UDC 512.53+512.64

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ON TRANSITIVITY COEFFICIENTS FOR MINIMAL POSETS WITH NON-POSITIVE QUADRATIC TITS FORM

Combinatorial properties of finite posets connected with the positivity of their quadratic Tits form (playing an important role in the theory of matrix representations of posets) are explored. The coefficients of transitivity for all minimal posets with non-positive quadratic Tits form (such posets are called P-critical and their number is 75 up to isomorphism and duality) are calculated. Some relationships between these coefficients and the heights of posets are established.

Key words: *poset, duality, height, neighboring elements, Hasse diagram, Dynkin diagram, quadratic Tits form, positivity, P-critical poset, coefficient of transitivity.*

Introduction. In the study of the representations of quivers (directed graphs) introduced by him, P. Gabriel [15] introduced an integer quadratic form $q_Q(z) = q_Q(z_1, \ldots, z_n)$ for a finite quiver $Q = (Q_0, Q_1)$ with the set of vertices Q_0 and the set of arrows Q_1, $n = |Q_0| < \infty$, as follows:

$$q_Q(z) := \sum_{i \in Q_0} z_i^2 - \sum_{i \to j} z_i z_j ,$$

where $i \to j$ runs through Q_1. This form was called the *quadratic Tits form of the quiver* Q. P. Gabriel proved that the quiver Q is of finite representation type over a field k (i.e., has finitely many indecomposable representations, up to equivalence) if and only if its Tits form is positive. This Gabriel's result laid the foundations of a new direction in the theory of algebra dealing with the investigation of the relationships between the properties of representations of various objects and the properties of quadratic forms associated with these objects (see, e.g., [5, 8, 9, 11–13, 19, 21, 24]).

In [4] Yu. A. Drozd showed that a (finite) poset S is of finite representation type if and only if its quadratic Tits form

$$q_S(z) = z_0^2 + \sum_{i \in S} z_i^2 + \sum_{\substack{i<j \\ i,j \in S}} z_i z_j - z_0 \sum_{i \in S} z_i$$

is weakly positive, i.e., positive on the nonzero vectors with non-negative coordinates (representations of posets were introduced by L. A. Nazarova and A. V. Roiter in [5]). In contrast to quivers, the posets with weakly positive Tits forms and posets with positive Tits forms do not coincide. Since the connected quivers having positive quadratic Tits form coincide with the quivers whose underlying graphs are (simply faced) Dynkin diagrams, the posets with positive Tits form are analogs of the Dynkin diagrams. Therefore investigations related to posets with positive Tits form are natural (see, e.g., [2, 4, 6, 15, 18, 23]).

In particular, in [2] the authors classified all the posets with positive quadratic Tits form and the minimal posets with non-positive Tits form, which were called P-*critical*.

The present paper is devoted to the investigation of combinatorial properties of P-critical posets.

1. Formulation of the main result. Partially ordered sets arise in the study of many problems in various areas of mathematics and their applications. Among such problems, the study of which continues to this day, an

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important role is played by combinatorial problems associated with the study of their discrete parameters, the relationship between them and with graph theory (see, e.g., [1, 7, 10 , 16, 17, 20]).

Throughout the paper, all posets (partially ordered sets) are finite. In considering a poset $S = (A, <)$ the set A will not be written and we keep to the following conventions: by a subset S' of S we mean a subset A' of A together with the induced order relation (which is denoted by the same symbol $<$), and we write $x \in S$ instead of $x \in A$, etc.

Linear ordered sets of order n are also called *chains of length* n, and the greatest length $h(S)$ among the lengths of all chains of a poset S is called its *height*.

A poset S is called *positive* if its Tits quadratic form

$$q_S(z) = z_0^2 + \sum_{i\in S} z_i^2 + \sum_{\substack{i<j\\ i,j\in S}} z_i z_j - z_0 \sum_{i\in S} z_i$$

is positive (see Introduction), and P-*critical* if its Tits quadratic form is not positive, but that of any proper subset of S is positive. Notice that the poset S and the dual to it poset S^{op} are simultaneously positive or non-positive (by definition, $S^{\mathrm{op}} = S$ as usual sets and $x < y$ in S^{op} if $x > y$ in S).

