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AXISYMMETRIC THERMOELASTIC DEFORMATION OF A MULTILAYER FOUNDATION WITH IMPERFECT THERMAL CONTACT OF ITS LAYERS

A solution to an axisymmetric stationary problem of thermoelasticity for a multilayer foundation with imperfect thermal contact between its layers is solved by using the method of compliance functions along with the Hankel transform. The recurrence relations for the auxiliary functions and the compliance functions of neighboring layers of the foundation are constructed. The influence of the coefficient of thermal resistance on the distribution of normal and tangential stresses and temperature at the points of the lower boundary of the upper layer for a two-layer foundation subjected to the action of thermal loads is analyzed.

Keywords: *multilayer foundation, compliance functions, Hankel transformation, stresses, temperature.*

Introduction. Elastic multilayer foundations are widely used for modeling pavement-road surfaces, airfield pavements, plant floors, rolling mills, and other engineering structures of stratified nature. Under certain operational conditions, they may undergo significant thermal impacts. Therefore, the temperature effects must be taken into account when evaluating the strength of layered structures. By making use of the method of compliance functions [13], solutions of plane and spatial thermoelasticity problems for multilayer foundations and plates with perfect thermal contact of layers were obtained in [1, 4–6]. A solution to the plane thermoelastic problem for a multilayer foundation with imperfect thermal contact between its layers was derived in [14].

A number of studies [10–12] are concerned with axisymmetric thermoelastic problems for solids under imperfect thermal contact. A solution to an axisymmetric contact problem of the elasticity theory for a three-layer elastic cylinder under the perfect one-sided mechanical contact and imperfect thermal contacts is given in [3] by implementing an iterative algorithm based on the finite element method. In [8, 15], a mathematical model has been suggested for the heat exchange in a piecewise inhomogeneous layer through a thin inclusion. The solution of the corresponding heat transfer problem was constructed for the case of imperfect thermal interaction of the "layer–inclusion" system. Using the spline collocation method, an axisymmetric problem on the analysis of stress-strain state in a multilayer hollow cylinder of finite length under the action of internal pressure and temperature was reduced in [7] to a one-dimensional problem. By employing the Fourier – Bessel integral transform method, a solution to a non-stationary heat-transfer problem for a two-layer space with imperfect thermal contact was derived in [2].

The method of compliance functions was proven to be an efficient tool for the analysis of multilayer structures [13]. Herein, this method is extended for the solution of an axisymmetric stationary problem of thermoelasticity for a multilayer foundation with a imperfect thermal contact between its layers.

1. Statement of the problem. We consider the axisymmetric thermoelastic deformation of a multilayer foundation. The foundation is presented by a package of n layers resting upon a rigid half-space. We enumerate the layers from top to bottom so that the very top layer receives the number 1, while the half-space has the number $n+1$. In every layer including the half-space, we introduce a local cylindrical coordinate system $O_j\rho z_j$, $j=1,\ldots,n+1$, as shown in Fig. 1. Each layer is characterized by the thickness h_j, the Poisson

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ratio ν_j, the Young modulus E_j, the shear modulus μ_j, the heat-conduction coefficient $\alpha_{T,j}$, and the coefficient of thermal expansion $k_{T,j}$, $j = 1,\dots,n$.

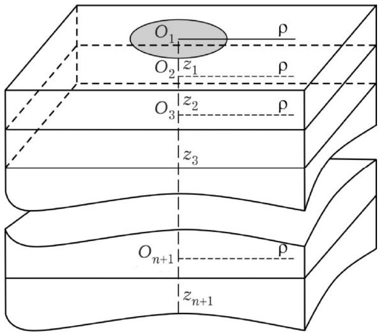

Fig. 1

The top limiting plane of the considered assembly is subjected to given temperature and force loadings. The interface between the n th layer and the rigid half-space is kept at a constant temperature that equals to zero. This fact is indicated in the boundary conditions of the following form:

$$\sigma_{z,1}(\rho,0) = \sigma(\rho), \quad \tau_{\rho z 1}(\rho,0) = \tau(\rho), \quad T_1(\rho,0) = f(\rho),$$

$$u_{z,n+1}(\rho,0) = 0, \quad u_{\rho,n+1}(\rho,0) = 0, \quad T_{n+1}(\rho,0) = 0, \tag{1}$$

where $\sigma(\rho)$, $\tau(\rho)$, and $f(\rho)$ are given functions.

