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MODELING OF THERMOELASTOPLASTIC DEFORMATION OF REINFORCED PLATES. 1. STRUCTURAL MODEL OF THE REINFORCED MEDIUM

A numerical-analytical structural model of thermoelastoplastic deformation of a composite material cross-reinforced with fibers in arbitrary directions is developed on the basis of the time steps algorithm. The materials of the constituents of the composition are isotropic; their plastic deformation is described by the theory of flow with isotropic hardening, taking into account the dependence of the loading function on temperature. Conditions, determining thermoelastic deformation, unloading, neutral and active loading of the thermosensitive constituents of the composition are obtained. The coupled problems of the thermophysical and mechanical behavior of the reinforced material are considered. Structural relationships that are necessary for solving the thermophysical component of the investigated problem are presented. The developed structural model is focused on the use of explicit numerical integration schemes for both elastoplastic and thermophysical problems.

Key words: *fiber reinforcement, structural models, thermoelastoplasticity, thermal sensitivity, flow theory, effective relations, step-by-step algorithm, explicit numerical schemes.*

Thin-walled structural elements made of composite materials (CM) are widely used in engineering applications [9, 13, 18, 20, 22, 23, 26]. Often, CM products are subjected to high-intensity both force and thermal loading [9, 13], which can deform plastically [21, 23] the materials of the composition. Consequently, the modeling of thermoelastoplastic deformation of CM structures is an urgent problem that is now at the stage of formation [21, 23]. The elastic-plastic behavior of dispersion-hardened CMs was modeled in [19, 27], a similar behavior of fibrous media at large and small deformations of the components of the composition was modeled in [3, 17]. However, it is known, that under intense thermal action on many modern CM structures [9], the materials of the composition change their mechanical properties [4, 5]. This circumstance can significantly affect the inelastic behavior of reinforced thin-walled elements under high-intensity loads. Structural models of CM, with regard to the thermal effect on the elastoplastic deformation of the components of the composition (within the framework of the theory of flow), have not yet been constructed. In this case, it is necessary to take into account the connectedness of the temperature and elastoplastic problems.

To take into account the poor resistance of thin-walled CM structures to transverse shear, the nonclassical theories of Reissner [1, 6, 13, 25], Reddy [24], or Ambartsumian [2, 17] are usually used. Less commonly are used theories based on the broken line hypothesis [12]. Numerical solutions of physically and geometrically nonlinear problems of the dynamics of thin-walled structural elements, as a rule, are constructed using explicit schemes [1, 10, 17].

According to the above, this work is devoted to modeling thermoelastoplastic deformation of flexible reinforced plates taking into account their poor resistance to transverse shear. The numerical solution of the coupled thermomechanical problem, arising in this case, is supposed to be constructed using explicit step-by-step schemes.

1. Numerical and analytical modeling of thermoelastoplastic deformation of CM. As in [1, 7, 10, 17], we assume that small strains ε_{ij} of the isotro-

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pic component of the composition can be represented as a sum of elastic e_{ij}, incompressible plastic p_{ij} and temperature components $\delta_{ij}\varepsilon_\Theta$:

$$\varepsilon_{ij} = e_{ij} + p_{ij} + \delta_{ij}\varepsilon_\Theta, \quad i,j = 1,2,3, \quad p_{ii} = 0, \quad \varepsilon_\Theta = \int_{t_0}^{t} \alpha\dot{\Theta}\, dt, \tag{1}$$

where Θ is the temperature of the material; α denotes coefficient of linear thermal expansion; t_0 is the initial moment of time t; δ_{ij} is the Kronecker symbol; point denotes a derivative with respect to time t.

Plastic flow of a material is associated with a loading surface $f = 0$ corresponding to the von Mises yield condition [7, 10]:

$$f(T,\chi,\Theta) \equiv T^2 - \tau_s^2(\chi,\Theta) = 0, \tag{2}$$

where

$$T = \sqrt{\frac{1}{2}s_{ij}s_{ij}}, \qquad \chi = \int_{t_0}^{t} \sqrt{2\dot{p}_{ij}\dot{p}_{ij}}\, dt, \qquad s_{ij} = \sigma_{ij} - \delta_{ij}\sigma_0,$$

$$\sigma_0 = \frac{1}{3}\sigma_{\ell\ell}, \qquad i,j = 1,2,3, \tag{3}$$

σ_{ij} are components of stress tensor; χ is the Odqvist parameter; τ_s denotes yield point at pure shear. The initial loading surface $T = \tau_s(\Theta) \equiv \tau_s(0,\Theta)$ is the usual temperature-dependent Θ yield point [4, 5]. (In this section, unless otherwise stated, summation is performed over repeated indices from 1 to 3.)

Based on the associated law of plastic flow, in view of (2) and (3) under active loading, we obtain [7, 10]

$$\dot{p}_{ij} = \frac{s_{ij}}{2\tau_s^2(\chi,\Theta)} s_{m\ell}\dot{p}_{m\ell}, \qquad i,j = 1,2,3. \tag{4}$$

Within the framework of the theory of plasticity with isotropic hardening, the power of plastic deformations $W_p = s_{m\ell}\dot{p}_{m\ell}$ is expressed in terms of the power of shape deformations $W = s_{m\ell}\dot{\bar{\varepsilon}}_{m\ell}$, i.e. assume that [10]

$$W_p = æW, \qquad s_{m\ell}\dot{p}_{m\ell} = æ s_{m\ell}\dot{\bar{\varepsilon}}_{m\ell} = æ s_{m\ell}\dot{\varepsilon}_{m\ell}, \tag{5}$$

where

$$\bar{\varepsilon}_{m\ell} = \varepsilon_{m\ell} - \delta_{m\ell}\varepsilon_0, \qquad \varepsilon_0 = \frac{1}{3}\varepsilon_{ii}, \qquad m,\ell = 1,2,3, \tag{6}$$

$æ = æ(\chi)$ is proportionality factor, which depends on the hardening parameter χ and is expressed in terms of the shear modulus $\tilde{G}$ and the tangent modulus of the material in the pure shear diagram. Relation (5) is considered to be true for any type of stress-strain state (SSS) of a material that is not sensitive to temperature changes. In particular, for a pure shear, the energy relation (5), with regard to (3) and (6), takes the form:

$$\tau\dot{\gamma}_p = æ(\chi)\tau\dot{\gamma}, \qquad \dot{\gamma}_p = æ(\chi)\dot{\gamma}, \tag{7}$$

where

$$\chi = \int_{t_0}^{t} \dot{\gamma}_p\, dt = \int_0^{\gamma_p} d\gamma_p = \gamma_p, \tag{8}$$

and τ is a shear stress at pure shear; γ denotes complete angular deformation; γ_p is the plastic component of the quantity γ. Relation (7) is true when equality (2) is satisfied, where $T = \tau$ and $\partial\tau_s / \partial\Theta \equiv 0$.