Let S be a poset and $S_<^2 := \{(x,y) \mid x,y \in S,\ x < y\}$. If $(x,y) \in S_<^2$ and there is no z satisfying $x < z < y$, then x and y are called *neighboring*. Put $n_w = n_w(S) := |S_<^2|$ and denote by $n_e = n_e(S)$ the number of pairs (x,y) of neighboring elements of S. On the language of the Hasse diagram of S (that represents S in the plane), n_e is equal to the number of all its edges and n_w is equal to the number of all its paths, up to parallelity, going bottom-up (two paths are called parallel if they start and terminate at the same vertices). The ratio $k_t = k_t(S)$ of the numbers $n_w - n_e$ and n_w we call the *coefficient of transitivity of* S; for $n_w = 0$, it is assumed that $k_t = 0$ [9]. Obviously, dual posets have the same coefficient of transitivity.

It is clear, that the coefficient of transitivity of S is the probability that comparable elements of S are not neighboring.

The main result of this paper is the following theorem:

Theorem 1. *Let S and T be P-critical posets. Then*

(*1*) $\quad k_t(T) > k_t(S) \quad$ *if* $\quad h(T) > h(S) + 1$;

(*2*) $\quad k_t(T) > k_t(S) - \dfrac{1}{10} \quad$ *if* $\quad h(T) = h(S) + 1$.

2. Classification of P-critical posets. For subsets X, Y of a poset S, by $X \sqcup Y$ we mean the subset $X \bigcup Y$ if $X \bigcap Y = \varnothing$. From Dilworth's theorem it follows that any poset can be represented in the form $\sqcup_i^m X_i$, where X_i, $i = 2, \ldots, m$, are chains, with additional relations $y < z$ for y and y belonging to different components. By A_s, B_s, C_s we denote, respectively, the chains $a_1 < \ldots < a_s$, $b_1 < \ldots < b_s$, $c_1 < \ldots < c_s$.

The P-critical posets were classified by the authors in [2, Theorem 3] (see also Section 5 below). The following theorem is a set-theoretic reformulation of this result (in [2] the classification is expressed in terms of Hasse graphs).

Theorem 2. *The P-critical sets are exhausted, up to isomorphism and duality, by the following posets*:

1) $A_2 \sqcup B_2,\ a_1 < b_2,\ b_1 < a_2$; **2**) $A_3 \sqcup B_3,\ a_2 < b_2$;

3) $A_2 \sqcup B_4$, $a_2 < b_3$;

4) $A_3 \sqcup B_3$, $a_1 < b_1$, $a_3 < b_3$;

5) $A_1 \sqcup B_6$, $a_1 < b_4$;

6) $A_2 \sqcup B_5$, $a_1 < b_1$, $a_2 < b_4$;

7) $A_2 \sqcup B_5$, $a_1 < b_3$;

8) $A_3 \sqcup B_4$, $a_1 < b_2$;

9) $A_3 \sqcup B_4$, $a_3 < b_4$;

10) $A_1 \sqcup B_7$, $a_1 < b_3$;

11) $A_2 \sqcup B_6$, $a_1 < b_1$, $a_2 < b_3$;

12) $A_3 \sqcup B_5$, $a_2 < b_1$, $a_3 < b_3$;

13) $A_1 \sqcup B_7$, $a_1 < b_6$;

14) $A_2 \sqcup B_6$, $a_1 < b_1$, $a_2 < b_6$;

15) $A_2 \sqcup B_6$, $a_1 < b_2$;

16) $A_2 \sqcup B_6$, $a_1 < b_5$;

17) $A_2 \sqcup B_6$, $a_2 < b_6$;

18) $A_3 \sqcup B_5$, $a_1 < b_4$;

19) $A_3 \sqcup B_5$, $a_2 < b_5$;

20) $A_4 \sqcup B_4$, $a_1 < b_3$;

21) $A_2 \sqcup B_6$, $a_1 < b_2$, $a_2 < b_3$;

22) $A_3 \sqcup B_5$, $a_1 < b_1$, $a_2 < b_2$, $a_3 < b_3$;