The imperfect thermal contact is assumed on the layer interfaces [9] that implies the following interface conditions:

$$\sigma_{z,j+1}(\rho,0) = \sigma_{z,j}(\rho,h_j), \quad \tau_{\rho z,j+1}(\rho,0) = \tau_{\rho z,j}(\rho,h_j),$$

$$u_{z,j+1}(\rho,0) = u_{z,j}(\rho,h_j), \quad u_{\rho,j+1}(\rho,0) = u_{\rho,j}(\rho,h_j),$$

$$k_{T,j}\left.\frac{\partial T_j(\rho,z)}{\partial z}\right|_{z=h_j} = \frac{1}{R_j}\left(T_{j+1}(\rho,0) - T_j(\rho,h_j)\right),$$

$$k_{T,j+1}\left.\frac{\partial T_{j+1}(\rho,z)}{\partial z}\right|_{z=0} = k_{T,j}\left.\frac{\partial T_j(\rho,z)}{\partial z}\right|_{z=h_j}, \tag{2}$$

where R_j is the coefficient of thermal resistance, $j = 1,\dots,n$.

Our objective herein is to find the distributions of stresses, displacements, and temperature within the layers of the foundation considered.

2. Method of solution. The problem is to be solved by implementing the method of compliance functions [13] in the mapping domain of the Hankel transform:

$$\bar{v}^m(p) = \int_0^{+\infty} \rho v(\rho) J_m(p\rho)\, d\rho, \tag{3}$$

$$v(\rho) = \int_0^{+\infty} p \bar{v}^m(p) J_m(p\rho)\, dp, \tag{4}$$

where $\bar{v}^m(p)$ is the Hankel transforms of order m, J_m is the m-order Bessel function of the first kind.

The Hankel transforms of displacements, stresses, and temperature at the points of the j th layer can be represented by linear combinations of six auxiliary functions $\alpha_j = \alpha_j(p)$, $\beta_j = \beta_j(p)$, $\gamma_j = \gamma_j(p)$, $\delta_j = \delta_j(p)$, $\eta_j = \eta_j(p)$, $\varepsilon_j = \varepsilon_j(p)$, which are introduced through the following formulas [4]:

$$\alpha_j(p) = \bar{\sigma}_{z,j}(p,0), \quad \beta_j(p) = \mu_j p W_j(p,0), \quad \gamma_j(p) = \mu_j p U_j(p,0),$$

$$\delta_j(p) = \bar{\tau}_{\rho z,j}(p,0), \quad \eta_j(p) = \bar{T}_j(p,0), \quad \varepsilon_j(p) = \frac{1}{p}\left.\frac{d\bar{T}_j(p,z)}{dz}\right|_{z=0}, \tag{5}$$

where $\bar{\sigma}_{z,j}(p,z) = \bar{\sigma}^0_{z,j}(\rho,z)$, $\bar{\tau}_{\rho z,j}(p,z) = \bar{\tau}^1_{\rho z,j}(p,z)$, $U_j(p,z) = \bar{u}^1_{\rho,j}(p,z)$, $W_j(p,z) = \bar{u}^0_{z,j}(p,z)$, $\bar{T}_j(p,z) = \bar{T}^0_j(p,z)$.

The transforms of stresses, displacements and temperature can be given in the following form [4]:

$$
\begin{aligned}
2\mu p W(p,z) &= ((2-\omega)\sinh pz - \omega pz\cosh pz)\alpha + \\
&\quad + 2(\cosh pz - \omega pz\sinh pz)\beta + \\
&\quad + 2((1-\omega)\sinh pz - \omega pz\cosh pz)\gamma - \omega pz\delta\sinh pz + \\
&\quad + E\omega\alpha_T((\sinh pz + pz\cosh pz)\eta + pz\varepsilon\sinh pz), \\
2\mu p U(p,z) &= \omega pz\alpha\sinh pz + 2((1-\omega)\sinh pz + \omega pz\cosh pz)\beta + \\
&\quad + 2(\cosh pz + \omega pz\sinh pz)\gamma + \\
&\quad + ((2-\omega)\sinh pz + \omega pz\cosh pz)\delta - \\
&\quad - E\omega\alpha_T(pz\,\eta\sinh pz + (\sinh pz - pz\cosh pz))\varepsilon, \\
\bar{\sigma}_z(p,z) &= (\cosh pz - \omega pz\sinh pz)\alpha + 2\omega(\sinh pz - pz\cosh pz)\beta - \\
&\quad - 2\omega pz\gamma\sinh pz - ((1-\omega)\sinh pz + \omega pz\cosh pz)\delta + \\
&\quad + E\omega\alpha_T(pz\eta\sinh pz + (pz\cosh pz - \sinh pz)\varepsilon), \\
\bar{\tau}_{\rho z}(p,z) &= (\omega pz\cosh pz - (1-\omega)\sinh pz)\alpha + 2\omega pz\beta\sinh pz + \\
&\quad + 2\omega(\sinh pz + pz\cosh pz)\gamma + \\
&\quad + (\cosh pz + \omega pz\sinh pz)\delta - \\
&\quad - E\omega\alpha_T((\sinh pz + pz\cosh pz)\eta + pz\varepsilon\sinh pz), \\
\bar{T}(p,z) &= \eta\cosh pz + \varepsilon\sinh pz, \qquad (6)
\end{aligned}
$$

where $\omega = 0.5(1-\nu)^{-1}$ and subscript j is omitted for the sake of simplicity.

In such manner, the problem is reduced to finding the six auxiliary functions (5) for each layer of the foundation.

In order to construct the auxiliary functions of the neighboring layers, we implement the following recursive procedure. We apply the Hankel integral transform (3) of the corresponding order to conditions (2), which yields:

$$\bar{\sigma}_{z,j+1}(p,0) = \bar{\sigma}_{z,j}(p,h_j), \quad \bar{\tau}_{\rho z,j+1}(p,0) = \bar{\tau}_{\rho z,j}(p,h_j),$$

$$\mu_{j+1}W_{j+1}(p,0) = \mu_{j+1}W_j(p,h_j), \quad \mu_{j+1}S_{j+1}(p,0) = \mu_{j+1}S_j(p,h_j),$$

$$\bar{T}_{j+1}(p,0) = L_j \left.\frac{d\bar{T}_j(p,z)}{dz}\right|_{z=h_j} + \bar{T}_j(p,h_j),$$

$$k_{T,j+1}\frac{1}{p}\left.\frac{d\bar{T}_{j+1}(p,z)}{dz}\right|_{z=0} = k_{T,j}\frac{1}{p}\left.\frac{d\bar{T}_j(p,z)}{dz}\right|_{z=h_j},$$

where $L_j = R_j k_{T,j}$, $j = 1,\dots,n$.

By using formulas (5) with the left-hand sides of the latter relations and formulas (6) for $z = h_j$ with their right-hand sides, we can get the following expressions:

$$\boldsymbol{\alpha}_{j+1} = \mathbf{M}_{11,j}\boldsymbol{\alpha}_j + \mathbf{M}_{12,j}\boldsymbol{\beta}_j + \mathbf{M}_{13,j}\eta_j , \tag{7}$$

$$\boldsymbol{\beta}_{j+1} = \mathbf{M}_{21,j}\boldsymbol{\alpha}_j + \mathbf{M}_{22,j}\,\boldsymbol{\beta}_j + \mathbf{M}_{23,j}\eta_j , \tag{8}$$

$$\eta_{j+1} = (C_j + L_j p S_j)\eta_j + (S_j + L_j p C_j)\varepsilon_j , \tag{9}$$

$$\varepsilon_{j+1} = \Delta_j (S_j \eta_j + C_j \varepsilon_j) , \tag{10}$$

where $j = 1, \ldots, n$, and

$$\Delta_j = \frac{k_{T,j}}{k_{T,j+1}}, \quad \bar{\mu}_j = \frac{\mu_j}{\mu_{j+1}}, \quad p_j = p h_j ,$$

$$\boldsymbol{\alpha}_j = \begin{pmatrix} \alpha_j \\ \delta_j \end{pmatrix}, \quad \boldsymbol{\beta}_j = \begin{pmatrix} \beta_j \\ \gamma_j \end{pmatrix}, \quad S_j = \sinh p_j, \quad C_j = \cosh p_j ,$$

