

**ДВОЕТАПНА ТРАНСПОРТНА ЗАДАЧА
З ОБМЕЖЕННЯМ НА КІЛЬКІСТЬ
ПРОМІЖНИХ ПУНКТІВ**

[1].

[1].

().

AMPL-

gurobi.

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a_1, \dots, a_m , b_1, \dots, b_n , D_1, \dots, D_l , d
 $(1 \leq d \leq l)$, c_{ik} , c_{kj}
 A_i , D_k , B_j
 $x = \{x_{ik}\}_{i=1, \dots, m}^{k=1, \dots, l}$, $y = \{y_{kj}\}_{k=1, \dots, l}^{j=1, \dots, n}$, $z = \{z_k\}_{k=1, \dots, l}$, d

$$f_{xyz}^* = f_{xyz}(x^*, y^*, z^*) = \min_{x, y, z} \left\{ f(x, y) = \sum_{i=1}^m \sum_{k=1}^l c_{ik} x_{ik} + \sum_{k=1}^l \sum_{j=1}^n c_{kj} y_{kj} \right\} \quad (1)$$

$$\sum_{k=1}^l x_{ik} = a_i, \quad i = 1, \dots, m, \quad (2)$$

$$\sum_{k=1}^l y_{kj} = b_j, \quad j = 1, \dots, n, \quad (3)$$

$$\sum_{i=1}^m x_{ik} - \sum_{j=1}^n y_{kj} = 0, \quad k = 1, \dots, l, \quad (4)$$

$$\sum_{i=1}^m x_{ik} \leq z_k \sum_{i=1}^m a_i, \quad k = 1, \dots, l, \quad (5)$$

$$\sum_{k=1}^l z_k \leq d, \quad (6)$$

$$x_{ik} \geq 0, y_{kj} \geq 0, z_k = 0 \vee 1, \quad i = 1, \dots, m, k = 1, \dots, l, j = 1, \dots, n. \quad (7)$$

$(m+n) \times l$ x y , l z , $m+n+2l+1$
 a_1, \dots, a_m

$$(3) - \quad b_1, \dots, b_n \quad (4)$$

$$, \quad ; \quad , \quad d \quad , \quad (6) \quad , \quad (5)$$

$$(1) - (7) \quad d = l, \quad , \quad l$$

(5) (6)

[2, 3].

$$1. \quad d = l, \quad (1) - (7)$$

$$f_{xy}^* = f_{xy}(x^*, y^*) = \min_{x,y} \left\{ f(x, y) = \sum_{i=1}^m \sum_{k=1}^l c_{ik} x_{ik} + \sum_{k=1}^l \sum_{j=1}^n c_{kj} y_{kj} \right\} \quad (8)$$

$$\sum_{k=1}^l x_{ik} = a_i, \quad i = 1, \dots, m, \quad (9)$$

$$\sum_{k=1}^l y_{kj} = b_j, \quad j = 1, \dots, n, \quad (10)$$

$$\sum_{i=1}^m x_{ik} - \sum_{j=1}^n y_{kj} = 0, \quad k = 1, \dots, l, \quad (11)$$

$$x_{ik} \geq 0, y_{kj} \geq 0, \quad i = 1, \dots, m, k = 1, \dots, l, j = 1, \dots, n. \quad (12)$$

$$2 [4]. \quad (9) - (12) - \quad , \quad \sum_{i=1}^m a_i \neq \sum_{j=1}^n b_j .$$

$$(8) - (12)$$

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j, \quad (13)$$

$$(13)$$

:

A_{m+1}

$$a_{m+1} = \sum_{j=1}^n b_j - \sum_{i=1}^m a_i \quad ;$$

B_{n+1}

$$b_{n+1} = \sum_{i=1}^m a_i - \sum_{j=1}^n b_j .$$

2. (1) – (7), $1 \leq d \leq l$.

[1, l]

l , $d=1, \dots, d$
 [1, l],
 (1) – (7)
 (13), $d=1$

$$x^* = \{x_{i1}^* = a_i\}_{i=1, \dots, m}, \quad y^* = \{y_{1j}^* = b_j\}_{j=1, \dots, n},$$

$$f_{xy}^* = \sum_{i=1}^m c_{i1} a_i + \sum_{j=1}^n c_{1j} b_j.$$

$$1 < d < l, \quad (1) - (7)$$

3.

(2) – (7)

$$\sum_{i=1}^m a_i \neq \sum_{j=1}^n b_j.$$

$$1 < d < l,$$

$$l-d$$

(6).