23) $A_4 \sqcup B_4$, $a_2 < b_1$, $a_3 < b_2$, $a_4 < b_3$;

24) $A_2 \sqcup B_6$, $a_1 < b_2$, $a_2 < b_6$;

25) $A_3 \sqcup B_5$, $a_1 < b_1$, $a_2 < b_2$;

26) $A_2 \sqcup B_6$, $a_1 < b_5$, $a_2 < b_6$;

27) $A_3 \sqcup B_5$, $a_1 < b_1$, $a_2 < b_5$;

28) $A_3 \sqcup B_5$, $a_1 < b_4$, $a_2 < b_5$;

29) $A_4 \sqcup B_4$, $a_1 < b_3$, $a_2 < b_4$;

30) $A_1 \sqcup B_2 \sqcup C_1$, $a_1 < b_2$, $c_1 < b_2$;

31) $A_2 \sqcup B_2 \sqcup C_2$;

32) $A_1 \sqcup B_2 \sqcup C_3$, $b_2 < c_3$;

33) $A_1 \sqcup B_3 \sqcup C_2$, $a_1 < b_3$, $b_2 < c_2$;

34) $A_2 \sqcup B_2 \sqcup C_2$, $a_1 < b_2$, $b_1 < c_2$, $c_1 < a_2$;

35) $A_1 \sqcup B_3 \sqcup C_3$;

36) $A_1 \sqcup B_1 \sqcup C_5$, $b_1 < c_4$;

37) $A_1 \sqcup B_2 \sqcup C_4$, $b_1 < c_1$, $b_2 < c_4$;

38) $A_2 \sqcup B_1 \sqcup C_4$, $b_1 < c_4$;

39) $A_1 \sqcup B_2 \sqcup C_4$, $a_1 < b_2$, $b_1 < c_3$;

40) $A_1 \sqcup B_3 \sqcup C_3$, $a_1 < b_2$, $b_1 < c_2$;

41) $A_2 \sqcup B_2 \sqcup C_3$, $a_2 < b_2$, $b_1 < c_3$;

42) $A_1 \sqcup B_2 \sqcup C_5$;

43) $A_1 \sqcup B_1 \sqcup C_6$, $b_1 < c_3$;

44) $A_1 \sqcup B_2 \sqcup C_5$, $b_1 < c_1$, $b_2 < c_3$;

45) $A_1 \sqcup B_3 \sqcup C_4$, $b_2 < c_1$, $b_3 < c_3$;

46) $A_1 \sqcup B_1 \sqcup C_6$, $b_1 < c_6$;

47) $A_1 \sqcup B_2 \sqcup C_5$, $b_1 < c_2$;

48) $A_1 \sqcup B_2 \sqcup C_5$, $b_1 < c_5$;

49) $A_2 \sqcup B_1 \sqcup C_5$, $b_1 < c_3$;

50) $A_2 \sqcup B_2 \sqcup C_4$, $b_1 < c_1$, $b_2 < c_3$;

51) $A_3 \sqcup B_1 \sqcup C_4$, $b_1 < c_3$;

52) $A_3 \sqcup B_2 \sqcup C_3$, $b_1 < c_1$, $b_2 < c_3$;

53) $A_4 \sqcup B_1 \sqcup C_3$, $b_1 < c_3$;

54) $A_4 \sqcup B_2 \sqcup C_2$, $b_1 < c_2$;

55) $A_1 \sqcup B_2 \sqcup C_5$, $a_1 < b_2$, $b_1 < c_2$;

56) $A_1 \sqcup B_2 \sqcup C_5$, $a_1 < b_2$, $b_1 < c_5$;

57) $A_1 \sqcup B_3 \sqcup C_4$, $a_1 < b_2$, $b_1 < c_4$;

58) $A_1 \sqcup B_3 \sqcup C_4$, $a_1 < b_3$, $b_1 < c_1$;

59) $A_1 \sqcup B_3 \sqcup C_4$, $a_1 < b_3$, $b_1 < c_4$;

60) $A_1 \sqcup B_4 \sqcup C_3$, $a_1 < b_2$, $b_1 < c_3$;