Let us now assume that the diagram of material deformation at pure shear depends on temperature, i.e. according to (2) we have

$$\tau = \tau_s(\chi, \Theta) = \tau_s(\gamma_p, \Theta), \tag{9}$$

where χ is determined by expression (8). Using (9), we obtain an energy relation similar to (7). Differentiating (9) in time, under active loading we will have

$$\dot{\tau} = \tau_\chi \dot{\chi} + \tau_\Theta \dot{\Theta} = \tau_\chi \dot{\gamma}_p + \tau_\Theta \dot{\Theta}, \tag{10}$$

where

$$\tau_\chi \equiv \frac{\partial \tau_s}{\partial \chi} = \frac{\partial \tau_s}{\partial \gamma_p} \equiv \bar{G}, \qquad \tau_\Theta \equiv \frac{\partial \tau_s}{\partial \Theta}, \tag{11}$$

$\tau_\chi = \tau_\chi(\gamma_p, \Theta)$, $\tau_\Theta = \tau_\Theta(\gamma_p, \Theta)$ are functions known from experiment; $\tau_\chi \equiv \bar{G}$ is the tangent modulus in the diagram $\tau \sim \gamma_p$ at constant temperature Θ.

On the other hand, the change of shear stress at pure shear, taking into account the thermal sensitivity of the material, can be expressed through Hooke's law ($\tau = G(\Theta)\gamma_e$) [7]:

$$\dot{\tau} = G\dot{\gamma}_e + G_\Theta \gamma_e \dot{\Theta} = G\dot{\gamma}_e + G^{-1} G_\Theta \tau \dot{\Theta}, \tag{12}$$

where

$$G_\Theta(\Theta) \equiv \frac{dG(\Theta)}{d\Theta}, \qquad \gamma_e = \gamma - \gamma_p. \tag{13}$$

After substitution the expression (12) into the left-hand side of equality (10) and in view of the second relation (13), we get

$$\dot{\gamma}_p = \frac{1}{G + \bar{G}} \left[G\dot{\gamma} + \frac{1}{G}(\tau G_\Theta - \tau_\Theta G)\dot{\Theta} \right].$$

After multiplying this equality by τ, we have

$$\tau\dot{\gamma}_p = \frac{1}{G + \bar{G}} \left[G\tau\dot{\gamma} + \frac{1}{G}(\tau^2 G_\Theta - \tau\tau_\Theta G)\dot{\Theta} \right]. \tag{14}$$

If the material is insensitive to temperature changes ($\tau_\Theta \equiv 0$, $\tau = \tau_s(\chi) = \tau_s(\gamma_p)$ and $G_\Theta \equiv 0$), then relation (7) follows from (14) with $æ = G/(G + \bar{G})$. We assume that the energy relation (14) must be true for any SSS. Therefore we obtain (see (2) and (3))

$$s_{m\ell}\dot{p}_{m\ell} = \frac{1}{G + \bar{G}} \left[G s_{m\ell} \dot{\bar{\varepsilon}}_{m\ell} + \frac{1}{G} T(TG_\Theta - \tau_\Theta G)\dot{\Theta} \right] =$$

$$= \frac{1}{G + \bar{G}} \left[G s_{ml} \dot{\varepsilon}_{ml} + \frac{1}{G} \tau_s(\tau_s G_\Theta - \tau_\Theta G)\dot{\Theta} \right]. \tag{15}$$

Equality (15) generalizes relation (5). Let us substitute (15) into the right-hand side of (4), and on the left-hand side of (4) we express plastic deformations in terms of elastic ones (see (1) and (6)) and use Hooke's law for a thermosensitive material: $\bar{e}_{ij} = s_{ij}\dfrac{1}{2G}$, whence $\dot{\bar{e}}_{ij} = \left(s_{ij} - s_{ij} G_\Theta \dot{\Theta} \dfrac{1}{G} \right) \dfrac{1}{2G}$, where $\bar{e}_{ij} = e_{ij} - \delta_{ij}\varepsilon_0$. Then, in view of (2), we have

$$\dot{\bar{\varepsilon}}_{ij} = \frac{\dot{s}_{ij}}{2G} - \frac{s_{ij} G_\Theta}{2G^2}\dot{\Theta} + \frac{s_{ij}}{2(G + \bar{G})\tau_s^2} \left[G s_{m\ell} \dot{\varepsilon}_{m\ell} + \frac{\tau_s}{G}(\tau_s G_\Theta - \tau_\Theta G)\dot{\Theta} \right],$$

$$i, j = 1, 2, 3,$$

from which follows

$$\dot{s}_{ij} = 2G\dot{\varepsilon}_{ij} - GAs_{ij}s_{m\ell}\dot{\varepsilon}_{m\ell} + \frac{1}{G}[G_\Theta - \tau_s(\tau_s G_\Theta - \tau_\Theta G)A]s_{ij}\dot{\Theta}, \quad i,j = 1,2,3, \tag{16}$$

where

$$A = \frac{Gc}{(G + \bar{G})\tau_s^2(\chi, \Theta)}, \tag{17}$$

c is switching parameter: $c = 0$ under thermoelastic deformation, unloading and neutral loading of the material, and $c = 1$ under active loading and thermoelastoplastic deformation.