$$\mathbf{M}_{11,j} = \begin{pmatrix} C_j - \omega_j p_j S_j & -(1-\omega_j) S_j - \omega_j p_j C_j \\ -(1-\omega_j) S_j + \omega_j p_j C_j & C_j + \omega_j p_j S_j \end{pmatrix},$$

$$\mathbf{M}_{12,j} = \begin{pmatrix} 2\omega_j (S_j - p_j C_j) & -2\omega_j p_j S_j \\ 2\omega_j p_j S_j & 2\omega_j (S_j + p_j C_j) \end{pmatrix},$$

$$\mathbf{M}_{13,j} = \alpha_{T,j} E_j \omega_j \begin{pmatrix} p_j S_j + r_j (S_j - p_j C_j) \\ -(S_j + p_j C_j) + r_j p_j S_j \end{pmatrix},$$

$$\mathbf{M}_{21,j} = \frac{1}{2\bar{\mu}_j} \begin{pmatrix} (2-\omega_j) S_j - \omega_j p_j C_j & -\omega_j p_j S_j \\ \omega_j p_j S_j & (2-\omega_j) S_j + \omega_j p_j C_j \end{pmatrix},$$

$$\mathbf{M}_{22,j} = \frac{1}{\bar{\mu}_j} \begin{pmatrix} -\omega_j p_j S_j + C_j & (1-\omega_j) S_j - \omega_j p_j C_j \\ (1-\omega_j) S_j + \omega_j p_j C_j & \omega_j p_j S_j + C_j \end{pmatrix},$$

$$\mathbf{M}_{23,j} = \frac{\alpha_{T,j} E_j \omega_j}{2\bar{\mu}_j} \begin{pmatrix} S_j + p_j C_j - r_j p_j S_j \\ -p_j S_j - r_j (S_j - p_j C_j) \end{pmatrix}.$$

By following the strategy presented in [14], it can be proved that

$$\varepsilon_j = -r_j \eta_j, \quad \boldsymbol{\beta}_j = \mathbf{A}_j \boldsymbol{\alpha}_j + \mathbf{B}_j \eta_j , \tag{11}$$

where matrices $\mathbf{A}_j$ and $\mathbf{B}_j$ are the compliance matrices. The elements of these matrices along with functions r_j are called the compliance functions of j th layer for the foundation.

By making use of the interface conditions for the n th layer and the half-space and relations (7)–(11), the recursions for the compliance functions can be given in the following form:

$$r_n = \coth p_n, \quad \mathbf{A}_n = -\mathbf{M}_{22,n}^{-1}\mathbf{M}_{21,n}, \quad \mathbf{B}_n = -\mathbf{M}_{22,n}^{-1}\mathbf{M}_{23,n}$$

$$r_j = \frac{\Delta_j S_j + r_{j+1}(C_j + L_j p S_j)}{\Delta_j C_j + r_{j+1}(S_j + L_j p C_j)} ,$$

$$\mathbf{A}_j = (\mathbf{A}_{j+1}\mathbf{M}_{12,j} - \mathbf{M}_{22,j})^{-1}(\mathbf{M}_{21,j} - \mathbf{A}_{j+1}\mathbf{M}_{11,j}) ,$$

$$\mathbf{B}_j = (\mathbf{A}_{j+1}\mathbf{M}_{12,j} - \mathbf{M}_{22,j})^{-1}(\mathbf{M}_{23,j} - \mathbf{A}_{j+1}\mathbf{M}_{13,j} -$$

$$-\left(C_j + L_j p S_j - r_j\left(S_j + L_j p C_j\right)\right)\mathbf{B}_{j+1}\Big),$$

$$j = 1, \ldots, n-1. \tag{12}$$

By substituting the right-hand sides of relations (11) into (7) and (9), we can derive the following formulas:

$$\boldsymbol{\alpha}_{j+1} = \left(\mathbf{M}_{11,j} + \mathbf{M}_{12,j}\mathbf{A}_j\right)\boldsymbol{\alpha}_j + \left(\mathbf{M}_{12,j}\mathbf{B}_j + \mathbf{M}_{13,j}\right)\boldsymbol{\eta}_j,$$

$$\boldsymbol{\eta}_{j+1} = \left(C_j + L_j p S_j - r_j\left(S_j + L_j p C_j\right)\right)\boldsymbol{\eta}_j. \tag{13}$$