61) $A_1 \sqcup B_4 \sqcup C_3$, $a_1 < b_3$, $b_1 < c_3$;

62) $A_1 \sqcup B_5 \sqcup C_2$, $a_1 < b_3$, $b_1 < c_2$;

63) $A_1 \sqcup B_6 \sqcup C_1$, $a_1 < b_3$, $b_1 < c_1$;

64) $A_1 \sqcup B_6 \sqcup C_1$, $a_1 < b_6$, $b_1 < c_1$; **65**) $A_2 \sqcup B_2 \sqcup C_4$, $a_1 < b_2$, $b_1 < c_2$;

66) $A_3 \sqcup B_2 \sqcup C_3$, $a_2 < b_2$, $b_1 < c_2$; **67**) $A_1 \sqcup B_2 \sqcup C_5$, $b_1 < c_2$, $b_2 < c_3$;

68) $A_1 \sqcup B_3 \sqcup C_4$, $b_1 < c_1$, $b_2 < c_2$, $b_3 < c_3$; **69**) $A_2 \sqcup B_2 \sqcup C_4$, $b_1 < c_2$, $b_2 < c_3$;

70) $A_2 \sqcup B_3 \sqcup C_3$, $b_1 < c_1$, $b_2 < c_2$, $b_3 < c_3$; **71**) $A_3 \sqcup B_2 \sqcup C_3$, $b_1 < c_2$, $b_2 < c_3$;

72) $A_1 \sqcup B_3 \sqcup C_4$, $a_1 < b_3$, $b_1 < c_1$, $b_2 < c_2$;

73) $A_2 \sqcup B_3 \sqcup C_3$, $a_2 < b_3$, $b_1 < c_1$, $b_2 < c_2$;

74) $A_3 \sqcup B_3 \sqcup C_2$, $a_3 < b_3$, $b_1 < c_1$, $b_2 < c_2$; **75**) $A_1 \sqcup B_1 \sqcup C_1 \sqcup D_1$.

3. Lemmas on coefficients of transitivity. Let S be a (finite) poset. Recall (see Section 2) that $n_w(S)$ denotes the number of pairs (x,y) of elements $x,y \in S$ satisfying $x < y$, and $n_e(S)$ the number of pairs of neighboring elements. In addition, we denote by $n_w^S(X,Y)$, where X, Y are subposets of S, the number of pairs (x,y) of elements $x \in X$, $y \in Y$ satisfying $x < y$. If $a < b$ in S, we denote by $S \setminus (a,b)$ the poset with the same elements as S for which $x < y$ if $x < y$ in S except $x \le a < b \le y$. For a chain $\{x_1 < \ldots < x_k\}$, we denote by $[x_i, x_j]$, $i \le j$, its subchain $\{x_i < \ldots < x_j\}$; it is assumed that $[x_i, x_i] = \{x_i\}$.

For proving Theorem 1 we need the following lemmas.

Lemma 1. *Let* $S = S_1 \sqcup S_2$. *Then*

$$n_e(S) = n_e(S_1) + n_e(S_2), \qquad n_w(S) = n_w(S_1) + n_w(S_2).$$

The p r o o f is obvious. ♦

Lemma 2. *Let* $S = A_m$. *Then*

$$n_e(S) = m - 1, \qquad n_w(S) = \frac{(m-1)m}{2}.$$

The lemma is proved by simple combinatorial calculations. ♦

Lemma 3. *Let* $S = \{A_m \sqcup B_n, a_i < b_j\}$. *Then*

(***a***) $n_e(S) = m + n - 1$;

(***b***) $n_w(S) = \dfrac{(m-1)m + (n-1)n}{2} + i(n - j + 1)$.

The lemma follows from the equalities

$$n_e(S) = n_e(A_m) + n_e(B_n) + 1,$$

$$n_w(S) = n_w(A_m) + n_w(B_n) + n_w^S(A_m, B_n),$$

$$n_w^S(A_m, B_n) = i(n - j + 1)$$

and Lemmas 1, 2 (with using simple combinatorial calculations). ♦

Lemma 4. *Let* $S = \{A_m \sqcup B_n, a_i < b_j, a_{i'} < b_{j'}\}$, *where* $i < i'$, $j < j'$. *Then*

(***a***) $n_e(S) = m + n$;

(***b***) $n_w(S) = \dfrac{(m-1)m + (n-1)n}{2} + i'(n - j' + 1) + i(j' - j)$.