Let us determine the conditions under which $c = 0$ and $c = 1$. According to (2), thermoelastic deformation is determined by the condition $f(T, \chi, \Theta) < 0$, i.e.

$$T < \tau_s(\chi, \Theta). \tag{18}$$

The beginning of unloading from the loading surface is characterized by the conditions

$$f(T, \chi, \Theta) = 0, \qquad \dot{f} < 0. \tag{19}$$

Thus, using (2) and in view of (3) and (11), we calculate the time derivative of the loading function

$$\dot{f} = \frac{\partial f}{\partial s_{ij}}\dot{s}_{ij} + \frac{\partial f}{\partial \chi}\dot{\chi} + \frac{\partial f}{\partial \Theta}\dot{\Theta} = s_{ij}\dot{s}_{ij} - 2\tau_s(\bar{G}\dot{\chi} + \tau_\Theta\dot{\Theta}). \tag{20}$$

According to (3), we have $\dot{\chi} = \sqrt{2\dot{p}_{ij}\dot{p}_{ij}} \geq 0$, therefore during unloading from the loading surface ($f = 0$) there is no increase in plastic deformations ($\dot{\chi} = 0$). Therefore, taking into account $\dot{f} < 0$ (see (19)), from (20) we obtain the unloading conditions

$$T = \tau_s(\chi, \Theta), \qquad s_{ij}\dot{s}_{ij} - 2\tau_s\tau_\Theta\dot{\Theta} < 0. \tag{21}$$

Under the conditions of neutral loading, we have $f = 0$ and $\dot{f} = 0$, while there is no increase in plastic deformations ($\dot{\chi} = 0$), therefore, from (20) follow the similar to (21) conditions for neutral loading, where the “less” sign must be replaced by the “equal” sign. Under conditions of active loading during plastic deformation the equalities $f = 0$ and $\dot{f} = 0$ are still satisfied, but at the same time $\dot{\chi} > 0$. Taking into account that according to the experimental data [4] $\tau_s > 0$ and $\bar{G} > 0$, we obtain from (2) and (20) the conditions for active loading

$$T = \tau_s(\chi, \Theta), \qquad s_{ij}\dot{s}_{ij} - 2\tau_s\tau_\Theta\dot{\Theta} > 0. \tag{22}$$

The inequality in (22) with regard to (3) can be rewritten as

$$(T^2)^{\cdot} - 2\tau_s\tau_\Theta\dot{\Theta} > 0 \qquad \Rightarrow \qquad 2T\dot{T} - 2\tau_s\tau_\Theta\dot{\Theta} > 0.$$

Therefore, for $T = \tau_s > 0$ (see (2)) conditions (22) can be written as follows:

$$T = \tau_s(\chi, \Theta), \qquad \dot{T} - \tau_\Theta\dot{\Theta} > 0. \tag{23}$$

It is known from experiments [4], that usually (except for steels in the temperature range from 0 to $110 \div 120\,^\circ\mathrm{C}$ [5]) $\tau_\Theta < 0$ (see (11)). Therefore from (23) we obtain: ***1***) in the isothermal case ($\dot{\Theta} \equiv 0$) an plastic deformations increment is possible only with an increase in the intensity of shear stresses

($\dot{\Theta} > 0$); ***2***) at a constant T ($\dot{T} \equiv 0$) an plastic deformations increment is possible only with an increase in temperature ($\dot{\Theta} > 0$). These results are in complete agreement with the known experimental data [4, 5].

Relations (21) and (22) can be written in a different form. Let us substitute expression (16) into (21). Under thermoelastic deformation, unloading and neutral loading, we have $c = 0$ and $A = 0$ (see (17)). Therefore, from (21) we obtain

$$Gs_{ij}\dot{\varepsilon}_{ij} + \tau_s\left(\tau_s G_\Theta \frac{1}{G} - \tau_\Theta\right)\dot{\Theta} \le 0 \qquad (24)$$

Under active loading, from Hooke's law for a thermosensitive material, in view of (4), we have

$$\dot{s}_{ij} = 2G\dot{\bar{e}}_{ij} + s_{ij}G_\Theta \frac{1}{G}\dot{\Theta} = 2G(\dot{\bar{\varepsilon}}_{ij} - \dot{p}_{ij}) + s_{ij}G_\Theta \frac{1}{G}\dot{\Theta} =$$

$$= 2G\dot{\bar{\varepsilon}}_{ij} + s_{ij}G_\Theta \frac{1}{G}\dot{\Theta} - G\frac{1}{\tau_s^2}s_{ij}s_{m\ell}\dot{p}_{m\ell}, \qquad i, j = 1, 2, 3. \qquad (25)$$

According to Drucker's postulate, the inequality $s_{m\ell}\dot{p}_{m\ell} > 0$ holds. Therefore, after substituting (25) into (22) and with regard to $G > 0$, we obtain the conditions for active loading in another form:

$$T = \tau_s(\chi, \Theta), \qquad Gs_{ij}\dot{\varepsilon}_{ij} + \tau_s\left(\tau_s G_\Theta \frac{1}{G} - \tau_\Theta\right)\dot{\Theta} > 0. \qquad (26)$$

Taking into account (1), (3), and (6), relations (16) can be rewritten as

$$\dot{\sigma}_{ij} = 2G\dot{\varepsilon}_{ij} + \lambda\delta_{ij}\dot{\varepsilon}_{\ell\ell} - GAs_{ij}s_{m\ell}\dot{\varepsilon}_{m\ell} + \left\{\delta_{ij}\left[K_\Theta \frac{1}{3K}\sigma_{\ell\ell} - 3K\alpha\right] + \right.$$

$$\left. + \frac{1}{G}[G_\Theta - \tau_s(\tau_s G_\Theta - \tau_\Theta G)A]s_{ij}\right\}\dot{\Theta}, \qquad i, j = 1, 2, 3, \qquad (27)$$

where $K_\Theta(\Theta) \equiv \dfrac{dK(\Theta)}{d\Theta}$; $K = K(\Theta)$ denotes bulk modulus of elasticity; $\lambda = \lambda(\Theta)$ is the Lame parameter; coefficient A was calculated by formula (17), in which the switching parameter according to (2), (24) and (26) is expressed as

$$c = 0 \quad \text{if} \quad T < \tau_s(\chi, \Theta) \quad \text{or} \quad T = \tau_s(\chi, \Theta),$$

$$\text{and} \quad Gs_{ij}\dot{\varepsilon}_{ij} + \tau_s\left(\tau_s G_\Theta \frac{1}{G} - \tau_\Theta\right)\dot{\Theta} \le 0,$$

$$c = 1 \quad \text{if} \quad T = \tau_s(\chi, \Theta) \quad \text{and} \quad Gs_{ij}\dot{\varepsilon}_{ij} + \tau_s\left(\tau_s G_\Theta \frac{1}{G} - \tau_\Theta\right)\dot{\Theta} > 0, \qquad (28)$$

