0}^{t} \rho^*(t, \tau, s, v(s)) d\tau \ge 1 \right\}. \quad (16)$$

Theorem 2. Let Pontryagin's condition be satisfied for the conflict-controlled process (1), (2) and let the set-valued mapping $M(t)$, $t >= t_0$, be convex-valued.

Then, if for the initial state (t_0, z_0) there is a measurable in s the selection $\gamma(t, s)$ of the mapping $W(t, s)$, $t_0 \le s \le t < +\infty$, such that $T^*(z_0) \ne \varnothing$, $T^* \in T^*(z_0)$ and Condition 2 is satisfied, then the problem of approaching the trajectory of (1) with the terminal set (2) is solvable at the moment T^* in the class of quasi-strategies.

Proof. Let $v(t)$, $t \in [t_0, T^*]$, be an admissible control of the second player and τ – the moment of the failure of control devices of the first player.

Consider the test function

$$f^*(t, v(\cdot)) = 1 - \int_{t_0}^{t} \rho^*(T^*, \tau, s, v(s)) ds.$$

It is continuous, does not increase and $f^*(t_0, v(\cdot)) = 1$. From the inclusion $T^* \in T^*(z_0)$ and the inequality in (16), it follows that there exists a moment, t^*, $t^* \le T^*$ such that $f^*(t^*, v(\cdot)) = 0$. Evidently, $t^* \in (t_0, \tau) \cup (\tau + \delta, T^*] = \Delta$.

Consider the set-valued mapping

$$U^*(s, v) =$$

$$= u \in U(s): \pi\Phi(T^*, s)(u - v) - \gamma(T^*, s) \in \rho^*(s, v)[M(T^*, \tau) - \xi(T^*)], \quad (17)$$

where

$$\rho^*(s, v) = \begin{cases} \rho^*(T^*, \tau, s, v), & s \le t^*, \quad s \in \Delta, \quad \xi(T^*) \overline{\in} M(T^*, \tau), \\ 0, & s > t^*, \quad s \in \Delta, \quad \xi(T^*) \overline{\in} M(T^*, \tau), \\ 0, & \qquad\qquad\qquad \xi(T^*) \in M(T^*, \tau). \end{cases}$$

The mapping $U^*(s, v)$ is jointly $L \times B$ measurable in (s, v) and closed-valued. With the help of its superposition measurable selection, we determine the control of the first player

$$u^*(s) = u^*(s, v(s)), \quad s \in \Delta. \quad (18)$$

Then, substituting the control law of the first player, (17), (18) in the Cauchy formula and taking account of the assumptions of the theorem and formulas (14)–(16), we obtain the inclusion $\pi z(T^*) \in M(T^*)$.

ILLUSTRATIVE EXAMPLE

A simple example was deliberately chosen, when all the quantities appearing in the reasoning can be found explicitly, without resorting to approximate calculations.

Let a conflict-controlled process be given

$$\dot z = \alpha_\tau(t) u - v, \; z \in E^n, \; \|u\| \le a > 1, \; \|v\| \le 1, \; t_0 = 0, \; z(0) = z_0,$$

the terminal set is the ε-vicinity of the origin

$$M^*(t) = z : \| z \| \le \varepsilon,$$

which does not change in time.

Evidently, $M_0 = 0$, $M(t) = \varepsilon S$, where S is the unit ball in Euclidian space E^n centered at the origin. Then $L = M_0^{\perp} = E^n$ is the orthogonal complement to M_0 and the orthoprojector π is the operator of an identity transformation, which is given by the unit matrix. Since, in this example, $C(t)$ is the zero matrix, then $\Phi(t, t_0)$ is the unit matrix of order n.

Following the scheme of the method and taking into account the properties of the geometric subtraction, we obtain

$$W(t, s, v) = aS - v, \; W(t, s) = (a-1)S.$$

Since $a > 1$, then Pontryagin's condition is fulfilled, therewith $0 \in W(t, s)$. Setting the selection $\gamma(t, s)$ equal to zero, we have $\xi(t) = z_0$.