Hence, the proposed solution scheme can be verbalized as follows: first step is to find the compliance functions by means of formulas (12); next step is to determine the functions α_1, δ_1, η_1 from the boundary conditions (1); then it is to find all the auxiliary functions by means of (13) and (11); next, to substitute the obtained auxiliary functions into the formulas for the transforms of stresses, displacements and temperature (6) and, finally, to use the inverse Hankel transform (4) of corresponding order to compute the stresses, displacements, and temperature in the physical domain.

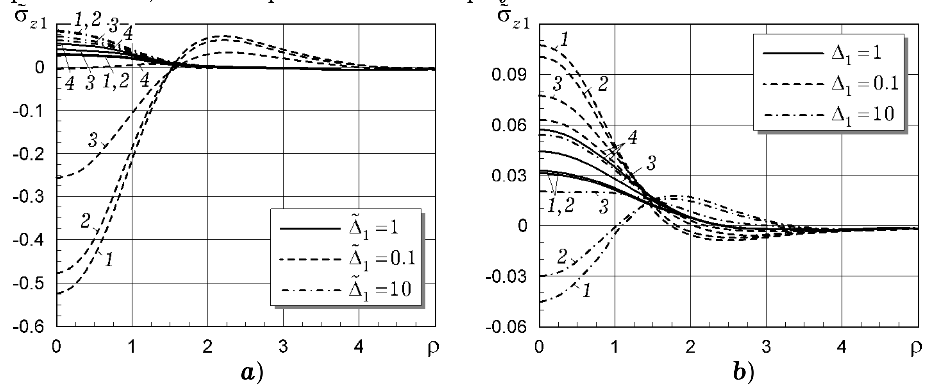

Fig. 2. The normal stress $\tilde{\sigma}_{z,1}(\rho, h_1)$ on the lower boundary of the upper layer in the two-layer foundation for $\Delta_1 = 1$ and different coefficients of thermal expansion (***a***) and for $\tilde{\Delta}_1 = 1$ and different thermal conductivities (***b***) under the perfect thermal contact (curves *1*), at $R_1 = 0.1$ (curves *2*), at $R_1 = 1$ (curves *3*), and at $R_1 = 10$ (curves *4*).

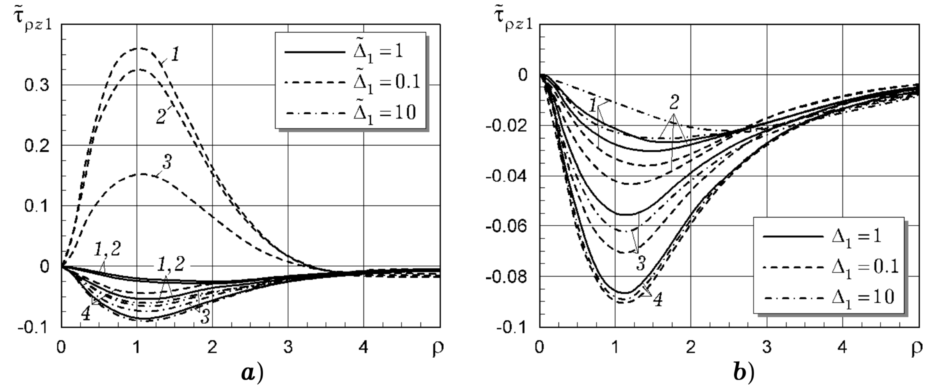

Fig. 3. The tangential stress $\tilde{\tau}_{\rho z,1}(\rho, h_1)$ on the lower boundary of the upper layer in the two-layer foundation for $\Delta_1 = 1$ and different coefficients of thermal expansion (***a***) and for $\tilde{\Delta}_1 = 1$ and different thermal conductivities (***b***) under the perfect thermal contact (curves *1*), at $R_1 = 0.1$ (curves *2*), at $R_1 = 1$ (curves *3*), and at $R_1 = 10$ (curves *4*).