In the case of isothermal deformation ($\dot{\Theta} = 0$), relations (27), (28) are reduced to the corresponding relations given in [10], where the thermal effect does not take into account.

As in [7, 17], for the convenience of further presentation, the governing equations for the k-th material of the composition can be written in matrix form (see (17), (27), (28)):

$$\dot{\boldsymbol{\sigma}}_k = \mathbf{Z}_k\dot{\boldsymbol{\varepsilon}}_k + \boldsymbol{\beta}_k\dot{\Theta}, \qquad \mathbf{Z}_k = \bar{\mathbf{Z}}_k - G^{(k)}\bar{\bar{\mathbf{Z}}}_k, \qquad k = 0, 1, 2, \ldots, N, \qquad (29)$$

where

$$\boldsymbol{\sigma}_k = \{\sigma_1^{(k)}\ \sigma_2^{(k)}\ \sigma_3^{(k)}\ \sigma_4^{(k)}\ \sigma_5^{(k)}\ \sigma_6^{(k)}\}^\top \equiv \{\sigma_{11}^{(k)}\ \sigma_{22}^{(k)}\ \sigma_{33}^{(k)}\ \sigma_{23}^{(k)}\ \sigma_{31}^{(k)}\ \sigma_{12}^{(k)}\}^\top,$$

$$\boldsymbol{\varepsilon}_k = \{\varepsilon_1^{(k)}\ \varepsilon_2^{(k)}\ \varepsilon_3^{(k)}\ \varepsilon_4^{(k)}\ \varepsilon_5^{(k)}\ \varepsilon_6^{(k)}\}^\top \equiv \{\varepsilon_{11}^{(k)}\ \varepsilon_{22}^{(k)}\ \varepsilon_{33}^{(k)}\ 2\varepsilon_{23}^{(k)}\ 2\varepsilon_{31}^{(k)}\ 2\varepsilon_{12}^{(k)}\}^\top, \tag{30}$$

$\bar{\mathbf{Z}}_k = (\bar{z}_{ij}^{(k)})$, $\bar{\bar{\mathbf{Z}}}_k = (\bar{\bar{z}}_{ij}^{(k)})$ are symmetric 6×6-matrices, $\boldsymbol{\beta}_k = \{\beta_i^{(k)}\}$ is a six-component column vector whose nonzero elements are defined as:

$$\bar{z}_{ij}^{(k)} = 2\delta_{ij}G^{(k)} + \lambda^{(k)}, \quad \bar{z}_{\ell\ell}^{(k)} = G^{(k)},$$

$$\beta_i^{(k)} = \frac{K_\Theta^{(k)}}{3K^{(k)}}\sum_{m=1}^{3}\sigma_m^{(k)} - 3K^{(k)}\alpha^{(k)} + \frac{s_i^{(k)}}{G^{(k)}}\left[G_\Theta^{(k)} - \tau_s^{(k)}(\tau_s^{(k)}G_\Theta^{(k)} - \tau_\Theta^{(k)}G^{(k)})A^{(k)}\right],$$

$$\beta_\ell^{(k)} = \frac{s_\ell^{(k)}}{G^{(k)}}\left[G_\Theta^{(k)} - \tau_s^{(k)}(\tau_s^{(k)}G_\Theta^{(k)} - \tau_\Theta^{(k)}G^{(k)})A^{(k)}\right], \quad i,j = 1,2,3, \quad \ell = 4,5,6,$$

$$\bar{\bar{z}}_{ij}^{(k)} = A^{(k)}s_i^{(k)}s_j^{(k)}, \quad i,j = 1,2,\ldots,6, \quad A^{(k)} = \frac{c^{(k)}\tau_s^{(k)2}(\chi^{(k)},\Theta)}{(1+\text{æ}^{(k)})}, \quad \text{æ}^{(k)} = \frac{\bar{G}^{(k)}}{G^{(k)}},$$

$$G^{(k)} = \frac{E^{(k)}}{2(1+\nu^{(k)})}, \quad K^{(k)} = \frac{E^{(k)}}{3(1-2\nu^{(k)})}, \quad \lambda^{(k)} = \frac{\nu^{(k)}E^{(k)}}{(1+\nu^{(k)})(1-2\nu^{(k)})},$$

$$c^{(k)} = \begin{cases} 0 & \text{if} \quad T^{(k)} < \tau_s^{(k)} \quad \text{or} \quad T^{(k)} = \tau_s^{(k)}, \quad W^{(k)} \le 0, \\ 1 & \text{if} \quad T^{(k)} = \tau_s^{(k)}, \quad W^{(k)} > 0, \end{cases}$$

$$W^{(k)} \equiv G^{(k)}\mathbf{s}_k^\top\dot{\boldsymbol{\varepsilon}}_k + \frac{\tau_s^{(k)}}{G^{(k)}}(\tau_s^{(k)}G_\Theta^{(k)} - \tau_\Theta^{(k)}G^{(k)})\dot{\Theta}, \quad T^{(k)2} = \frac{1}{2}\sum_{i=1}^{3}s_i^{(k)2} + \sum_{i=4}^{6}s_i^{(k)2},$$