Next,

$$\Theta_* = \min\{s: s > \tau + \delta, \; \int_{\tau}^{\tau+\delta} S ds \subset \int_{\tau+\delta}^{s} (a-1) S ds\} =$$

$$= \min\{s: s > \tau + \delta, \;\; \delta S \subset (s - \tau - \delta)(a-1)S\} =$$

$$= \min\{s: s > \tau + \delta, \; (s - \tau - \delta)(a-1) - \delta \ge 0\} = \tau + \delta + \frac{\delta}{a-1}.$$

Here we used the fact that convex compact sets in the Aumann's integral are integrated as the constants.

Note that the time allotted for the elimination of the breakdown is equal $\dfrac{\delta}{a-1}$.

The resolving function is

$$\rho(z_0, v) = \rho(t, \tau, \; s, v) = \max\{\rho \ge 0: [aS - v] \cap \rho[\varepsilon S - z_0] \ne \varnothing\}.$$

The intersection of sets is non-empty if their difference contains zero. Taking into account also the central symmetry of the ball centered at zero, we obtain:

$$\rho(z_0, v) = \max\{\rho \ge 0: \rho z_0 - v \in (a + \rho\varepsilon)S\}.$$

The extreme element lies, obviously, on the boundary of the ball, therefore, it is the largest positive root of the quadratic equation for ρ

$$\| v - \rho z_0 \| = a + \rho\varepsilon$$

and has the form

$$\rho(z_0, v) = \frac{(z_0, v) + a\varepsilon + \sqrt{[(z_0, v) + a\varepsilon]^2 + (\| z_0 \|^2 - \varepsilon^2)(a^2 - \| v \|^2)}}{\| z_0 \|^2 - \varepsilon^2}.$$

Using, for example, the Lagrange multiplier method [25], it is easy to show that

$$\min_{\| v \| \le 1} \rho(z_0, v) = \frac{a-1}{\| z_0 \| - \varepsilon},$$

Moreover, the minimum is attained on the element $v = -\dfrac{z_0}{\| z_0 \|}$.

Since the resolving function $\rho(t, \tau, s, v)$ in this example does not depend on t, s, and τ, and in the inequality of formula (5) the operations of taking the minimum in $v(\cdot)$ and integration can be reversed, the duration of the active period of the approach is equal

$$\frac{\| z_0 \| - \varepsilon}{a-1}.$$

Together with the time for repair of accident δ and the time for elimination of the consequences $\frac{\delta}{a-1}$, the total approach time is

$$T_0 = \min T(z_0) = \frac{\| z_0 \| + \delta - \varepsilon}{a-1} + \delta.$$

Let us define the controls of the first player on each of the special sections of the segment $[0, T_0]$. On the interval $[\tau, \tau + \delta]$ it is absent since the control devices have failed.

Let us focus on the section of time $\left(\tau + \delta, \tau + \delta + \frac{\delta}{a-1}\right)$, where consequences of the accident are eliminated.

Beginning from the moment $\tau + \delta$ the first player knows the value

$$\int_{\tau}^{\tau+\delta} v(s) ds = \omega.$$

From the definition of the moment Θ_* and inclusion (10), it follows that

$$\omega \in \int_{\tau+\delta}^{\tau+\delta+\frac{\delta}{a-1}} (a-1) S ds.$$

Calculating the Aumann integral of a set-valued mapping with constant direct images $(a-1)S$, we obtain

$$\omega \in \delta S.$$

Let us choose the selection $\omega(s)$ of the set-valued mapping $(a-1)S$ in the form

$$\omega(s) = (a-1) \omega/\delta, \ \ \omega \in \delta S.$$

Then, under the formula (11), control of the first player on the interval $\left(\tau + \delta, \tau + \delta + \frac{\delta}{a-1}\right)$ takes the form

$$u(s) = v(s) + (a-1) \omega/\delta.$$

Consider the interval $[t_0, \tau) \cup (\Theta_*, T_0]$. We choose control depending on where the zero of the test function

$$f(t, v(\cdot)) = 1 - \int_{t_0}^{t} \rho(z_0, v(s)) ds$$

is located. By definition of the resolving function $\rho(z_0, v)$,

$$[aS - v] \cap \rho(z_0, v)[\varepsilon S - z_0] \neq \varnothing. \qquad (19)$$