The lemma follows from the equalities

$$n_e(S) = n_e(A_m) + n_e(B_n) + 2\,,$$

$$n_w(S) = n_w\big(S \setminus (a_{i'}, b_{j'})\big) + n_w^S([a_{i+1}, a_{i'}], [b_{j'}, b_n]) =$$

$$= n_w\big(S \setminus (a_{i'}, b_{j'})\big) + (i' - i)(n - j + 1)$$

and Lemmas 2, 3. ♦

Lemma 5. *Let* $S = \{A_m \sqcup B_n \sqcup C_s,\ a_i < b_j,\ b_{j'} < c_k\}$, *where* $j > j'$. *Then*

(***a***) $n_e(S) = m + n + s - 1$;

(***b***) $n_w(S) = \dfrac{(m-1)m + (n-1)n + (s-1)s}{2} + i(n - j + 1) + j'(s - k + 1)$.

The lemma follows from the equalities

$$n_e(S) = n_e(A_m) + n_e(B_n) + n_e(C_s) + 2\,,$$

$$n_w(S) = n_w\big(A_m \sqcup B_n,\ a_i < b_j\big) + n_w\big(B_n \sqcup C_s,\ b_{j'} < c_k\big) - n_w(B_n)$$

and Lemmas 2, 3. ♦

Lemma 6. *Let* $S = \{A_m \sqcup B_n,\ a_i < b_j,\ a_{i+1} < b_{j+1},\ a_{i+2} < b_{j+2}\}$. *Then*

(***a***) $n_e(S) = m + n + 1$;

(***b***) $n_w(S) = \dfrac{(m-1)m + (n-1)n}{2} + (i+2)n - i(j-1) - (2j+1)$.

The lemma follows from the equalities

$$n_e(S) = n_e(A_m) + n_e(B_n) + 3\,,$$

$$n_w(S) = n_w\big(S \setminus (a_{i+2}, b_j)\big) + n_w^S(\{a_{i+2}\}, [b_{j+2}, b_n]) =$$

$$= n_w\big(S \setminus (a_{i+2}, b_j)\big) + n - j - 1$$

and Lemmas 2, 4. ♦

Lemma 7. *Let* $S = \{A_m \sqcup B_n \sqcup C_s,\ a_i < b_j,\ b_{j'} < c_k,\ b_{j'+1} < c_{k+1}\}$, *where* $j > j' + 1$. *Then*

(***a***) $n_e(S) = m + n + s$;

(***b***) $n_w(S) = \dfrac{(m-1)m + (n-1)n + (s-1)s}{2} + i(n - j + 1) + (j' + 1)(s - k) + j'$.

The lemma follows from the equalities

$$n_e(S) = n_e(A_m) + n_e(B_n) + n_e(C_s) + 3\,,$$

$$n_w(S) = n_w\big(A_m \sqcup B_n,\ a_i < b_j\big) +$$

$$+ n_w\big(B_n \sqcup C_s,\ b_{j'} < c_k,\ b_{j'+1} < c_{k+1}\big) - n_w(B_n)$$

and Lemmas 2–4. ♦

4. Calculation of the transitivity coefficients. Proof of Theorem 1. We first calculate the coefficients of transitivity k_t of the P-critical posets, numbered $N = 1, 2, \ldots, 75$. In the table below the posets are not in increasing order of their numbers, but in nondecreasing order of their heights (denoted by h), and if the heights are equal, in nondecreasing order of the transitivity coefficients.