$$\mathbf{s}_k = \{s_1^{(k)}\ s_2^{(k)}\ s_3^{(k)}\ s_4^{(k)}\ s_5^{(k)}\ s_6^{(k)}\}^\top \equiv \{s_{11}^{(k)}\ s_{22}^{(k)}\ s_{33}^{(k)}\ s_{23}^{(k)}\ s_{31}^{(k)}\ s_{12}^{(k)}\}^\top,$$

$$0 \le k \le N, \tag{31}$$

where N is the number of families of reinforcing fibers; $E^{(k)} = E^{(k)}(\Theta)$, $\nu^{(k)} = \nu^{(k)}(\Theta)$ denote Young's modulus and Poisson's ratio of the k-th component of the composition ($k = 0$ is a binder, $k = 1,2,\ldots,N$, denote reinforcement of the k-th family); superscript «$\top$» denotes the operation of transposition. The remaining quantities in (31) have the same meaning after dropping the index k. There is no summation over the repeated index ℓ in equalities (31).

As noted in the Introduction, the solution of the considered problem is supposed to be constructed using explicit numerical schemes [7, 10, 17], therefore, the values of unknown functions will be calculated at discrete time moments $t_{n+1} = t_n + \Delta$, $n = 0,1,2,\ldots$, where $\Delta = \text{const} > 0$ is the time step. Thus, we assume that for $t = t_{n-1}, t_n$, the values of the following quantities are already known:

$$\overset{n-1}{\dot{\Theta}}(\mathbf{r}) \equiv \dot{\Theta}(t_{n-1},\mathbf{r}), \quad \overset{m}{\Theta}(\mathbf{r}) \equiv \Theta(t_m,\mathbf{r}), \quad m = n-1,n, \quad \mathbf{r} = \{x_1,x_2,x_3\}, \tag{32}$$

where $\mathbf{r}$ is the location vector. We transform the second term on the right-hand side of (29) using the trapezoid formula, which has according to [8] the second order of accuracy relative to Δ:

$$\overset{n}{\Theta} - \overset{n-1}{\Theta} = \Delta(\overset{n}{\dot{\Theta}} + \overset{n-1}{\dot{\Theta}})/2,$$

whence follows

$$\overset{n}{\dot{\Theta}} = 2(\overset{n}{\Theta} - \overset{n-1/2}{\Theta}) / \Delta , \tag{33}$$

where

$$\overset{n-1/2}{\Theta} \equiv \overset{n-1}{\Theta} + \Delta \overset{n-1}{\dot{\Theta}} / 2 . \tag{34}$$

From equality (34), in view of assumptions (32), we obtain that the quantities $\overset{n-1/2}{\Theta}$ and $\overset{n}{\Theta}$ on the right-hand side of relation (33) are already known at the current time t_n.

Let us substitute the expression (33) into the right-hand side of equality (29). Then, with regard to the notation similar to (32), for $t = t_n$ we have

$$\overset{n}{\dot{\boldsymbol{\sigma}}}_k = \overset{n}{\mathbf{Z}}_k \overset{n}{\dot{\boldsymbol{\varepsilon}}}_k + \overset{n}{\mathbf{p}}_k , \qquad 0 \le k \le N , \tag{35}$$

where

$$\overset{n}{\mathbf{p}}_k \equiv \frac{2}{\Delta} (\overset{n}{\Theta} - \overset{n-1/2}{\Theta}) \overset{n}{\boldsymbol{\beta}}_k , \qquad 0 \le k \le N . \tag{36}$$

Matrix equality (35) is the governing equation for the thermoelastoplastic isotropic k-th material of the composition. Since the elements of the matrix $\mathbf{Z}_k$ and the column vector $\boldsymbol{\beta}_k$ depend on the solution of the problem (see (29), (31)), then relation (35) with allowance for (36) is nonlinear. For its linearization, as in [17], we use the method of variable parameters of elasticity [14]. Then, at $t = t_n$ the 6×6 matrix $\mathbf{Z}_k = (z_{ij}^{(k)})$ and the six-component column vector $\mathbf{p}_k = \{p_i^{(k)}\}$, $i, j = 1, 2, \ldots, 6$, according to (32), (34), and (36) will be known in the governing equation (35) on the current iteration of this method.

Linearized matrix equality (35) formally coincides with the Duhamel – Neumann relations for an anisotropic medium [11, 16]. Using the initial assumptions for the CM, similar to those adopted in [16, 17], and repeating the reasoning from these works taking into account (35), at the moment of time t_n in the current iteration we obtain the following linearized matrix equality characterizing the thermoelastoplastic state of the CM:

$$\overset{n}{\dot{\boldsymbol{\sigma}}} = \overset{n}{\mathbf{B}} \overset{n}{\dot{\boldsymbol{\varepsilon}}} + \overset{n}{\mathbf{p}}, \qquad n = 0, 1, 2, \ldots , \tag{37}$$

Where

$$\mathbf{B} \equiv \left(\omega_0 \mathbf{Z}_0 + \sum_{k=1}^{N} \omega_k \mathbf{Z}_k \mathbf{E}_k \right) \mathbf{H}^{-1}, \quad \mathbf{H} \equiv \omega_0 \mathbf{I} + \sum_{k=1}^{N} \omega_k \mathbf{E}_k ,$$

$$\mathbf{p} \equiv \mathbf{f} - \mathbf{B}\mathbf{g}, \quad \mathbf{f} \equiv \omega_0 \mathbf{p}_0 + \sum_{k=1}^{N} \omega_k (\mathbf{p}_k + \mathbf{Z}_k \mathbf{r}_k), \quad \mathbf{g} \equiv \sum_{k=1}^{N} \omega_k \mathbf{r}_k ,$$

$$\omega_0 \equiv 1 - \sum_{k=1}^{N} \omega_k , \quad \mathbf{r}_k \equiv \mathbf{D}_k^{-1} \boldsymbol{\zeta}_k , \quad \mathbf{E}_k \equiv \mathbf{D}_k^{-1} \mathbf{C}_k , \tag{38}$$

