UDC 512.56

I. Yu. Raievska[1,2], M. Yu. Raievska[1,2✉]

BRACES AND SKEW-BRACES VIA GAP: YANGBAXTER AND LOCALNR PACKAGES

Using the YangBaxter package the classifications of braces (up to order 32) and skew-braces (up to order 16) according to the additive or multiplicative groups which are Miller – Moreno p-groups is performed. Moreover, some information concerning nearring braces from the LocalNR package is presented.

Key words: *brace, skew-brace, nearring, local nearring, Miller – Moreno group.*

Preliminaries.

Definition 1. A triple $A = (A, +, \circ)$, where $(A, +)$ and $(A, \circ)$ are two abelian groups such that $a \circ (b + c) = a \circ b - a + a \circ c$ holds for all $a, b, c \in A$ is called a *finite brace*.

If the additive group is cyclic, the brace A is said to be *cyclic*.

Definition 2. A triple $(A, +, \circ)$, where $(A, +)$ and $(A, \circ)$ are two (not necessarily abelian) groups such that the compatibility $a \circ (b + c) = a \circ b - a + a \circ c$ holds for all $a, b, c \in A$ is called a *skew-brace*.

A quaternion brace is a brace whose multiplicative group is isomorphic to some generalized quaternion group Q_{2^m}. Rump [9] established a complete classification of quaternion braces of order $2^n \geq 32$.

Rump [8] classified the braces of the following two types: cyclic p-group $(A, +)$ and cyclic group $(A, +)$ such that $(A, \circ)$ is also cyclic.

In view of the results mentioned above the next natural step is to consider braces and skew-braces whose additive or multiplicative groups are minimal non-abelian or, in a different terminology, Miller – Moreno groups [4].

Recall that a finite group is called a *Miller – Moreno group* if it is non-abelian and all its proper subgroups are abelian. The structure of these groups is completely described by the well-known Rédei's theorem [7].

Theorem 1. *Finite Miller – Moreno groups are groups of the following types*:

1°) *the quaternion group* Q_8;

2°) *the group* $G = \langle a \rangle \rtimes \langle b \rangle$ *of order* p^{m+n} *with* $a^{p^m} = b^{p^n} = 1$ *and* $b^{-1}ab = a^{1+p^{m-1}}$, *where* p *is prime,* $m \geq 2$ *and* $n \geq 1$;

3°) *the group* $G = (\langle a \rangle \times \langle c \rangle) \rtimes \langle b \rangle$ *of order* p^{m+n+1} *with* $a^{p^m} = b^{p^n} = c^p = 1$, $b^{-1}ab = ac$ *and* $b^{-1}cb = c$, *where* $m \geq n \geq 1$ *and* $m + n > 2$ *for* $p = 2$;

4°) *the group* $G = P \rtimes \langle b \rangle$ *of order* $p^r q^s$ *with an elementary abelian subgroup* P *of order* p^r *in which the element* b *induces an irreducible automorphism of prime order* q *and, in addition,* $b^{q^s} = 1$ *and* $\langle b^q \rangle = Z(G)$, *where* p *and* q *are different prime numbers and* r, s *are natural numbers*.

In what follows, we focus on braces and skew-braces of order p^n which we also call p-*braces* and p-*skew-braces*, respectively. We say that a p-bra-

✉ raemarina@imath.kiev.ua

ce (p-skew-brace) $(A,+,\circ)$ is *Miller − Moreno* if its multiplicative (adjoint) group $(A,\circ)$ is a Miller − Moreno p-group.

Definition 3. A non-empty set R with two binary operations "$+$" and "$\cdot$" is a *nearring* if:

1) $(R,+)$ is a group with neutral element 0;

2) $(R,\cdot)$ is a semigroup;

3) $x\cdot(y+z)=x\cdot y+x\cdot z$ for all $x,y,z\in R$.

Such a nearring is called a *left nearring*. If axiom **3**) is replaced by an axiom $(x+y)\cdot z=x\cdot z+y\cdot z$ for all $x,y,z\in R$, then we get a *right nearring*. For a nearring R, the group $(R,+)$ is denoted by R^{+} and called the *additive group* of R. Furthermore, R is a *nearring with an identity* i if the semigroup $(R,\cdot)$ is a monoid with identity element i. In the latter case the group of all invertible elements of the monoid $(R,\cdot)$ is denoted by R^{*} and called the *multiplicative group* of R.

***Definition 4* [3].** A nearring R with identity is called *local* if the set L of all non-invertible elements of R forms a subgroup of the additive group R^{+} and a *nearfield* if $L=0$.

Local near-rings with Miller − Moreno multiplicative group were studied in [1].

***Definition 5* [10].** We say that a local nearring N *splits* if there is a subnearfield K of N with $N=K\oplus J(N)$ as additive groups. If K can be chosen to be a subnearfield of the centre $Z(N):=\{r\in N \mid \forall s\in N: rs=sr\}$, we say that N *totally splits*.

***Remark 1* [10].** If N is totally split, then any nearfield $K\subset Z(N)$ is commutative, hence it is a field.

1. Braces from the YangBaxter package. The YangBaxter package [11] provides functionality for constructing braces and skew-braces. It also includes a database of braces and skew-braces of small orders.

