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PRINCIPLE OF TIME STRETCHING IN GAME DYNAMIC PROBLEMS

Introduction. *There exists a wide range of mechanical, economical and biological processes evolving in condition of conflict and uncertainty, which can be described by various kind dynamic systems, depending on the process nature. This paper deals with the dynamic games of pursuit, described by a system of general form, encompassing a wide range of the functional-differential systems. The deciding factor in study of dynamic games is availability of information on current state of the process. In real systems information, as a rule, arrives with delay in time. Also, there are a number of problems for which Pontryagin's condition, reflecting an advantage of the pursuer over the evader in control resources, does not hold. Establishment of close relation between its time-stretching modification and the effect of variable information delay offers much promise for solving the above mentioned problems.*

The purpose of the paper is *to deduce, sufficient conditions for termination of the games, for which Pontryagin's condition does not hold, by the use of the effect of information delay, to specify these conditions for the case of integro-differential dynamics, and to illustrate the obtained result with the model example.*

Methods. *For investigation of the dynamic game of pursuit we apply the scheme of Pontryagin's First Direct method providing bringing of the trajectory of conflict-controlled process to the cylindrical terminal set at a finite moment of time. In so doing, construction of the pursuer's control is accomplished on the basis of the Filippov-Castaing theorem on measurable choice that insures realization of the process of pursuit in the class of stroboscopic strategies by Hajek. To deduce solution of the conflict-controlled integro-differentional system in the Cauchy form, the method of successive approximation is used.*

Results. *It is shown that the dynamic game of pursuit with separated control blocks of the players and variable delay of information is equivalent to certain perfect information game. Based on this fact, the principle of time stretching is developed to study the games with complete information for which classic Pontryagin's condition, lying at the heart of all direct methods of pursuit, does not hold. The time-stretching modification of this condition, proposed in the paper, makes it feasible to obtain sufficient conditions for bringing the game trajectory to the terminal set at a finite moment of time. In so doing, the control of pursuer, providing achievement of the game goal, is constructed. These conditions are specified for the integro-diffential game of pursuit. By way of illustration, an example of integro-differential game of pursuit is analyzed in detail. The time stretching function, providing fulfillment of the modified Pontryagin's condition is found. Simple relationships between* *dynamics parameters and control resources of the players are deduced that provide feasibility of capture of the evader by the pursuer, under arbitrary initial states of the players.*

Conclusion. *Thus, in the paper an efficient tool is developed for analysis of conflict situation, for example, interception of a mobile target by controlled object in condition of conflict counteraction. The situation is analyzed, when the pursuing object lacks conventional advantage in control resources over the evading counterpart, that is, the classic Pontryagin's condition does not hold. Suggested approach makes it feasible to realize the process of pursuit with the help of appropriate Krasovskii' counter-controls.*

Keywords: *dynamic game, time-variable information delay, Pontryagin's condition, Aumann's integral, principle of time stretching, Minkowski' difference, integro-differential game.*

INTRODUCTION

In the theory of dynamic games a number of efficient methods are created to make decision in conditions of conflict and uncertainty. They originated in fundamental works of R. Isaacs [1], L.S. Pontryagin [2], N.N. Krasovskii [3], L. Berkovitz [4], A. Friedman [5], O. Hajek [6], B.N. Pshenitchny [7] and their disciples, which are based on various mathematical ideas respective to availability of information to opposing sides in the course of game [8–10]. There exists a wide range of mechanical, economical and biological processes which can be described by dynamic systems of various kind, in particular, by the ordinary differential, difference, difference-differential, integral, integral-differential, partial differential and fractional equations, as well as by impulse systems, depending on the process nature [11]. Any disturbance, counteraction or inaccuracy readily leads to game situation. The deciding factor in study of dynamic games is availability of information on current state of the process, its prehistory or various kind counter-parts discrimination, that results in the problems of pursuit-evasion by position or in the class of stroboscopic, quasi- or ε-strategies.

Sometimes, in real systems information arrives with delay in time. It is shown that the dynamic game of pursuit with variable information delay is equivalent to certain perfect-information game with the changed dynamics and terminal set. It was first proved for the linear differential games with constant delay of information, then for the case of variable information delay [12]. This effect of information delay opened up possibilities for application of classic methods to analyze the games with delay of information [12], [13].

At the heart of the First Direct method, developed for solving the linear differential pursuit games, lies Pontryagin's condition [2]. This condition reflects an advantage of the pursuer over the evader in control resources in terms of the game parameters. However, there are a number of problems for which this condition does not hold, e.g. the problems of soft meeting (simultaneous coincidence of geometric coordinates and velocities of objects), pursuit problems for oscillatory processes or different inertia systems etc [14], [15], [16].

Analysis of Pontryagin's condition performed by Nikolskij [14] greatly advanced its understanding and was a contributory factor to the modification of this condition [17], prescribing construction of the pursuer's control on the basis of evader's one in the past. Unfortunately, analysis of the model example contains slips.

Establishment of close relation of the modified condition with the passage from the original game with perfect information to an auxiliary one with delayed information [18], [19] gave impetus to the development of efficient approach (the principle of time stretching) to solving complicated games of pursuit, namely, those for which Pontryagin's condition does not hold [19], [20], [21].

This paper deals with the dynamic games of pursuit, described by a system of general form that encompasses a wide range of the functional-differential systems. The gist of this approach consists in artificial abandoning the availability of information on the current evader's control to the pursuer. In fact, the passage is made from the original game with complete information to the game with the same dynamics and the terminal set, yet with special kind information availability delay. This delay is a function of time, decreasing as the game trajectory approaches the terminal set and vanishing as it hits the target. The central idea of the time stretching principle consists in introduction of certain function, called the time stretching function, in which terms the time delay is expressed in explicit form. Then the obtained game with delayed information is analyzed on the basis of its equivalence to the perfect-information game with the changed dynamics, for which Pontryagin's condition includes the time stretching function.

The time stretching principle proved its efficiency in solving the problems of soft meeting in various cases of second-order dynamics, for which formula for the time stretching function is deduced in explicit form, in their number, in the case of different-kind dynamics of the pursuer and the evader [20–22]. Simple conditions on the games parameters insuring feasibility of the pursuit termination are deduced. The geometric-descriptive situation of finding 'tracks' of the evader is studied in detail, that provides realization of the time stretching principle by the way of the pursuer following the evader's trajectory with delay in time [20].