$\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$ are six-component column vectors of averaged stresses σ_{ij} and strains ε_{ij} in the composition, similar in structure to (30); $\mathbf{I}$ is the unit 6×6-matrix; $\mathbf{B}$, $\mathbf{E}_k$, $\mathbf{C}_k$ are 6×6-matrices; $\mathbf{D}_k^{-1}$, $\mathbf{H}^{-1}$ are matrices inverse to 6×6-matrices $\mathbf{D}_k$ and $\mathbf{H}$; $\mathbf{p}$, $\mathbf{f}$, $\mathbf{g}$, $\mathbf{r}_k$, $\boldsymbol{\zeta}_k$ are six-component column vectors; ω_k are densities of reinforcement with fibers of the k-th family. Elements of matrices $\mathbf{C}_k = (c_{ij}^{(k)})$, $\mathbf{D}_k = (d_{ij}^{(k)})$, and column vectors $\boldsymbol{\zeta}_k = \{\zeta_i^{(k)}\}$ are calculated by the formulas:

$$c_{1j}^{(k)} = d_{1j}^{(k)} = q_{1j}^{(k)}, \qquad c_{ij}^{(k)} = \sum_{\ell=1}^{6} g_{i\ell}^{(k)} z_{\ell j}^{(0)}, \qquad d_{ij}^{(k)} = \sum_{\ell=1}^{6} g_{i\ell}^{(k)} z_{\ell j}^{(k)},$$

$$\zeta_1^{(k)} = 0, \quad \zeta_i^{(k)} = \sum_{\ell=1}^{6} g_{i\ell}^{(k)} (p_\ell^{(0)} - p_\ell^{(k)}), \quad i = 2,3,\ldots,6, \quad j = 1,2,\ldots,6,$$
$$1 \le k \le N, \tag{39}$$

$$g_{11}^{(k)} = q_{11}^{(k)} = \ell_{11}^{(k)}\ell_{11}^{(k)}, \qquad g_{12}^{(k)} = q_{12}^{(k)} = \ell_{12}^{(k)}\ell_{12}^{(k)}, \quad \ldots,$$

$$g_{16}^{(k)} = 2q_{16}^{(k)} = 2\ell_{12}^{(k)}\ell_{11}^{(k)}, \quad \ldots, \quad 2g_{61}^{(k)} = q_{61}^{(k)} = 2\ell_{21}^{(k)}\ell_{11}^{(k)}, \quad \ldots,$$

$$g_{66}^{(k)} = q_{66}^{(k)} = \ell_{11}^{(k)}\ell_{22}^{(k)} + \ell_{12}^{(k)}\ell_{21}^{(k)}, \qquad 1 \le k \le N, \tag{40}$$

$$\ell_{11}^{(k)} = \sin\theta_k \cos\varphi_k, \qquad \ell_{12}^{(k)} = \sin\theta_k \sin\varphi_k, \qquad \ell_{13}^{(k)} = \cos\theta_k,$$

$$\ell_{21}^{(k)} = -\sin\varphi_k, \qquad \ell_{22}^{(k)} = \cos\varphi_k, \qquad \ell_{23}^{(k)} = 0,$$

$$\ell_{31}^{(k)} = -\cos\theta_k \cos\varphi_k, \quad \ell_{32}^{(k)} = -\cos\theta_k \sin\varphi_k, \quad \ell_{33}^{(k)} = \sin\theta_k,$$
$$1 \le k \le N. \tag{41}$$

Matrix elements of 6×6-matrices $\mathbf{G}_k = (g_{ij}^{(k)})$ and $\mathbf{Q}_k = (q_{ij}^{(k)})$ not written out in (40) are given in Tables (21.40) and (21.44) in [11]. The matrices $\mathbf{G}_k$ and $\mathbf{Q}_k$ determine the transformations of the column vectors $\boldsymbol{\sigma}_k$ and $\boldsymbol{\varepsilon}_k$ (see (30)) during the transition from the global rectangular coordinate system x_j to the local rectangular system $x_i^{(k)}$ associated with the fibers of the k-th family. In this case, the axis $x_1^{(k)}$ is assumed to be directed along the reinforcement and is specified by two angles of the spherical coordinate system θ_k and φ_k (Fig. 1). The direction cosines $\ell_{ij}^{(k)}$ between the axes $x_i^{(k)}$ and x_j, $i,j = 1,2,3$, are determined by relations (41). (In equalities (38) and (39), the superscript n is omitted.)

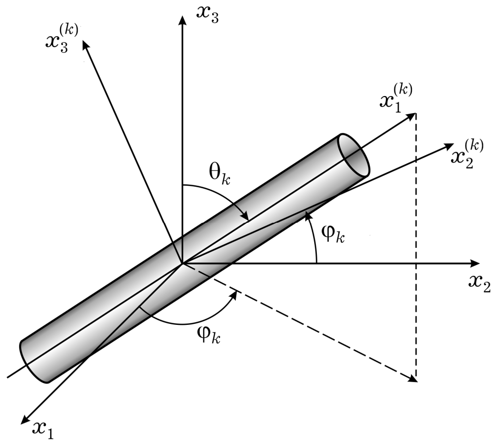

Fig. 1

As in [16, 17], in deriving relations (37) and (38), additionally we obtain the linearized matrix equalities

$$\overset{n}{\dot{\boldsymbol{\varepsilon}}}_0 = \overset{n}{\mathbf{H}}{}^{-1}\, \overset{n}{\dot{\boldsymbol{\varepsilon}}} - \overset{n}{\mathbf{H}}{}^{-1}\, \overset{n}{\mathbf{g}}, \qquad \overset{n}{\dot{\boldsymbol{\varepsilon}}}_k = \overset{n}{\mathbf{E}}_k\, \overset{n}{\dot{\boldsymbol{\varepsilon}}}_0 + \overset{n}{\mathbf{r}}_k, \qquad 1 \le k \le N. \tag{42}$$

The first equality (42) at $t = t_n$ at a given iteration expresses the strain rates $\dot{\boldsymbol{\varepsilon}}_0$ of the binder material through the rates $\dot{\boldsymbol{\varepsilon}}$ of averaged CM strains. The second relation (42) determines the strain rates $\dot{\boldsymbol{\varepsilon}}_k$ of reinforcement of the k-th family through the strain rates $\dot{\boldsymbol{\varepsilon}}_0$ of the binder.