(5)

(2) – (7)

$$\sum_{k=1}^d x_{ik} = a_i, \quad i = 1, \dots, m, \quad (14)$$

$$\sum_{k=1}^d y_{kj} = b_j, \quad j = 1, \dots, n, \quad (15)$$

$$\sum_{i=1}^m x_{ik} - \sum_{j=1}^n y_{kj} = 0, \quad k = 1, \dots, d, \quad (16)$$

$$x_{ik} \geq 0, y_{kj} \geq 0, \quad i = 1, \dots, m, k = 1, \dots, d, j = 1, \dots, n. \quad (17)$$

$$\bar{y} = \{\bar{y}_{kj}, k=1, \dots, d, j=1, \dots, n\}, \quad \bar{x} = \{\bar{x}_{ik}, i=1, \dots, m, k=1, \dots, d\} \quad (14) - (17).$$

$$\bar{x} \geq 0 \quad \bar{y} \geq 0 \quad :$$

$$\sum_{k=1}^d \bar{x}_{ik} = a_i, \quad i=1, \dots, m, \quad (18)$$

$$\sum_{k=1}^d \bar{y}_{kj} = b_j, \quad j=1, \dots, n, \quad (19)$$

$$\sum_{i=1}^m \bar{x}_{ik} - \sum_{j=1}^n \bar{y}_{kj} = 0, \quad k=1, \dots, d. \quad (20)$$

$$(18) \quad i=1, \dots, m, \quad (19) \quad j=1, \dots, n,$$

$$\sum_{i=1}^m \sum_{k=1}^d \bar{x}_{ik} = \sum_{i=1}^m a_i, \quad (21)$$

$$\sum_{j=1}^n \sum_{k=1}^d \bar{y}_{jk} = \sum_{j=1}^n b_j. \quad (22)$$

$$(21) \quad (22) \quad ,$$

$$\sum_{k=1}^d \sum_{i=1}^m \bar{x}_{ik} - \sum_{k=1}^d \sum_{j=1}^n \bar{y}_{jk} = \sum_{i=1}^m a_i - \sum_{j=1}^n b_j. \quad (23)$$

$$(20) \quad k=1, \dots, d, \quad :$$

$$\sum_{k=1}^d \sum_{i=1}^m \bar{x}_{ik} - \sum_{k=1}^d \sum_{j=1}^n \bar{y}_{kj} = 0. \quad (24)$$

$$\sum_{i=1}^m a_i \neq \sum_{j=1}^n b_j \quad (23) \quad \sum_{k=1}^d \sum_{i=1}^m \bar{x}_{ik} - \sum_{k=1}^d \sum_{j=1}^n \bar{y}_{kj} \neq 0,$$

$$(24) \quad , \quad \bar{x} \quad \bar{y}$$

$$(14) - (17),$$

$$\sum_{i=1}^m a_i \neq \sum_{j=1}^n b_j \quad (14) - (17) - \quad ,$$

$$d \quad ,$$

$$\sum_{i=1}^m a_i \neq \sum_{j=1}^n b_j \quad , \quad (2) - (7) - \quad .$$

$$(1) - (7). \quad 3 \quad , \quad (1) - (7) - \quad , \quad (2) - (7) - \quad ,$$

(1)–(7). « (1)–(7) ».

3. AMPL- (1)–(7) AMPL (A Mathematical Programming Language) [5] gurobi [6].

```

AMPL- (1)–(7) :
param m>=2; # (A)
param l>=1; # (D)
param n>=2; # (B)
param d>=1<=m; # D
# :
param cik{i in 1..m, k in 1..l} >= 0; # A D
param ckj{k in 1..l, j in 1..n} >= 0; # D B
#
param a{i in 1..m} >= 0; # A
param b{j in 1..n} >= 0; # B
check: sum{i in 1..m} a[i] = sum{j in 1..n} b[j]; # 1
# ( )
var x{i in 1..m, k in 1..l} >=0; # A D
var y{k in 1..l, j in 1..n} >=0; # D B
var z{k in 1..l} binary; # 1- D, 0- D
minimize f_opt: # :
sum{i in 1..m, k in 1..l} cik[i,k]*x[i,k]+ # A D
sum{k in 1..l, j in 1..n} ckj[k,j]*y[k,j]; # D B
subject to #
con2 {i in 1..m}: # A D
    sum{k in 1..l}x[i,k] = a[i];
con3 {j in 1..n}: # D
    sum{k in 1..l}y[k,j] = b[j];
con4 {k in 1..l}: #
    sum{i in 1..m}x[i,k]-sum{j in 1..n}y[k,j]=0;
con5 {k in 1..l}: # ./
    sum{i in 1..m}x[i,k]<=z[k]*sum{i in 1..m}a[i];
con6: # d
    sum{k in 1..l}z[k]<=d;

    param ;
var - ;
    check - (2)–(7) 2,
    AMPL- (1)–(7)
    gurobi