In the paper the time stretching principle is applied to the dynamic games, described by a system of general form, encompassing a wide scope of the functional-differential systems. The result of investigation is specified for the integral-differential games of pursuit. To this end, we derive solution to the integro-differential system in the Cauchy form. In so doing, the method of successive approximations is used to solve the Volterra integro-differential equation of second order. To support the suggested technique, an example of integro-differential pursuit game is examined in detail.

EFFECT OF INFORMATION DELAY IN THE DYNAMIC GAMES OF PURSUIT

Statement of the pursuit game usually includes a system of equations, describing the conflict-controlled process. But subsequent analysis of the game, as a rule, employs only presentation of the system solution. In the case of differential game it is the Cauchy formula. To begin with, we recall the impact of information delay in the dynamic games of pursuit, that is, the equivalence of the game with variable information delay to certain complete-information game with the changed object's dynamics and the terminal set and, on its basis, outline the time stretching principle.

Let a trajectory of the conflict-controlled process be given in the form:

$$z(t)=g(t)-\int_{t_0}^{t}\left(f_1(t,\theta,u(\theta))-f_2(t,\theta,v(\theta))\right)d\theta\,,\, t\in[t_0,+\infty). \tag{1}$$

Here $z(t)\in R^n$, where R^n is the real $n-$dimensional Euclidean space, $g:[t_0,+\infty)\to R^n$ is continuous vector-function. Controls u and v are picked by the players at each instant of time from the compacts U and V in a way their realizations in time be Lebesgue measurable functions. Functions $f_1(t,\theta,u)$ and $f_2(t,\theta,v)$, $f_1:\Delta(t_0)\times U \to R^n$, $f_2:\Delta(t_0)\times V \to R^n$, where $\Delta(t_0)=\{(t,\theta): 0\le t_0<\theta\le t\le +\infty\}$ are assumed to be Lebesgue measurable both in t and θ and continuous in u and v, respectively; $U\in K(R^n)$, $V\in K(R^n)$, where by $K(R^n)$ is denoted the set of all non-empty compacts from R^n.

Besides, a terminal set M_* having a cylindrical form is given:

$$M_* = M_0 + M . \tag{2}$$

Here M_0 is a linear subspace of R^n and M — a convex compact from the orthogonal complement to M_0 in R^n, i.e. $M \in coK(L)$. By $coK(L)$ is meant the set of all convex sets from $K(R^n)$.

Let us denote by Ω_U and Ω_V the sets of all measurable functions taking their values in the compacts U and V, respectively. In the sequel, they are referred to as the sets of admissible controls of the pursuer and the evader, respectively.

We analyze the game, standing on the pursuer side (u). The goal of the pursuer is at a finite moment of time to bring a trajectory of the system (1) to the terminal set M_*, under arbitrary admissible control of the evader (v). By the moment of the game termination is meant the first moment of time t when $z(t)\in M_*$. Such dynamic game is called the game of pursuit.

Let π be the operator of orthogonal projection from R^n onto L, $\pi: R^n \to L$. Then bringing the system trajectory to the terminal set is equivalent to the inclusion $\pi z(t)\in M$. It is supposed that $g(t_0)\notin M_*$ and the players know the parameters of conflict-controlled process (1), (2), namely vector- functions $g(t)$, functions $f_1(t,\theta,u)$ and $f_2(t,\theta,v)$, the control domains U, V, and the terminal set M^*.

Let us suppose that current information on the game state become available to the pursuer with the delay in time $\tau(t)$. The function $\tau: [t_0+\tau_0,+\infty)\to R$, $\tau(t_0+\tau_0)=\tau_0$ is assumed to be piecewise-continuous, besides, it can have at most countable number of discontinuities and all discontinuities are of the first order, and is absolutely continuous on the intervals of its continuity. What is more, $\dot\tau(t)<1$ at the points at which the derivative $\dot\tau(t)$ exists. The last condition provides for access of fresh information in the course of the game.

The game starts at the moment t_0 but information on the evader control becomes available to the pursuer only beginning from the moment $t_0+\tau_0$, $\tau_0>0$. In the course of the game, i.e. at each current instant of time t, $t\ge t_0+\tau_0$, the pursuer has access to information on the evader control at the moment $t-\tau(t)$. Denote by $u^t(\cdot)$ realization of the pursuer control on the half-interval $[t-\tau(t),t)$,

$$u^t(s) = \{u(s) : s \in [t - \tau(t), t)\}.$$

We name the pair $(g(t), u^t(\cdot))$ by the position of game at the moment t. Suppose that on the initial half-interval $[t_0, t_0 + \tau_0)$ the pursuer applies some admissible control $u^{t_0+\tau_0}(\cdot)$,

$$u^{t_0+\tau_0}(\cdot) = \{u(s) : s \in [t_0, t_0 + \tau_0)\},\ u^{t_0+\tau_0}(\cdot) \in \Omega_U.$$

The pair $(g(t_0), u^{t_0+\tau_0}(\cdot))$ is referred to as the initial position of the delayed-information game.

Definition1. Let X and Y be non-empty sets from R^n. The geometric difference of sets is defined by the formula

$$X \underline{*} Y = \{z : z + Y \subset X\} = \bigcap_{y \in Y} (X - y).$$

In what follows, the notion of Aumann integral of set-valued mapping is used [23].

Definition 2. Let $F(t)$ be a measurable mapping, $F : [t_0, T] \to P(R^n)$, where $P(R^n)$ is a set of all closed subsets of the space R^n. The union of integrals, taken over all its measurable selections $f(t)$, $f(t) \in F(t)$, namely,

$$\bigcup_{f(\cdot) \in F(\cdot)} \int_{t_0}^{T} f(t)dt$$

is called the Aumann integral of set-valued mapping $F(t)$ and usually denoted by $\int_{t_0}^{T} F(t)dt$.

Let us take into consideration the following set-valued mappings

$$V(\tau(t)) = \left\{ x : x = \int_0^{\tau(t)} f_2(t, t - \theta, v(\theta))d\theta,\ v(\cdot) \in \Omega_V \right\},$$

$$M(\tau(t)) = M \underline{*} V(\tau(t))..$$

Condition1. The set-valued mapping $M(\tau(t))$ has non-empty images for all $t \geq t_0 + \tau_0$.