Moreover, the intersection of the two balls occurs at a single point.

Let us determine the point

$$m(z_0, v) = m \in \varepsilon S: \rho(z_0, v)(m - z_0) \in aS - v =$$

$$= m \in \varepsilon S: \rho(z_0, v) m + v - \rho(z_0, v)\, z_0 \in aS = -\varepsilon \frac{v - \rho(z_0, v)\, z_0}{\| v - \rho(z_0, v)\, z_0 \|},$$

lying in the boundary of the set εS. Vector $v-\rho(z_0, v)\, z_0$ connects centers v and $\rho(z_0, v)\, z_0$ of the touching balls $aS - v$ and $\rho(z_0, v)[\varepsilon S - z_0]$.

Then control of the first player on the active section $[t_0, \tau) \cup (\Theta_*, T_0] \cap [t_0, t_*)$, with an account of formula (19), has the form

$$u(s) = v(s) - \rho(z_0, v(s))(m(z_0, v(s)) - z_0),$$

where $m(z_0, v)$ is the current aiming point. On the passive section $\{[t_0, \tau) \cup (\Theta_*, T_0]\} \cap [t_*, T_0]$, $u(s) = v(s)$, since the resolving function $\rho(z_0, v)$ is set equal to zero.

It should be noted that since in this case a simple motion is considered, then at the moment t_* there is already hitting, and on the passive section, the trajectory is only kept on the set εS.

Let us illustrate the second approach to solving the problem at hand. We have

$$M(t, \tau) = \begin{cases} \varepsilon S \overset{*}{-} \delta S, & t > \tau + \delta > 0, \\ \varepsilon S, & 0 \le t \le \tau + \delta, \end{cases} = \begin{cases} (\varepsilon - \delta) S, & t > \tau + \delta > 0, \\ \varepsilon S, & 0 \le t \le \tau + \delta. \end{cases}$$

Condition 2 is fulfilled if $\varepsilon \ge \delta$.

The resolving function is

$$\rho^*(z_0, v) = \max\{\rho^* \ge 0 : [aS - v] \cap \rho^*[(\varepsilon - \delta)S - z_0] \ne \varnothing\}, \ s > \tau + \delta,$$

$$\rho^*(z_0, v) = \rho^*(t, \tau, s, v) = 0, \quad s \in [\tau, \tau + \delta],$$

$$\rho^*(z_0, v) = \rho^*(t, \tau, s, v) = \max\{\rho^* \ge 0 : [aS - v] \cap \rho^*[\varepsilon S - z_0] \ne \varnothing\}, \ s < \tau.$$

It makes sense to consider only the case when $t > \tau + \delta$. Then

$$\rho^*(z_0, v) = \max\{\rho^* \ge 0 : [aS - v] \cap \rho^*[(\varepsilon - \delta)S - z_0] \ne \varnothing\}$$

is the greatest positive root of the equation for ρ^*

$$\| v - \rho^* z_0 \| = a + \rho^*(\varepsilon - \delta).$$

Then

$$\rho^*(z_0, v) = \frac{(z_0, v) + a(\varepsilon - \delta) + \sqrt{[(z_0, v) + a(\varepsilon - \delta)]^2 + (\| z_0 \|^2 - (\varepsilon - \delta)^2 (a^2 - \| v \|^2)}}{\| z_0 \|^2 - (\varepsilon - \delta)^2}$$

whence

$$\min_{\|v\| \le 1} \rho^*(z_0, v) = \frac{a - 1}{\| z_0 \| - \varepsilon + \delta}.$$