The YangBaxter package includes the *AllSmallSkewbraces* database of skew-braces and the *AllBraces* database of braces of small orders. In these libraries, braces are arranged according to their order and, for example, the commands $B := AllBraces(n)$ and $S := AllSmallSkewbraces(n)$ define the lists of braces B and skew-braces S of order n, respectively.

The functions for obtaining the underlying multiplicative and additive groups of braces (skew-braces) are called *UnderlyingMultiplicativeGroup* and *UnderlyingAdditiveGroup*, respectively. The package has the functions to list all braces and skew-braces with a given order, i.e. *AllSmallBraces* and *AllSmallSkewbraces*, respectively.

Let $[n,i]$ be the ith group of order n in the SmallGroups library in the computer algebra system GAP [2]. We denote by C_n, D_n and Q_n the cyclic, dihedral and quaternion groups of order n, respectively.

The following lemmas and propositions are obtained using a computer algebra system GAP and the YangBaxter package.

Lemma 1. *Let $(A,+,\circ)$ be a Miller − Moreno p-brace up to order 32. Then $(A,\circ)$ is isomorphic to one of the following groups*:

1) D_8 $[8,3]$;

2) Q_8 $[8,4]$;

3) $(C_4\times C_2)\rtimes C_2$ $[16,3]$;

4) $C_4\rtimes C_4$ $[16,4]$;

5) $C_8 \rtimes C_2$ [16,6];

6) $(C_4 \times C_2) \rtimes C_4$ [32,2];

7) $C_8 \rtimes C_4$ [32,4];

8) $(C_8 \times C_2) \rtimes C_2$ [32,5];

9) $C_4 \rtimes C_8$ [32,12];

10) $C_{16} \rtimes C_2$ [32,17].

Proposition 1. *There exist* 8 *Miller – Moreno braces with* $(A,\circ) \cong D_8$ [8,3], *i.e.*:

1) 1 *brace with the additive group* C_8 [8,1];

2) 5 *braces with the additive group* $C_4 \times C_2$ [8,2];

3) 2 *braces with the additive group* $C_2 \times C_2 \times C_2$ [8,5].

Proposition 2. *There exist* 3 *Miller – Moreno braces with* $(A,\circ) \cong Q_8$ [8,4], *i.e.*:

1) 1 *brace with the additive group* C_8 [8,1];

2) 1 *brace with the additive group* $C_4 \times C_2$ [8,2];

3) 1 *brace with the additive group* $C_2 \times C_2 \times C_2$ [8,5].

Proposition 3. *There exist* 65 *Miller – Moreno braces with* $(A,\circ) \cong (C_4 \times C_2) \rtimes C_2$ [16,3], *i.e.*:

1) 19 *braces with the additive group* $C_4 \times C_4$ [16,2];

2) 2 *braces with the additive group* $C_8 \times C_2$ [16,5];

3) 36 *braces with the additive group* $C_4 \times C_2 \times C_2$ [16,10];

4) 8 *braces with the additive group* $C_2 \times C_2 \times C_2 \times C_2$ [16,14].

Proposition 4. *There exist* 45 *Miller – Moreno braces with* $(A,\circ) \cong C_4 \rtimes C_4$ [16,4], *i.e.*:

1) 14 *braces with the additive group* $C_4 \times C_4$ [16,2];

2) 4 *braces with the additive group* $C_8 \times C_2$ [16,5];

3) 23 *braces with the additive group* $C_4 \times C_2 \times C_2$ [16,10];

4) 4 *braces with the additive group* $C_2 \times C_2 \times C_2 \times C_2$ [16,14].

Proposition 5. *There exist* 11 *Miller – Moreno braces with* $(A,\circ) \cong C_8 \rtimes C_2$ [16,6], *i.e.*:

1) 1 *brace with the additive group* C_{16} [16,1];

2) 2 *braces with the additive group* $C_4 \times C_4$ [16,2];

3) 6 *braces with the additive group* $C_8 \times C_2$ [16,5];

4) 1 *brace with the additive group* $C_4 \times C_2 \times C_2$ [16,10];

5) 1 *brace with the additive group* $C_2 \times C_2 \times C_2 \times C_2$ [16,14].

Proposition 6. *There exist* 296 *Miller – Moreno braces with* $(A,\circ) \cong (C_4 \times C_2) \rtimes C_4$ [32,2], *i.e.*:

1) 6 *braces with the additive group* $C_8 \times C_4$ [32,3];

2) 125 *braces with the additive group* $C_4 \times C_4 \times C_2$ [32,21];

3) 8 *braces with the additive group* $C_8 \times C_2 \times C_2$ [32,36];

4) 135 *braces with the additive group* $C_4 \times C_2 \times C_2 \times C_2$ [32,45];

5) 22 *braces with the additive group* $C_2 \times C_2 \times C_2 \times C_2 \times C_2$ [32,51].