Let us take a look at the pursuit game with complete information, starting at the moment $t_0 + \tau_0$, which current state is defined by the formula

$$\tilde{z}(t) = \tilde{g}(t) - \int_{t_0+\tau_0}^{t} f_1(t, \theta, u(\theta))d\theta + \int_{t_0+\tau_0}^{t} (1 - \dot{\tau}(\theta)) f_2(t, \theta - \tau(\theta), v(\theta - \tau(\theta)))d\theta, \qquad (3)$$

$$\tilde{z}(t_0+\tau_0)=\tilde{g}(t_0+\tau_0),\quad \tilde{g}(t)=g(t)-\int_{t_0}^{t_0+\tau_0} f_1(t,\theta,u(\theta))d\theta$$

and the terminal set $M_*(t)$ having the form of set-valued mapping,

$$M_*(t)=M_0+M(\tau(t)). \tag{4}$$

We observe that, by virtue of Condition 1, the terminal set $M_*(t)$ has the solid set component $M(\tau(t))$.

Theorem1. Let in the dynamic game (1), (2) with variable information delay $\tau(t)$ Condition 1 hold. Then this game can be terminated at the moment T, $T \geq t_0+\tau_0$, starting from the initial position $\left(g(t_0), u^{t_0+\tau_0}(\cdot)\right)$, if and only if the game (3), (4) with complete information can be terminated at the same time T.

PRINCIPLE OF TIME STRETCHING IN DYNAMIC GAMES OF PURSUIT

Now we outline an approach to solving games of pursuit in the frames of the First Direct method [2] and derive sufficient conditions insuring the game termination at some finite time, which, generally speaking, is not optimal. In such case they say about guaranteed result. It is achieved by using counter-controls by Krasovskii [3], provided by stroboscopic strategies by O. Hajek [6]. We name this approach by the time stretching principle.

The First Direct method, created to study the linear differential games of pursuit is based on the Pontryagin's condition that reflects an advantage of the pursuer over the evader expressed in terms of the game parameters.

The linear differential game is described by the system of linear differential equations

$$\dot{z}(t)=Az(t)-u(t)+v(t),$$

$z \in R^n$, $z(0)=z_0$, A is a quadratic matrix of order n. It presents a particular case of the conflict-controlled process (1) with

$$g(t)=e^{tA}z_0,\ f_1(t,\theta,u)=e^{(t-\theta)A}u,\ f_2(t,\theta,v)=e^{(t-\theta)A}v$$

(by e^{tA} is denoted exponent of the matrix tA).

Condition 2 (Pontryagin's condition). The set-valued mapping

$$W(t)=\pi e^{tA}U \overset{*}{-} \pi e^{tA}V \tag{5}$$

has non-empty images for all $t \in [0,+\infty)$.

Note, that in the case of linear differential game the state variable $\tilde{z}(t)$ of the equivalent game with complete information, evolving on the half-axis $[\tau_0,+\infty)$, satisfies the differential equation

$$\dot{\tilde{z}}(t)=A\tilde{z}(t)-u(t)+(1-\dot{\tau}(t))e^{\tau(t)A}v(t-\tau(t)),$$

$$\tilde{z}(\tau_0) = e^{\tau_0 A} z_0 - \int_0^{\tau_0} e^{(\tau_0 - \theta)A} u(\theta) d\theta$$

and the terminal set of the equivalent game appears as the set $M_*(t)$ (4) with

$$V(\tau(t)) = \left\{ x : x = \int_0^{\tau(t)} e^{\theta A} v(\theta) d\theta,\ v(\cdot) \in \Omega_V \right\}.$$

The set-valued mapping $W(t)$ (5) is applied in the First Direct Method as follows. It is shown that if at some time t_1 the initial state of the game z_0 satisfies the inclusion

$$\pi e^{t_1 A} z_0 \in \int_0^{t_1} W(\theta) d\theta,$$

then the game of pursuit can be terminated at the moment t_1 under arbitrary admissible controls of the evader.

Below we give generalization of Pontryagin's condition to the case of dynamics of general form (1).

Condition 3. $W(t,\theta) = \pi f_1(t_0 + t, \theta, U) \underline{*} \pi f_2(t_0 + t, \theta, V) \neq \varnothing$, $t_0 \le \theta \le t < +\infty$.

In the case Condition 3 does not hold we propose its modification constructed with the help of the function of time stretching.

Definition 3. By the function of time stretching is named a non-negative, monotonically increasing function of time $I(t)$, $t \in [0, +\infty)$, $I(0) = 0$, $I(t) > t$, $t > 0$, which can have at most countable number of discontinuities and all discontinuities are of the first order, absolutely continuous on the intervals of its continuity, and such that $\sup\limits_{t \in [0,+\infty) \setminus \Delta} \dot{I}(t) < +\infty$, where Δ is the set of $I(t)$ discontinuity and non-differentiability points.

Condition 4. There exists time-stretching function $I(t)$, such that the set-valued mapping

$$W_1(t,\theta) = \pi f_1(t, t_0 + I(t) - \theta, U) \underline{*} \dot{I}(\theta) \pi f_2(t, t_0 + I(t) - I(\theta), V) \neq \varnothing,\ t_0 \le \theta \le t < +\infty.$$

Theorem 2. Let in the perfect-information game (1), (2) Condition 4 hold and let for the given initial state $g(t_0)$ there exist a finite moment of time t_1:

$$t_1 = \left\{ \min t : t \ge 0, \pi \left(g(t_0 + I(t)) - \int_{t_0}^{t_0 + I(t) - t} f_1(t_0 + I(t), \theta, U) d\theta \right) \right.$$
$$\left. \cap \left(M + \int_0^t W_1(t,\theta) d\theta \right) \neq \varnothing \right\}. \tag{6}$$

Then a trajectory of the process (1) can be brought by the pursuer to the terminal set M_* (2) at the moment of time $t_0 + I(t_1)$, under any admissible controls of the evader.

Proof. Let us set $\tau_0 = I(t_1) - t_1$ and divide the interval of time $[t_0, t_0 + I(t_1)]$ into two parts – the initial half-interval $[t_0, t_0 + \tau_0)$ and the closed interval $[t_0 + \tau_0, t_0 + I(t_1)]$. In view of the assumptions concerning non-emptiness of the intersection in definition of time t_1 (6) and non-emptiness of the images of the set-valued mapping $W_1(t,\theta)$ (Condition 4) there exist an admissible control $u^{t_0+\tau_0}(\theta)$, $\theta \in [t_0, t_0 + \tau_0)$, point m, $m \in M$, and measurable selection $w_1(t_1,\theta)$, $w_1(t_1,\theta) \in W_1(t_1,\theta)$, $\theta \in [0, t_1)$, such that the following equation is fulfilled:

$$\pi\left(g(t_0 + I(t_1)) - \int_{t_0}^{t_0+I(t_1)-t_1} f_1\left(t_0 + I(t_1), \theta, u^{t_0+\tau_0}(\theta)\right) d\theta\right) = m + \int_0^{t_1} w_1(t_1,\theta)\, d\theta . \tag{7}$$

We set the pursuer control on the initial time-interval $[t_0, t_0 + \tau_0)$ to be equal to $u^{t_0+\tau_0}(\cdot)$.