Theorem 3. *The following holds for* P *-critical posets* 1–75:

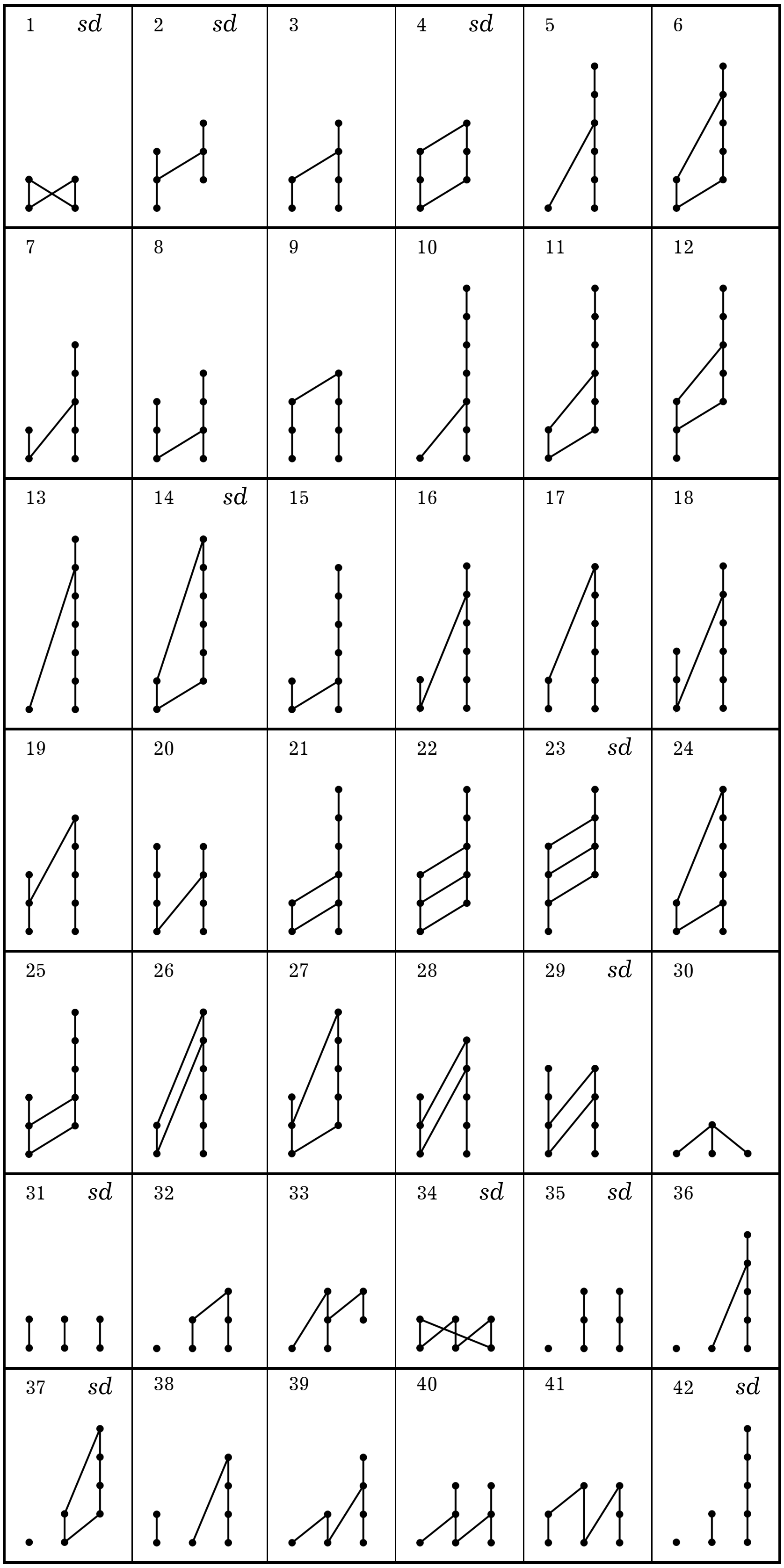
N	h	n_e	n_w	k_t	N	h	n_e	n_w	k_t	N	h	n_e	n_w	k_t
75	1	0	0	0	51	4	6	11	0.45454	47	5	6	15	0.6
1	2	4	4	0	60	4	7	13	0.46154	67	5	7	18	0.61111
30	2	3	3	0	69	4	7	13	0.46154	26	6	8	19	0.57895
31	2	3	3	0	29	4	8	15	0.46667	27	6	8	19	0.57895
34	2	6	6	0	2	4	5	10	0.5	64	6	7	17	0.58824
41	3	6	8	0.25	8	4	6	12	0.5	6	6	7	18	0.61111
33	3	5	7	0.28471	9	4	6	12	0.5	16	6	7	18	0.61111
71	3	7	10	0.3	20	4	7	14	0.5	17	6	7	18	0.61111
32	3	4	6	0.33333	3	4	5	11	0.54545	46	6	6	16	0.625
35	3	4	6	0.33333	56	5	7	13	0.46154	44	6	7	19	0.63158
66	3	7	11	0.36364	28	5	8	16	0.5	45	6	7	19	0.63158