```

$2, 3,$
 380
 $150, 50, 350, 250$
 $1, 2.$

$B_1, B_2, B_3, B_4,$
 $($

$: 1,$
 $200, 220$
 $D_1, D_2, D_3,$

1

$A \setminus D$	D_1	D_2	D_3
A_1	5	2	5
A_2	3	1	3
A_3	4	7	1

2

$D \setminus B$	B_1	B_2	B_3	B_4
D_1	3	2	5	2
D_2	7	1	3	1
D_3	8	5	6	5

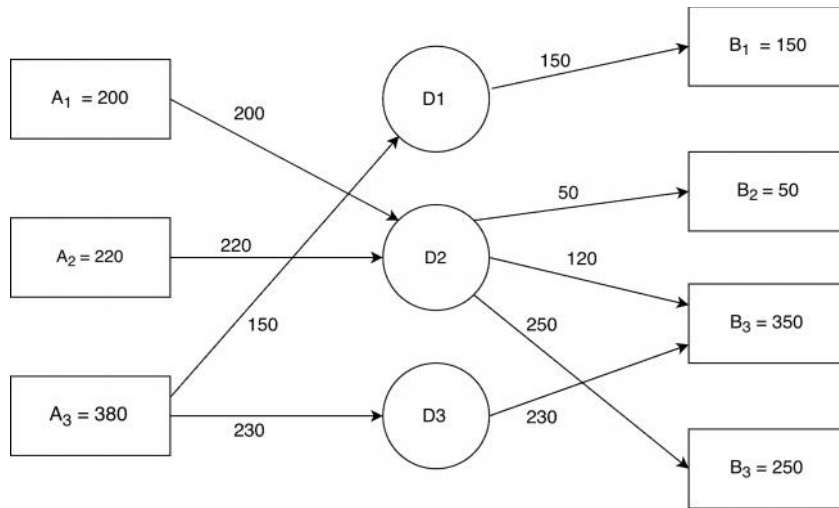
(1) – (7) AMPL- :

```

data; #
param m := 3; #
param l := 3; #
param n := 4; #
param d := 3; #
#
param cik: 1 2 3 := # ( ) D ( )
          1 5 2 5
          2 3 1 3
          3 4 7 1;
param ckj: 1 2 3 4 := # D ( ) B ( )
          1 3 2 5 2
          2 7 1 3 1
          3 8 5 6 5;
param a:= #
1 200 2 220 3 380; #A_1, A_2, A_3
param b:= #
1 150 2 50 3 350 4 250; #B_1, B_2, B_3, B_4
options solver gurobi;
solve; # (1)-(5)
#
display f_opt, _solve_time;
#
display x; display y; display z.

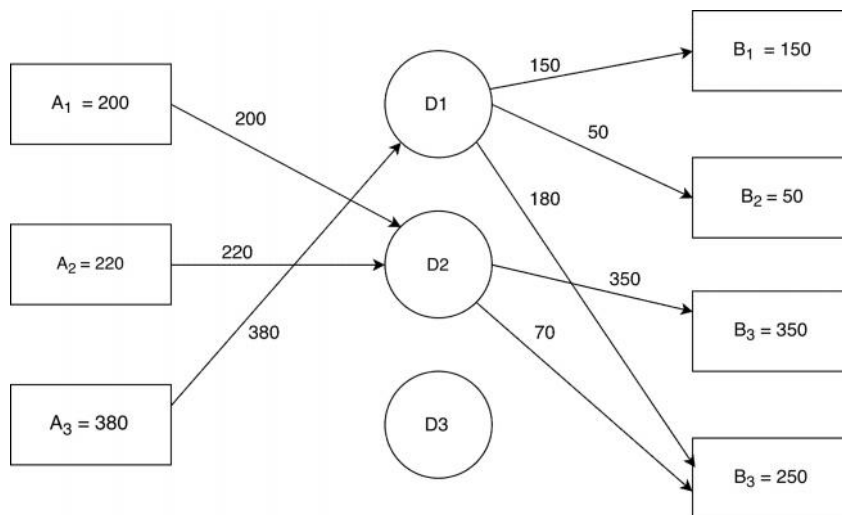
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. 1. (1) – (7) $d = 3$
 D_1 150 , D_2 – 420 ,
 D_3 – 230 .



. 1. $d = 3$

. 2. (1) – (7) $d = 2$
 D_1 380 , D_2 – 420 ,
 D_3 – 4170 .



. 2. $d = 2$

P.I. Stetsyuk, O.P. Bysaha, S.S. Tregubenko

TWO-STAGE TRANSPORTATION PROBLEM WITH CONSTRAINT ON THE NUMBER OF INTERMEDIATE LOCATIONS

A mathematical model of the two-stage transportation problem is proposed to determine the optimal plan for transportation of homogeneous products from suppliers to consumers if the number of intermediate locations is bounded above. The mathematical model is formulated as a Boolean linear programming problem. The conditions under which the problem has a solution are determined, and AMPL-code for solving the problem by state-of-the-art linear integer programming solvers is given. A demo example of calculation results using *gurobi* program is presented.

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01.11.2018

Про авторів:

-mail: stetsyukp@gmail.com

-mail: olehbusaha@gmail.com

-mail: info-cvni@ukr.net