The trajectory of the conflict-controlled process (1) on the interval $[t_0 + \tau_0, t_0 + I(t_1)]$ can be presented in the form:

$$z(t_0 + \tau_0 + t) = g(t_0 + \tau_0 + t) -$$

$$-\int_0^t \left(f_1(t_0 + \tau_0 + t, t_0 + \tau_0 + \theta, u(t_0 + \tau_0 + \theta)) - f_2(t_0 + \tau_0 + t, t_0 + \tau_0 + \theta, v(t_0 + \tau_0 + \theta))\right) d\theta .$$

Let us build current control of the pursuer at each instant of time $t_0 + \tau_0 + \theta$, $\theta \in [0, t_1]$ on the basis of the evader control at the instant $t_0 + I(t_1) - I(t_1 - \theta)$. It is easy to see that

$$t_0 + I(t_1) - I(t_1 - \theta) = t_0 + \tau_0 + \theta - \left(I(t_1 - \theta) - (t_1 - \theta)\right).$$

One can observe that in such case control of the pursuer at current instant $t_0 + \tau_0 + \theta, \theta \in [0, t_1]$, is constructed in accordance with the evader control at the moment on the time $I(t_1 - \theta) - (t_1 - \theta)$ earlier.

Thus, on the interval $[t_0 + \tau_0, t_0 + I(t_1)]$ transition is made from the original perfect-information game to the auxiliary game with the same dynamics and the same terminal set but with variable delay of information

$$\tau(t_0 + \tau_0 + \theta) = I(t_1 - \theta) - (t_1 - \theta),\ \theta \in [0, t_1]. \tag{8}$$

By Theorem1, this game with information delay is equivalent to the perfect-information game having the dynamics

$$\tilde{z}(t_0+\tau_0+t)=\tilde{g}(t_0+\tau_0+t)-\int_0^t f_1(t_0+\tau_0+t, t_0+\tau_0+\theta, u(t_0+\tau_0+\theta))d\theta+$$
$$+\int_0^t \dot{I}(t_1-\theta)f_2(t_0+I(t), t_0+I(t_1)-I(t_1-\theta), v(t_0+I(t_1)-I(t_1-\theta)))d\theta, \tag{9}$$

$$\tilde{g}(t_0+\tau_0+t)=g(t_0+\tau_0+t)-\int_{t_0}^{t_0+\tau_0} f_1\left(t_0+\tau_0+t, \theta, u^{t_0+\tau_0}(\theta)\right)d\theta.$$

The terminal set of the equivalent game has the form (4) with

$$M_*(t_0+\tau_0+t)=M_0+M(\tau(t_0+\tau_0+t)),$$
$$M(\tau(t_0+\tau_0+t))=M\underline{*}V(\tau(t_0+\tau_0+t))=M\underline{*}V(I(t_1-t)-(t_1-t)).$$

In view of the formula for the time delay (8), $\tau(t_0+I(t_1))=\tau(t_0+\tau_0+t_1)=0$, therefore, $M_*(t_0+I(t_1))=M_*$. Thus, at the moment of the game termination $t_0+I(t_1)$ the terminal set of the equivalent game is the set M_*.

Let us prescribe control of the pursuer on the interval $[t_0+\tau_0, t_0+I(t_1)]$ to be equal a measurable solution of the equation:

$$\pi f_1(t_0+I(t_1), t_0+\tau_0+\theta, u(t_0+\tau_0+\theta))=$$
$$=\dot{I}(t_1-\theta)\pi f_2(t_0+I(t_1), t_0+I(t_1)-I(t_1-\theta), v(t_0+I(t_1)-I(t_1-\theta)))+w(t_1, t_1-\theta)$$
$$\theta\in[0,t_1].$$

Such solution exists by virtue of the Filippov-Castaing theorem on a measurable choice [24].

With the help of the above formula, from the relationship (9) one can easily deduce that $\pi\tilde{z}(t_0+I(t_1))=m$, where $m\in M$. Therefore, at the moment $t_0+I(t_1)$ the trajectory of the original game (1) with complete information hits the terminal set M_*, under arbitrary control of the evader. In so doing, the pursuer constructs its current control on the basis of the evader's control in the past. The theorem is proved.

In case of the linear differential game Condition4 and Theorem2 take the forms of Condition5 and Theorem3, respectively.

Condition 5. [17]. There exists time-stretching function $I(t)$, such that the set-valued mapping

$$W_1(t)=\pi e^{tA}U\underline{*}\dot{I}(t)\pi e^{I(t)A}V$$

has non-empty images for all t, $0\le t<+\infty$.

Theorem 3. Let the linear differential game (5), (2) meet Condition 5 and suppose that for the given initial state z_0 there exists a finite instant of time t_1,

$$t_1 = t_1(z_0) = \min\left\{ t \geq 0 : \pi\left(e^{I(t)A} z_0 - \int_0^{I(t)-t} e^{(I(t)-\theta)A} U d\theta \right) \cap \left(M + \int_0^t W_1(\theta) d\theta \right) \neq \varnothing \right\}. \quad (10)$$

Then the game can be terminated by the pursuer at the time instant $I(t_1)$, under arbitrary admissible controls of the evader.

To this end, on the half-interval $[0, \tau_0)$ control of the pursuer is set equal to $u^{\tau_0}(\theta)$ and on the interval $[\tau_0, \tau_0 + t_1]$ it is built in the form of a measurable solution of the equation

$$\pi e^{(t_1-\theta)A} u(\tau_0 + \theta) =$$

$$= \dot{I}(t_1 - \theta) \pi e^{I(t_1-\theta)A} v(I(t_1) - I(t_1 - \theta)) + w_1(t_1 - \theta), \ \theta \in [0, t_1].$$

Note that control $u^{\tau_0}(\theta), \theta \in [0, \tau_0)$, where $\tau_0 = I(t_1) - t_1$ and measurable selection $w_1(\theta)$, $w_1(\theta) \in W_1(\theta)$, $\theta \in [0, t_1]$ are determined by the formula (10).