It is worth noting that if $R_j \to 0$ (i.e., the thermal contact between layers is perfect), then the formulas (13) coincide with the ones constructed in [4].

3. Numerical results. We consider a two layer foundation, where $h_1 = h_2 = h$, $E_1 = E_2 = E$, $\omega_1 = \omega_2 = 0.8$. The boundary conditions (1) are given as follows:

$$T_1(\rho, 0) = \begin{cases} T_0, & \rho < h, \\ 0, & \rho \ge h, \end{cases} \qquad \sigma_{z,1}(\rho, 0) = 0, \qquad \tau_{\rho z,1}(\rho, 0) = 0 .$$

We present plots for: $\tilde{\sigma}_{z,1}(\rho, h_1) = \dfrac{\sigma_{z,1}(\rho, h_1)}{\alpha_{T,1} T_0 E_1}$ in Fig. 2, $\tilde{\tau}_{\rho z,1}(\rho, h_1) = \dfrac{\overline{\tau}_{\rho z,1}(\rho, h_1)}{\alpha_{T,1} T_0 E_1}$ in Fig. 3, and $\tilde{T}_1(\rho, h_1) = \dfrac{T_1(\rho, h_1)}{T_0}$ in Fig. 4 at the points of the lower boundary of the upper layer in the two-layer foundation. The results allow for drawing the following conclusions:

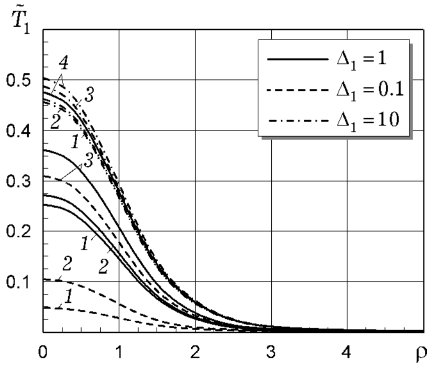

Fig. 4. The temperature $\tilde{T}_1(\rho, h_1)$ on the lower boundary of the upper layer in the two layer foundation under the perfect thermal contact (curves *1*), at $R_1 = 0.1$ (curves *2*), at $R_1 = 1$ (curves *3*); and at $R_1 = 10$ (curves *4*).

- the increment of thermal resistance coefficient R_1 increases $\tilde{T}_1(\rho, h_1)$ and decreases the tangential stress for the foundations of all considered kinds;
- the increment of R_1 decreases the normal stress for the foundations $\Delta_1 = 1$, $\tilde{\Delta}_1 = 10$ and $\Delta_1 = 0.1$, $\tilde{\Delta}_1 = 1$; for the considered foundations of all other kinds, the normal stress increases;
- the weakest influence on the temperature distribution is observed for the foundation with the coefficient of thermal conductivity of the upper layer being greater than the one for the lower layer;
- the most significant influence of the coefficient of thermal resistance on the distribution of normal and tangential stresses is observed for the foundations with the coefficient of thermal expansion of the upper layer being smaller than the one of the lower layer (for fixed values of other parameters);
- the normal stress is compressive for the foundations $\Delta_1 = 1$, $\tilde{\Delta}_1 = 0.1$ and $\Delta_1 = 10$, $\tilde{\Delta}_1 = 1$ (with exception for when $R_1 = 10$).

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ОСЕСИМЕТРИЧНА ТЕРМОПРУЖНА ДЕФОРМАЦІЯ БАГАТОШАРОВОЇ ОСНОВИ З НЕІДЕАЛЬНИМ ТЕПЛОВИМ КОНТАКТОМ МІЖ ШАРАМИ

Побудовано розв'язок стаціонарної осесиметричної задачі термопружності для багатошарової основи з неідеальним тепловим контактом між шарами. Для розв'язання задачі використано метод функцій податливості в просторі перетворення Ганкеля. Побудовано рекурентні співвідношення, які пов'язують допоміжні функції, та рекурентні формули для знаходження функцій податливості сусідніх шарів основи. Проаналізовано вплив коефіцієнта теплового опору на розподіл температури, нормальних та дотичних напружень у точках нижньої межі верхнього шару двошарової основи під дією теплового навантаження.

Ключові слова: *багатошарова основа, функції податливості, перетворення Ганкеля, напруження, температура.*

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