40	3	6	10	0.4	37	5	6	12	0.5	24	6	8	22	0.63636
54	4	6	9	0.33333	48	5	6	12	0.5	25	6	8	22	0.63636
42	4	7	11	0.36364	50	5	7	14	0.5	22	6	9	25	0.64
49	4	7	11	0.36364	58	5	7	14	0.5	23	6	9	25	0.64
38	4	5	8	0.375	72	5	8	17	0.52941	63	6	7	20	0.65
70	4	8	13	0.38461	18	5	7	15	0.53333	5	6	6	18	0.66667
74	4	8	13	0.38461	19	5	7	15	0.53333	15	6	7	21	0.66667
39	4	6	10	0.4	62	5	7	15	0.53333	21	6	8	25	0.68
53	4	6	10	0.4	42	5	5	11	0.54545	43	6	6	19	0.68421
57	4	7	12	0.41667	68	5	8	18	0.55556	14	7	8	23	0.65217
61	4	7	12	0.41667	55	5	7	16	0.5625	11	7	8	26	0.69231
65	4	7	12	0.41667	7	5	6	14	0.57143	12	7	8	26	0.69231
73	4	8	14	0.42857	49	5	6	14	0.57143	13	7	7	23	0.69565
4	4	6	11	0.45454	36	5	5	12	0.58333	10	7	7	26	0.73077