INTEGRO-DIFFERENTIAL GAME OF PURSUIT

Let us consider two controlled systems evolving in R^n according to the integro-differential equations, respectively:

$$\dot{x}(t) = A_1 x(t) + \lambda \int_0^t K(t,s) x(s) ds + f_1(u), u \in U, x(0) = x_0, \quad (11)$$

$$\dot{y}(t) = A_2 y(t) + \mu \int_0^t L(t,s) y(s) ds + f_2(v), v \in V, y(0) = y_0. \quad (12)$$

Here A_1 and A_2 are constant matrices, $K(t,s)$ and $L(t,s)$ are matrix functions whose elements are continuous on the set $\Delta = \{(t,s) : 0 \leq s < t < +\infty\}$; $f_1(u) : U \to R^n$ and $f_2(v) : V \to R^n$ are continuous vector- functions; u and v — control parameters of the pursuer and the evader, respectively; λ and μ are real numbers.

The goal of the pursuer is in the shortest time to achieve meeting with the evader, i.e. $x(t) = y(t)$, $t < +\infty$ and the evader tries to escape or maximally postpone the meeting.

We assume that the pursuer constructs its control on the basis of information about initial states of the players and current control of the evader, i.e. employs counter-controls by N.N. Krasovskii [3].

Solutions to the equations (11), (12), under the initial state $x(0) = x_0$, $y(0) = y_0$ and chosen controls $u(\theta)$, $v(\theta)$, $\theta \in [0, t]$ can be presented in the forms, respectively:

$$x(t)=e^{A_1 t}x_0+\int_0^t e^{A_1(t-\theta)}f_1(u(\theta))d\theta+\lambda\int_0^t e^{A_1(t-\theta)}\left(\int_0^\theta K(\theta,s)x(s)ds\right)ds,$$

$$y(t)=e^{A_2 t}x_0+\int_0^t e^{A_2(t-\theta)}f_2(v(\theta))d\theta+\mu\int_0^t e^{A_2(t-\theta)}\left(\int_0^\theta L(\theta,s)y(s)ds\right)ds.$$

Let us interchange, by virtue of the Dirichlet rule [25], the order of integration in the last terms of the both expressions. Then we obtain

$$\int_0^t e^{A_1(t-\theta)}\left(\int_0^\theta K(\theta,s)x(s)ds\right)d\theta=\int_0^t\left(\int_s^t e^{A_1(t-\theta)}K(\theta,s)d\theta\right)x(s)ds,$$

$$\int_0^t e^{A_2(t-\theta)}\left(\int_0^\theta L(\theta,s)y(s)ds\right)d\theta=\int_0^t\left(\int_s^t e^{A_2(t-\theta)}L(\theta,s)d\theta\right)y(s)ds.$$

Thus, we come to the linear integral Volterra equations of second order

$$x(t)=\lambda\int_0^t K(t,s)x(s)ds+g_1(t), \tag{13}$$

$$y(t)=\mu\int_0^t \hat{L}(t,s)y(s)ds+g_2(t). \tag{14}$$

Here

$$g_1(t)=e^{A_1 t}x_0+\int_0^t e^{A_1(t-\theta)}f_1(u(\theta))d\theta, \tag{15}$$

$$g_2(t)=e^{A_2 t}y_0+\int_0^t e^{A_2(t-\theta)}f_2(v(\theta))d\theta, \tag{16}$$

$$\hat{K}(t,s)=\int_0^t e^{A_1(t-\theta)}K(\theta,s)d\theta,$$

$$\hat{L}(t,s)=\int_0^t e^{A_2(t-\theta)}L(\theta,s)d\theta.$$

Using the method of successive approximations [26] one can find solutions to the equations (13), (14):

$$x(t) = \lambda \int_0^t \hat{R}_1(t, s; \lambda) g_1(s) ds + g_1(t), \tag{17}$$

$$y(t) = \mu \int_0^t \hat{R}_2(t, s; \mu) g_2(s) ds + g_2(t), \tag{18}$$

where $\hat{R}_1(t,s;\lambda)$ and $\hat{R}_2(t,s;\mu)$ are the resolvents of the integral equations (13), (14), defined by the Neumann rows

$$\hat{R}_1(t, s; \lambda) = \sum_{i=1}^{\infty} \lambda^{i-1} \hat{K}_i(t, s), \tag{19}$$

$$\hat{R}_2(t, s; \lambda) = \sum_{i=1}^{\infty} \mu^{i-1} \hat{L}_i(t, s). \tag{20}$$

These rows are absolutely converging and the iterated kernels $\hat{K}_i(t,s)$, $\hat{L}_i(t,s)$, $i = 1,2,...$ are given by the recursive formulas:

$$\hat{K}_1(t,s) = K(t,s), \hat{K}_i(t,s) = \int_s^t \hat{K}_1(t,\varsigma) \hat{K}_{i-1}(\varsigma, s) d\varsigma,$$

$$\hat{L}_1(t,s) = L(t,s) \ \hat{L}_i(t,s) = \int_0^t \hat{L}_1(t,\varsigma) \hat{L}_{i-1}(\varsigma, s) d\varsigma, \ i = 2,....$$

Let us substitute formulas (15), (16) for $g_1(t)$ and $g_2(t)$ into the expressions (17), (18). Then we have

$$x(t) = e^{A_1 t} x_0 + \int_0^t e^{A_1(t-\theta)} f_1(u(\theta)) d\theta + \lambda \int_0^t \hat{R}_1(t, s; \lambda) e^{A_1 s} ds \cdot x_0 +$$

$$+ \lambda \int_0^t \hat{R}_1(t, s; \lambda) \left(\int_0^s e^{A_1(s-\theta)} f_1(u(\theta)) d\theta \right) ds,$$

$$y(t) = e^{A_2 t} y_0 + \int_0^t e^{A_2(t-\theta)} f_2(v(\theta)) d\theta + \mu \int_0^t \hat{R}_2(t, s; \mu) e^{A_2 s} ds \cdot y_0 +$$

$$+ \mu \int_0^t \hat{R}_2(t, s; \mu) \left(\int_0^s e^{A_2(s-\theta)} f_2(v(\theta)) d\theta \right) ds.$$