The total time along with the time, allotted for the elimination of the breakdown is

$$T_0^* = \min T^*(z_0) = \frac{\| z_0 \| - \varepsilon + \delta}{a - 1} + \delta.$$

Let us define the control of the first player. On the set Δ we find the switching point t^* from the active section to the passivec one as zero of the test function

$$f^*(t, v(\cdot)) = 1 - \int_{t_0}^{t} \rho^*(z_0, v(s))\, ds.$$

By definition of the resolving function $\rho^*(z_0, v)$,

$$[aS - v] \cap \rho^*[(\varepsilon - \delta)S - z_0] \ne \varnothing.$$

Considering that the intersection of two balls occurs at a single point, the point of their touch, we get

$$m^*(z_0, \upsilon) = \{m \in (\varepsilon - \delta)S : \rho^*(z_0, \upsilon)(m - z_0) \in aS - \upsilon\} =$$

$$= -(\varepsilon - \delta)\frac{\upsilon - \rho^*(z_0, \upsilon)\, z_0}{\|\upsilon - \rho^*(z_0, \upsilon)\, z_0\|}.$$

The point $m^*(z_0, \upsilon)$ falls on the boundary of the set $(\varepsilon - \delta)S$, and the vector $\upsilon - \rho^*(z_0, \upsilon)\, z_0$ connects the centers υ and $\rho^*(z_0, \upsilon)\, z_0$ of touching balls $\rho^*(z_0, \upsilon)[(\varepsilon - \delta)S - z_0]$ and $aS - \upsilon$.

Eventually, the control of the first player has the form

$$u^*(s) = \upsilon(s) - \rho^*(s, \upsilon(s))(m^*(z_0, \upsilon(s)) - z_0), \;\; s \in \Delta, \;\; s \le t^*,$$

$$u^*(s) = \upsilon(s), \quad s > t^*.$$

CONCLUSIONS

The non-stationary game problem of control of moving objects with disturbances in dynamics, the cause of which is the failure of control devices or breakdown, is considered. The case is investigated when the moment of breakdown is unknown, and the time of its elimination is given. Based on the method of resolving functions, using the technique of set-valued mapping, sufficient conditions are obtained for bringing the trajectory of the conflict-controlled process to a given set in a certain guaranteed time. For the implementation of this procedure and choice of controls, relations are provided that allow the choice of $L \times B$-measurable selections of corresponding set-valued mappings Their superposition measurability makes it possible to construct the controls of the first player before and after switching in the form of counter-controls and, in general, in the form of quasi-strategies, since the moment of switching depends on control prehistory of the second player.

The results obtained are illustrated by a model example with simple motions with spherical control domains and a terminal set. The resolving functions are found explicitly, as well as, with their help, the guaranteed time for the game termination and the controls of the first player on the intervals of eliminating the breakdown and accumulating the resolving functions.

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ПРО ВІДМОВУ КЕРУВАЛЬНИХ ПРИСТРОЇВ РУХОМИХ ОБ'ЄКТІВ У КОНФЛІКТНІЙ СИТУАЦІЇ

Анотація. Розглянуто нестаціонарну ігрову задачу керування рухомими об'єктами з порушеннями в динаміці, причиною яких є відмова або поломка керувальних пристроїв. Досліджено ситуацію, коли момент поломки невідомий, а час на її ліквідацію є заданим. Встановлено достатні умови приведення траєкторії конфліктно-керованого процесу на задану множину за скінченний час. Отримані результати проілюстровано модельним прикладом з простим рухом.

Ключові слова: конфліктно-керований процес, багатозначне відображення, розв'язувальна функція, відмова керувальних пристроїв, стробоскопічна стратегія, умова Понтрягіна, інтеграл Ауманна.

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