The coefficients of transitivity k_t *are calculated up to the fifth decimal place. If the number of decimal places is less than five, then the decimal fraction is finite, and if it is five, then infinite. If two decimal fractions are equal up to five digits, then they are exactly equal.*

The p r o o f of Theorem 3 is carried out by direct calculations using Lemmas 1, 2 for $N = 31, 35, 42$, Lemma 3 for $N = 2,3,5,7,8,9,10,13,15,16,17,18,19,20$, Lemmas 1–3 for $N = 32,36,38,43,46,47,48,49,51,53,54$, Lemma 4 for $N = 4,6,11,12,14,21,24,25,26,27,28,29$, Lemmas 1, 2, 4 for $N = 37,44,45$, 50, 52, 67, 69, 71, Lemma 5 for $N = 33,39,40,41,55,56,57,58,59,60,61,62,63$, 64, 65, 66, Lemma 6 for $N = 22,23$, Lemma 7 for $N = 72,73,74$, Lemmas 1, 2, 7 for $N = 68,70$. ♦

Now Theorem 1 follows from Theorems 2, 3 and the fact that dual posets have the same transitivity coefficients. ♦

5. The table of the P **-critical posets (in terms of Hasse graphs)** We follow the paper [2]. The P -critical posets are written up to isomorphism and duality; their number is 75: $PC_1, PC_2, \ldots, PC_{75}$. Self-dual posets are marked (in the upper right corners) with "sd". If we add all the posets dual to unmarked ones, we obtain the classification of P -critical posets up to isomorphism; their number is 132: PC_k for $k = 1,2,\ldots,75$ and PC_s^{op} for $s \neq 1,2,4,14,23,29,31,34,35,37,42,45,52,54,64,66,70,75$.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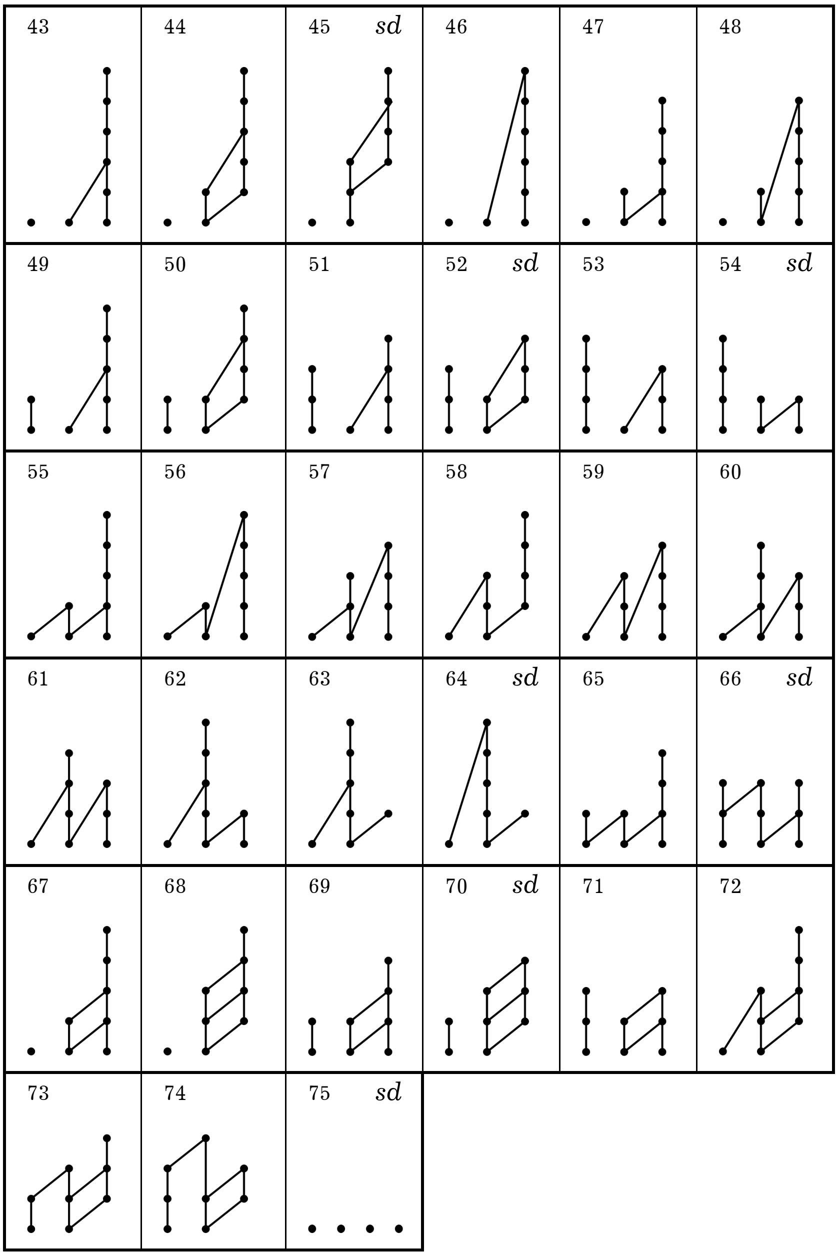

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ПРО КОЕФІЦІЄНТИ ТРАНЗИТИВНОСТІ ДЛЯ МІНІМАЛЬНИХ ЧАСТКОВО ВПОРЯДКОВАНИХ МНОЖИН З НЕДОДАТНОЮ КВАДРАТИЧНОЮ ФОРМОЮ ТІТСА

Досліджуються комбінаторні властивості скінченних частково впорядкованих множин, пов'язаних з додатністю їх квадратичної форми Тітса (яка відіграє важливу роль у теорії матричних зображень частково впорядкованих множин). Для всіх мінімальних частково впорядкованих множин з недодатною квадратичною формою Тітса (такі множини називають P-критичними, і їх кількість, з точністю до ізоморфізму і дуальності, дорівнює 75) *обчислено коефіцієнти транзитивності і встановлено деякі зв'язки між цими коефіцієнтами та висотами частково впорядкованих множин.*

Ключові слова: *частково впорядкована множина, дуальність, висота, сусідні елементи, діаграма Хассе, діаграма Динкіна, квадратична форма Тітса, додатність, P-критична частково впорядкована множина, коефіцієнт транзитивності.*

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Received 08.01.21