Upon application of the Dirichlet formula in the last terms of the above expressions we obtain

$$x(t)=\left(e^{A_1 t}+\lambda\int_0^t \hat{R}_1(t,s;\lambda)e^{A_1 s}ds\right)x_0+$$
$$+\int_0^t\left(e^{A_1(t-\theta)}+\lambda\int_\theta^t \hat{R}_1(t,s;\lambda)e^{A_1(s-\theta)}ds\right)f_1(u(\theta))d\theta, \tag{21}$$

$$y(t)=\left(e^{A_2 t}+\mu\int_0^t \hat{R}_2(t,s;\mu)e^{A_2 s}ds\right)y_0+$$
$$+\int_0^t\left(e^{A_2(t-\theta)}+\mu\int_\theta^t \hat{R}_2(t,s;\mu)e^{A_2(s-\theta)}ds\right)f_2(v(\theta))d\theta. \tag{22}$$

With the use of notations

$$\Phi_1(t,\theta)=e^{A_1(t-\theta)}+\lambda\int_\theta^t \hat{R}_1(t,s;\lambda)e^{A_1(s-\theta)}ds, \tag{23}$$

$$\Phi_2(t,\theta)=e^{A_2(t-\theta)}+\mu\int_\theta^t \hat{R}_2(t,s;\mu)e^{A_2(s-\theta)}ds, \tag{24}$$

the solutions to equations (11), (12) can be presented in explicit form:

$$x(t)=\Phi_1(t,0)x_0+\int_0^t \Phi_1(t,\theta)f_1(u(\theta))d\theta, \tag{25}$$

$$y(t)=\Phi_2(t,0)y_0+\int_0^t \Phi_2(t,\theta)f_2(v(\theta))d\theta. \tag{26}$$

Let us pass from the game under study (11), (12) to the equivalent game with the origin as the terminal set and the state vector $z=y-x$, evolving in R^n according to (1), with

$g(t)=\Phi_2(t,0)-\Phi_1(t,0)$, $f_1(t,\theta,u(\theta))=\Phi_1(t,\theta)u(\theta)$, $f_2(t,\theta,v(\theta))=\Phi_2(t,\theta)v(\theta)$,

and starting from the initial state

$$z_0=y_0-x_0.$$

Then Conditions 3, 4 and Theorem2 reduce to the Conditions 6, 7 and Theorem 4, respectively.

CONDITION 6. The set-valued mapping

$$W(t,\theta)=\Phi_1(I(t),\theta)f_1(U) \underline{*} \Phi_2(I(t),\theta)f_2(V)\neq\varnothing$$

has nonempty images at all $0\leq\theta\leq t<+\infty$.

CONDITION 7. There exists function of time stretching $I(t)$, such that the set-valued mapping

$$W(t,\theta)=\Phi_1(I(t),I(t)-\theta)f_1(U) \underline{*} \dot{I}(t)\Phi_2(I(t),I(t)-I(\theta))f_2(V)$$

has nonempty images at all $0\leq\theta\leq t<+\infty$.

Theorem 4. Let the parameters of pursuit game (11), (12) meet Condition7 and let for the given initial states x_0, y_0 there exist a finite moment t_1,

$$t_1=t_1(x_0,y_0)=$$

$$\left\{\min t: t\geq 0, \left(\Phi_2(I(t),0)y_0-\Phi_1(I(t),0)x_0-\int_0^{I(t)-t}\Phi_1(I(t),\theta)f_1(U)d\theta\right)\right. \tag{27}$$

$$\left.\cap\int_0^t W(t,\theta)d\theta\neq\varnothing\right\}.$$

Then the pursuer can terminate the game at the moment of time $I(t_1)$ for arbitrary admissible control of the evader.

Note that in the course of pursuit, beginning from the moment $\tau_0=I(t_1)-t_1$, the pursuer constructs its current control on the basis of the evader's control on the time $I(t_1-\theta)-(t_1-\theta)$ earlier, defined by the formula:

$$\Phi_1(I(t_1),\tau_0+\theta)f_1(u(\tau_0+\theta))=$$

$$=\dot{I}(t_1-\theta)\Phi_2(I(t_1),I(t_1)-I(t_1-\theta))f_2(v(I(t_1)-I(t_1-\theta)))+w(t_1,t_1-\theta)$$

$$\theta\in[0,t_1].$$

ILLUSTRATIVE EXAMPLE OF INTEGRO-DIFFERENTIAL PURSUIT GAME

Let us consider the game, in which motions of the pursuer and the evader in R^n are described by the equations, respectively [27]:

$$\dot{x}(t)=\lambda\int_0^t x(s)ds+u,\ u\in U,\ x(0)=x_0,\ \|u\|\leq\rho, \tag{28}$$

$$\dot{y}(t)=\mu\int_0^t y(s)ds+v,\ v\in V, y(0)=y_0,\ \|v\|\leq\sigma. \tag{29}$$

It is a particular case of the game (11), (12). We see that in the game under study $K_1(t,s)$, $K_2(t,s)$ are the unit matrices and A_1, A_2 — zero matrices. Using formulas for the iterated kernels one can easily evaluate

$$\hat{K}_1(t,s) = (t-s)E\,,\ \hat{L}_1(t,s) = (t-s)E\,.$$

Let us denote

$$\omega_1 = \sqrt{-\lambda}\,, \omega_2 = \sqrt{-\mu}\,.$$

In view of formulas (19), (20), the resolvents of the equations (28), (29), have the forms, respectively:

$$\hat{R}_1(t,s) = \frac{1}{\omega_1}\sin\omega_1(t-s)\,, \hat{R}_2(t,s) = \frac{1}{\omega_2}\sin\omega_2(t-s)\,, \text{ if } \lambda < 0\,,\ \mu < 0\,,$$

$$\hat{R}_1(t,s) = \frac{1}{\omega_2} sh\omega_2(t-s)\,, \hat{R}_2(t,s) = \frac{1}{\omega_1} sh\omega_1(t-s)\,, \text{ if } \lambda > 0\,,\ \mu > 0\,.$$

Then, by formulas (23), (24)

$$\Phi_1(t,\theta) = \begin{cases} \cos\omega_1(t-s)\cdot E & \textit{if } \lambda < 0 \\ ch\omega_1(t-s)\cdot E & \textit{if } \lambda > 0, \end{cases}$$

$$\Phi_2(t,\theta) = \begin{cases} \cos\omega_2(t-s)\cdot E & \textit{if } \mu < 0 \\ ch\omega_2(t-s)\cdot E & \textit{if } \mu > 0. \end{cases}$$

Here E is n-dimensional unit matrix.