According to formulas (31) and (38)–(41) at the time t_n in the current iteration the matrices $\mathbf{B}$, $\mathbf{H}^{-1}$, $\mathbf{E}_k$ and the column vectors $\mathbf{p}$, $\mathbf{g}$, $\mathbf{r}_k$ in equalities (37) and (42) are known. If the thermal effect is not taken into account ($\dot{\Theta} \equiv 0$), then from (34), (36), (38) and (39) we obtain that in (37) $\overset{n}{\mathbf{p}} \equiv 0$ and relation (37) is reduced to the governing equation for CM obtained in [17]] at the assumption about the elastoplastic behavior of the components of the composition. Consequently, relation (37) generalizes the structure equations derived in [17]].

Suppose that at the current moment of time t_n the iterative process has converged with the required accuracy, i.e. in relation (37), the strain rates $\overset{n}{\dot{\boldsymbol{\varepsilon}}}$ of the CM are known. Then, using formulas (42), we successively determine the rates of strains $\overset{n}{\dot{\boldsymbol{\varepsilon}}}_k$ of the components of the composition, and from equalities (35), the rates of stresses $\overset{n}{\dot{\boldsymbol{\sigma}}}_k$ in the same materials. Using the central finite differences in time on the three-point stencil, we get

$$\frac{1}{2\Delta}\left(\overset{n+1}{\boldsymbol{\sigma}}_k - \overset{n-1}{\boldsymbol{\sigma}}_k\right) = \overset{n}{\dot{\boldsymbol{\sigma}}}_k, \qquad \frac{1}{2\Delta}\left(\overset{n+1}{\boldsymbol{\varepsilon}}_k - \overset{n-1}{\boldsymbol{\varepsilon}}_k\right) = \overset{n}{\dot{\boldsymbol{\varepsilon}}}_k, \qquad 0 \le k \le N, \tag{43}$$

where the right-hand sides have already been calculated, and in the left-hand sides the column vectors $\overset{n-1}{\boldsymbol{\sigma}}_k$ and $\overset{n-1}{\boldsymbol{\varepsilon}}_k$ are assumed to be already known from the solution of the problem under consideration at the previous moment t_{n-1} in time. Therefore, from equalities (43), using an explicit scheme, we can determine the stresses $\overset{n+1}{\boldsymbol{\sigma}}_k$ and strains $\overset{n+1}{\boldsymbol{\varepsilon}}_k$ in the k-th component of the composition at the next moment t_{n+1} in time. After that, on the basis of Hooke's law, with regard to the correspondences (30) at $t = t_{n+1}$, we can also calculate the elastic strains:

$$\overset{n+1}{e}{}_{ii}^{(k)} = \frac{1}{E^{(k)}}\left(\overset{n+1}{\sigma}{}_{ii}^{(k)} - \nu^{(k)}\,\overset{n+1}{\sigma}{}_{jj}^{(k)} - \nu^{(k)}\,\overset{n+1}{\sigma}{}_{\ell\ell}^{(k)}\right), \qquad \overset{n+1}{e}{}_{ij}^{(k)} = \frac{1}{2G^{(k)}}\,\overset{n+1}{\sigma}{}_{ij}^{(k)},$$

$$i \ne j \ne \ell \ne i, \qquad i, j, \ell = 1, 2, 3, \qquad 0 \le k \le N, \tag{44}$$

where there does not summation over repeated indices.

From relation (1) at the moment t_{n+1} of time we have

$$\overset{n+1}{p}{}_{ij}^{(k)} = \overset{n+1}{\varepsilon}{}_{ij}^{(k)} - \overset{n+1}{e}{}_{ij}^{(k)} - \delta_{ij}\,\overset{n+1}{\bar{\alpha}}{}^{(k)}(\overset{n+1}{\Theta} - \overset{n}{\Theta}) - \delta_{ij}\,\overset{n}{\varepsilon}{}_{\theta}^{(k)}, \qquad \overset{n+1}{\bar{\alpha}}{}^{(k)} \equiv (\overset{n+1}{\alpha}{}^{(k)} + \overset{n}{\alpha}{}^{(k)})/2,$$

$$i, j = 1, 2, 3, \qquad 0 \le k \le N, \tag{45}$$

where the right-hand sides are known from equalities (43), (44) and the assumption that the temperature $\overset{n+1}{\Theta}$ is already determined from the heat balance equation for CM using an explicit numerical scheme (see below), and the strain $\overset{n}{\varepsilon}{}_{\theta}^{(k)}$ is known from the solution at the previous time moment t_{n-1}: $\overset{n}{\varepsilon}{}_{\theta}^{(k)} = \overset{n-1}{\varepsilon}{}_{\theta}^{(k)} + \overset{n}{\overline{\alpha}}{}^{(k)}(\overset{n}{\Theta} - \overset{n-1}{\Theta})$.

According to the second relation (3) for determining the Odqvist parameter $\chi^{(k)}$ at $t = t_{n+1}$, we have the equality

$$\overset{n+1}{\chi}{}^{(k)} = \int_0^{t_{n+1}} \sqrt{2\dot{p}_{ij}^{(k)}\dot{p}_{ij}^{(k)}}\, dt = \overset{n}{\chi}{}^{(k)} + \int_{t_n}^{t_{n+1}} \sqrt{2dp_{ij}^{(k)}dp_{ij}^{(k)}} \approx \overset{n}{\chi}{}^{(k)} + \sqrt{2\Delta \overset{n+1}{p}{}_{ij}^{(k)}\, \Delta \overset{n+1}{p}{}_{ij}^{(k)}}\,, \tag{46}$$

where

$$\Delta \overset{n+1}{p}{}_{ij}^{(k)} \equiv \overset{n+1}{p}{}_{ij}^{(k)} - \overset{n}{p}{}_{ij}^{(k)}, \qquad i, j = 1, 2, 3, \qquad 0 \le k \le N. \tag{47}$$

On the right-hand side of expressions (47), the values of plastic strains are already known (see (45)). Thus the last term in equality (46) is known. Therefore, using formula (46) and, in view of (43)−(45) and (47), it is possible to calculate the value of the Odqvist parameter $\overset{n+1}{\chi}{}^{(k)}$ at $t = t_{n+1}$. From a computational point of view, such a method for determining the value $\overset{n+1}{\chi}{}^{(k)}$ is convenient because at the next moment in time t_{n+1} it is unnecessary to refine the Odqvist parameter in the iterative procedure using the method of variable elastic parameters.