Let us analyze Condition 6 for various combinations of λ and μ signs:

1) $\lambda < 0, \mu > 0$. Condition 6 reduces to the form:

$$\rho\left|\cos\omega_1 t\right| - \sigma\left|ch\omega_2 t\right| \geq 0\,.$$

This inequality is not fulfilled for all $t \geq 0$, therefore Condition 6 does not hold. The principle of the time stretching is not applicable because

$$ch\omega_2 t = \frac{e^{\omega_2 t} + e^{-\omega_2 t}}{2} \to +\infty \text{ as } t \to +\infty\,.$$

2) $\lambda > 0$, $\mu < 0$. Condition 6 has the form:

$$\rho\left|ch\omega_1 t\right| - \sigma\left|\cos\omega_2 t\right| \geq 0 \quad \forall t \geq 0\,.$$

It holds if $\rho \geq 2\sigma$.

3) $\lambda > 0$, $\mu > 0$. Condition 7 has the form:

$$\rho|ch\omega_1 t| - \sigma|ch\omega_2 t| \geq 0 \quad \forall t \geq 0 .$$

It holds if $\rho \geq \sigma$ and $\omega_1 > \omega_2$.

4) $\lambda < 0$, $\mu = 0$. Condition 6 takes the form:

$$\rho|\cos\omega_1 t| - \sigma \geq 0 \;\; \forall t \geq 0 .$$

It does not hold and the time stretching is not applicable.

5) $\lambda = 0$, $\mu < 0$. Condition 6 reduces to the form:

$$\rho - \sigma|\cos\omega_2 t| \geq 0 \;\; \forall t \geq 0 .$$

It holds if $\rho \geq \sigma$.

6) $\lambda = 0$, $\mu > 0$. Condition 7 has form of the inequality $\rho - \sigma|ch\omega_2 t| \geq 0$ and does not hold $\forall t \geq 0$. The principle of the time stretching is not applicable since $ch\omega_2 t \to +\infty$ as $t \to +\infty$.

7) $\lambda > 0$, $\mu = 0$. Condition 6: takes the form $\rho|ch\omega_1 t| - \sigma \geq 0 \;\; \forall t \geq 0$ and holds if $\rho \geq \sigma$.

8) The case $\lambda < 0$, $\mu < 0$ presents special interest and is analyzed in detail. Condition 6 has the form:

$$\rho|\cos\omega_1 t| \cdot S \underline{*} \sigma|\cos\omega_2 t| \cdot S \neq \varnothing \;\; \forall t \geq 0 .$$

Here S, $S \in R^n$ is the ball of unit radius centered at the origin. Evidently, this condition does not hold.

Let us apply the time stretching principle. In the case under study Condition 7 (with the time stretching) looks as follows:

$$W_1(t) = \rho|\cos\omega_1 t| \cdot S \underline{*} \sigma\dot{I}(t)|\cos\omega_2 I(t)| \cdot S \neq \varnothing \;\; \forall t \geq 0 .$$

It is equivalent to the inequality

$$\rho|\cos\omega_1 t| - \sigma\dot{I}(t)|\cos\omega_2 I(t)| \geq 0 \quad \forall t \geq 0 . \tag{30}$$

Let us assume that

$$\omega_1 > \omega_2 \tag{31}$$

and let us select the function of time stretching

$$I(t) = \frac{\omega_1}{\omega_2} t .$$

Then the relation (30) reduces to the inequality:

$$\left(\rho-\sigma\frac{\omega_1}{\omega_2}\right)\left|\cos\omega_1 t\right|\geq 0. \qquad (32)$$

We impose, in addition to the condition (31), following constraint on the game parameters:

$$\frac{\rho}{\omega_1}\geq\frac{\sigma}{\omega_2}. \qquad (33)$$

It is easy to see that, under the conditions (31), (33), the inequality (32) holds for all $t\geq 0$, that means that Condition 7 is fulfilled.

Now we will show that assumption (27) of Theorem4 is satisfied for arbitrary initial states x_0, y_0 of the objects. To this end, we set control of the pursuer on the initial half-interval of time equal zero, that is $u^0(\theta)=0,\ \theta\in[0, I(t_1)-t_1)$. Then, in view of Theorem4, it remains to show that there exists an instant of time t_1, $0<t_1<+\infty$, at which the following inclusion is true:

$$\pi e^{I(t_1)A}z_0\in\int_0^{t_1}W_1(\theta)d\theta.$$

In the example under study this inclusion reduces to the form:

$$\cos\omega_2 I(t)y_0-\cos\omega_1 I(t)x_0\in\int_0^t\left(\rho\left|\cos\omega_1 t\right|-\sigma\dot{I}(t)\left|\cos\omega_2 I(t)\right|\right)d\theta\cdot S. \qquad (34)$$

Substituting $I(t)=\frac{\omega_1}{\omega_2}t$ into the inclusion (34), we convert the latter into the relationship

$$\left\|\cos\omega_1 t\cdot y_0-\cos\frac{\omega_1^2}{\omega_2}t\cdot x_0\right\|\leq\left(\rho-\sigma\frac{\omega_1}{\omega_2}\right)\int_0^t\left|\cos\omega_1 t\right|d\theta.$$

One can readily see that the left-hand part of the above inequality is less or equal to $\|x_0\|+\|y_0\|$, while the right-hand part is more or equal to $\left(\rho-\sigma\frac{\omega_1}{\omega_2}\right)\cdot\left[\frac{t}{\pi/\omega_1}\right]$, where by symbol $[\cdot]$ is denoted the integer part of a number. That is why, there exists a moment of time t_1 at which this inequality is satisfied and, therefore, the inclusion (34) holds.

Hence, by virtue of Theorem 4, under the conditions (31), (33) on the game parameters, the pursuer can catch the evader at a finite moment of time, for arbitrary initial states of the players.

CONCLUSION

An original approach is developed to study game dynamic problems of pursuit. It is applicable to a wide range of conflict-controlled functional-differential systems. In the paper, sufficient conditions for guaranteed capture are obtained in the case of integro-differential dynamics of objects. The time-stretching principle offers promise as an efficient tool for probing complicated problems of mobile objects conflict counteraction, for which classic condition for the instantantaneos advantage of the pursuer over the evader does not hold. Suggested approach makes it feasible to realize the process of pursuit with the help of appropriate counter-controls by Krasovskii.

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ПРИНЦИП РОЗТЯГУ ЧАСУ В ІГРОВИХ ЗАДАЧАХ ДИНАМІКИ

Вступ. Є широке коло процесів механіки, економіки і біології, які розвиваються в умовах конфлікту та невизначеності і можуть бути описані різного роду функціонально-диференційними системами залежно від природи процесу.

В роботі розглядаються динамічні ігри переслідування, які визначаються системою загального вигляду, яка охоплює широке коло функціонально-диференційних систем.

У дослідженні динамічних ігор вирішальним фактором є інформованість гравців про поточний стан процесу. У реальних системах інформація, як правило, надходить із запізненням у часі. Також, є низка ігрових завдань, для яких не виконується умова Понтрягіна, що відображає миттєву перевагу одного гравця над іншим в ресурсах керування. Встановлення тісного зв'язку між узагальненням цієї умови, пов'язаної з розтягуванням часу і ефектом змінного запізнювання інформації відкриває великі перспективи для розв’язання вищезазначених завдань.

Мета статті — на основі ефекту запізнювання інформації вивести достатні умови завершення гри, для якої є не виконаною умова Понтрягіна, конкретизувати їх на випадок інтегро-диференційної динаміки з подальшою ілюстрацією отриманого результату на модельному прикладі.

Методи. Для дослідження динамічної гри переслідування застосовується перший прямий метод Понтрягіна, що забезпечує приведення траєкторії конфліктно-керованого процесу на циліндрично термінальну множину в кінцевий момент часу. Водночас, побудова керування переслідувача здійснюється на основі теореми Філіппова-Каста про вимірний вибір, що забезпечує реалізацію процесу переслідування у класі стробоскопичних стратегій Хайєка. Для виведення рішення конфліктно-керованої інтегро-диференційної системи у формі Коші використовується метод послідовних апроксимацій.

Результати. Показано, що динамічна гра переслідування з розділеними блоками керування гравців і змінним запізненням інформації еквівалентна деякій грі з повною інформацією. На основі цього факту розроблено принцип розтягування часу для дослідження ігор з повною інформацією, для яких не виконано класичну умову Понтрягіна, що лежить в основі всіх прямих методів переслідування. У роботі пропонується модифікація цієї умови, пов'язана з розтягуванням часу, що дає змогу отримати достатні умови приведення траєкторії гри на термінальну множину в кінцевий момент часу. Описано спосіб побудови керування переслідувача, який приводить до досягнення мети.

Отримано конкретний вигляд цих умов для інтегро-диференційної гри переслідування. В якості ілюстрації проведено детальний аналіз конкретного прикладу інтегро-диференційної гри. Знайдено функцію розтягування часу, що забезпечує виконання узагальненої умови Понтрягіна. Виведено прості співвідношення між параметрами руху гравців і їхніми ресурсами керування, що забезпечують переслідувачеві можливість зловити гравців, які тікають, за будь-яких їхніх початкових положень.

Висновки. Розроблено ефективний засіб для аналізу конфліктних ситуацій, наприклад, перехоплення рухомої цілі керованим рухомим об'єктом. Проаналізовано ситуацію, коли об'єкт, який переслідує, не має миттєвої переваги у ресурсах керування перед гравцем, який тікає. Запропонований підхід дає змогу реалізувати процес переслідування за допомоги відповідних контр-керувань Красовського.

Ключові слова: *динамічна гра, змінне запізнення інформації, умова Понтрягіна, інтеграл Ауманна, принцип розтягу часу, різниця Міньковського, інтегро-диференційна гра.*

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ПРИНЦИП РАСТЯЖЕНИЯ ВРЕМЕНИ В ИГРОВЫХ ЗАДАЧАХ ДИНАМИКИ

Введение. Существует широкий круг процессов механики, экономики и биологии, развивающихся в условиях конфликта и неопределенности, которые могут быть описаны различного рода функционально-дифференциальными системами в зависимости от природы процесса. В работе рассматриваются динамические игры преследования, определяемые системой общего вида, охватывающей широкий круг функционально-дифференциальных систем.

При исследовании динамических игр решающим фактором является информированность игроков о текущем состоянии процесса. В реальных системах информация, как правило, поступает с запаздыванием во времени. Также существует ряд игровых задач, для которых не выполняется условие Понтрягина, отражающее мгновенное преимущество одного игрока над другим в ресурсах управления. Установление тесной связи между обобщением этого условия, связанного с растяжением времени и эффектом переменного запаздывания информации, открывает большие перспективы для решения вышеупомянутых задач.

Цель статьи — на основе эффекта запаздывания информации вывести достаточные условия завершения игры, для которой не выполнено условие Понтрягина, конкретизировать их на случай интегро-дифференциальной динамики с последующей иллюстрацией полученного результата на модельном примере.

Методы. Для исследования динамической игры преследования применяется первый прямой метод Понтрягина, обеспечивающий приведение траектории конфликтно-управляемого процесса на цилиндрическое терминальное множество в конечный момент времени. При этом, построение управления преследователя осуществляется на основе теоремы Филиппова-Кастена об измеримом выборе, что обеспечивает реализацию процесса преследования в классе стробоскопических стратегий Хайека. При выводе решения конфликтно-управляемой интегро-дифференциальной системы в форме Коши используется метод последовательных аппроксимаций.

Результаты. Показано, что динамическая игра преследования с разделенными блоками управлений игроков и переменным запаздыванием информации эквивалентна некоторой игре с полной информацией. На основе этого факта разработан принцип растяжения времени для исследования игр с полной информацией, для которых не выполнено классическое условие Понтрягина, лежащее в основе всех прямых методов преследования. В работе предлагается модификация этого условия, связанная с растяжением времени и позволяющая получить достаточные условия приведения траектории игры на терминальное множество в конечный момент времени. При этом описывается способ построения управления преследователя, приводящий к цели.

Получен конкретный вид этих условий для интегро-дифференциальной игры преследования. В качестве иллюстрации проведен подробный анализ конкретного примера интегро-дифференциальной игры. Найдена функция растяжения времени, обеспечивающая выполнение модифицированного условия Понтрягина. Выведены простые соотношения между параметрами движения игроков и их ресурсами управления, обеспечивающие преследователю возможность поймать убегающего при любых начальных положениях игроков.

Выводы. Разработано эффективное средство для анализа конфликтных ситуаций, например, перехвата подвижной цели управляемым движущимся объектом. Проанализирувана ситуация, когда преследующий объект не обладает мгновенным преимуществом в ресурсах управления перед убегающим играком. Предлагаемый подход позволяет реализовать процесс преследования с помощью подходящих контр-управлений Красовского.

Ключевые слова: *динамическая игра, переменное запаздывание информации, условие Понтрягина, интеграл Аумана, принцип растяжения времени, разница Минковского, интегро-дифференциальная игра*