At the modeling the dynamic thermoelastoplastic deformation of CM, it is necessary to take into account the coupling of the mechanical and thermophysical problems. Therefore, in addition to the defining mechanical relations (37), it is necessary to use the Fourier law for the CM, which can be written in matrix form as [15]:

$$\mathbf{q} = -\mathbf{\Lambda g}\,, \tag{48}$$

where

$$\mathbf{q} = \{q_1, q_2, q_3\}^{\top}, \quad \mathbf{g} = \{g_1, g_2, g_3\}^{\top} = \operatorname{grad}\Theta\,,$$

$$\mathbf{\Lambda} = \left(\omega_0\mathbf{\Lambda}_0 + \sum_{k=1}^{N}\omega_k\mathbf{L}_k^{\top}\mathbf{\Lambda}_k\overline{\mathbf{E}}_k\right)\overline{\mathbf{H}}, \quad \overline{\mathbf{H}} = \left(\omega_0\overline{\mathbf{I}} + \sum_{k=1}^{N}\omega_k\mathbf{L}_k^{\top}\overline{\mathbf{E}}_k\right)^{-1},$$

$$\overline{\mathbf{E}}_k = \mathbf{B}_k^{-1}\overline{\mathbf{C}}_k, \qquad 1 \le k \le N\,, \tag{49}$$

q_i, g_i are components of the vector of heat flux and temperature gradient Θ; $\mathbf{\Lambda}_k = (\lambda_{ij}^{(k)})$ is a symmetric 3×3-matrix of effective coefficients of thermal conductivity of CM; $\mathbf{\Lambda}_k = (\lambda_{ij}^{(k)})$ denotes the same for the k-th component of the composition (in the case of an isotropic material $\lambda_{ij}^{(k)} = \delta_{ij}\lambda_k$, $i, j = 1, 2, 3$, $0 \le k \le N$); $\overline{\mathbf{I}}$ is a unit 3×3-matrix; $\mathbf{L}_k = (\ell_{ij}^{(k)})$ is the orthogonal matrix of direction cosines $\ell_{ij}^{(k)}$ (see (41)); $\mathbf{B}_k^{-1}$ is the matrix inverse to the 3×3-matrix $\mathbf{B}_k$; $\overline{\mathbf{H}}$, $\overline{\mathbf{E}}_k$, $\overline{\mathbf{C}}_k$ are 3×3-matrices, and the elements $b_{ij}^{(k)}$ and $\overline{c}_{ij}^{(k)}$ of both matrices $\mathbf{B}_k$ and $\overline{\mathbf{C}}_k$ are calculated by the formulas

$$b_{11}^{(k)} = 1, \quad b_{1i}^{(k)} = 0, \quad b_{ij}^{(k)} = \lambda_{ij}^{(k)}, \quad \overline{c}_{1j}^{(k)} = \ell_{1j}^{(k)}, \quad \overline{c}_{ij}^{(k)} = \ell_{im}^{(k)}\lambda_{mj}^{(0)},$$
$$i = 2, 3, \quad j = 1, 2, 3, \qquad 0 \le k \le N. \tag{50}$$

If the materials of the composition are thermally sensitive, then $\lambda_{ij}^{(k)} = \lambda_{ij}^{(k)}(\Theta)$, $0 \le k \le N$. Therefore, according to (49), (50) the effective coefficients of the CM in (48) depend on the temperature ($\mathbf{\Lambda} = \mathbf{\Lambda}(\Theta)$, $\lambda_{ij} = \lambda_{ij}(\Theta)$, $i, j = 1, 2, 3$).

Conclusion. The developed structural model, focused on the use of explicit step-by-step schemes, makes possible numerically and analytically modelling the behavior of thermoelastoplastically deformed CMs of a fibrous structure within the framework of the theory of flow with isotropic hardening, when the loading function is sensitive to changes of the temperature in the composite material. The conditions for thermoelastic deformation, unloading, neutral and active loading of the components of the composition were obtained, with regard to the change of temperature over time. In this case, it is taken into account that the thermophysical problem for such a CM can be associated with a mechanical problem.

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МОДЕЛЮВАННЯ ТЕРМОПРУЖНОПЛАСТИЧНОГО ДЕФОРМУВАННЯ АРМОВАНИХ ПЛАСТИН. I. СТРУКТУРНА МОДЕЛЬ АРМОВАНОГО СЕРЕДОВИЩА

На основі алгоритму кроків за часом розроблено чисельно-аналітичну структурну модель термопружнопластичного деформування композитного матеріалу, перехресно армованого волокнами у довільних напрямках. Матеріали компонентів композиції є ізотропними, їхнє пластичне деформування описується теорією течії з ізотропним зміцненням при врахуванні залежності функції навантаження від температури. Отримано умови, що визначають термопружне деформування, розвантаження, нейтральне та активне навантаження термочутливих компонентів композиції. Розглядаються зв'язані задачі про теплофізичну та механічну поведінку армованого матеріалу. Наведено структурні співвідношення, необхідні для розв'язання теплофізичної складової досліджуваної проблеми. Розроблена структурна модель орієнтована на застосування явних чисельних схем інтегрування як пружнопластичної, так і теплофізиченої задач.

Ключові слова: *армування волокнами, структурні моделі, термопружнопластичність, термочутливість, теорія течії, ефективні співвідношення, покроковий алгоритм, явні чисельні схеми.*

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