Proposition 7. *There exist* 30 *Miller – Moreno braces with* $(A,\circ) \cong C_8 \rtimes C_4$ [32,4], *i.e.*:

1) 18 *braces with the additive group* $C_8 \times C_4$ [32,3];

2) 2 *braces with the additive group* $C_{16} \times C_2$ [32,16];

3) 10 *braces with the additive group* $C_8 \times C_2 \times C_2$ [32,36].

Proposition 8. *There exist* 101 *Miller – Moreno braces with* $(A,\circ) \cong (C_8 \times C_2) \rtimes C_2$ [32,5], *i.e.*:

1) 30 *braces with the additive group* $C_8 \times C_4$ [32,3];

2) 2 *braces with the additive group* $C_{16} \times C_2$ [32,16];

3) 15 *braces with the additive group* $C_4 \times C_4 \times C_2$ [32,21];

4) 36 *braces with the additive group* $C_8 \times C_2 \times C_2$ [32,36];

5) 12 *braces with the additive group* $C_4 \times C_2 \times C_2 \times C_2$ [32,45];

6) 6 *braces with the additive group* $C_2 \times C_2 \times C_2 \times C_2 \times C_2$ [32,51].

Proposition 9. *There exist* 48 *Miller – Moreno braces with* $(A,\circ) \cong C_4 \rtimes C_8$ [32,12], *i.e.*:

1) 26 *braces with the additive group* $C_8 \times C_4$ [32,3];

2) 2 *braces with the additive group* $C_{16} \times C_2$ [32,16];

3) 20 *braces with the additive group* $C_8 \times C_2 \times C_2$ [32,36].

Proposition 10. *There exist* 12 *Miller – Moreno braces with* $(A,\circ) \cong C_{16} \rtimes C_2$ [32,17], *i.e.*:

1) 1 *brace with the additive group* C_{32} [32,1];

2) 11 *braces with the additive group* $C_{16} \times C_2$ [32,16].

2. Skew-braces from the YangBaxter package.

Lemma 2. *Let* $(A,+,\circ)$ *be a Miller – Moreno* p*-skew-brace up to order* 16. *Then* $(A,\circ)$ *is isomorphic to one of the following groups*:

1) D_8 [8,3];

2) Q_8 [8,4];

3) $(C_4 \times C_2) \rtimes C_2$ [16,3];

4) $C_4 \rtimes C_4$ [16,4];

5) $C_8 \rtimes C_2$ [16,6].

Proposition 11. *There exist* 14 *Miller – Moreno skew-braces with* $(A,\circ) \cong D_8$ [8,3], *i.e.*:

1) 1 *skew-brace with the additive group* C_8 [8,1];

2) 5 *skew-braces with the additive group* $C_4 \times C_2$ [8,2];

3) 4 *skew-braces with the additive group* D_8 [8,3];

4) 2 *skew-braces with the additive group* Q_8 [8,4];

5) 2 *skew-braces with the additive group* $C_2 \times C_2 \times C_2$ [8,5].

Proposition 12. *There exist* 7 *Miller – Moreno skew-braces with* $(A,\circ) \cong Q_8$ [8,4], *i.e.*:

1) 1 *skew-brace with the additive group* C_8 [8,1];

2) 1 *skew-brace with the additive group* $C_4 \times C_2$ [8,2];

3) 2 *skew-braces with the additive group* D_8 [8,3];

4) 2 *skew-braces with the additive group* Q_8 [8,4];

5) 1 *skew-brace with the additive group* $C_2 \times C_2 \times C_2$ [8,5].

Proposition 13. *There exist* 189 *Miller – Moreno skew-braces with* $(A,\circ) \cong (C_4 \times C_2) \rtimes C_2$ [16,3], *i.e.*:

1) 19 *skew-braces with the additive group* $C_4 \times C_4$ [16,2];

2) 43 *skew-braces with the additive group* $(C_4 \times C_2) \rtimes C_2$ [16,3];

3) 36 *skew-braces with the additive group* $C_4 \rtimes C_4$ [16,4];

4) 2 *skew-braces with the additive group* $C_8 \times C_4$ [16,5];

5) 2 *skew-braces with the additive group* $C_8 \rtimes C_2$ [16,6];

6) 36 *skew-braces with the additive group* $C_4 \times C_2 \times C_2$ [16,10];

7) 30 *skew-braces with the additive group* $C_2 \times D_8$ [16,11];

8) 13 *skew-braces with the additive group* $C_2 \times Q_8$ [16,12];

9) 8 *skew-braces with the additive group* $C_2 \times C_2 \times C_2 \times C_2$ [16,14].

Proposition 14. *There exist* 162 *Miller – Moreno skew-braces with* $(A,\circ) \cong C_4 \rtimes C_4$ [16,4] , *i.e.*:

1) 14 *skew-braces with the additive group* $C_4 \times C_4$ [16,2];

2) 38 *skew-braces with the additive group* $(C_4 \times C_2) \rtimes C_2$ [16,3];

3) 40 *skew-braces with the additive group* $C_4 \rtimes C_4$ [16,4];

4) 4 *skew-braces with the additive group* $C_8 \times C_4$ [16,5];

5) 2 *skew-braces with the additive group* $C_8 \rtimes C_2$ [16,6];

6) 23 *skew-braces with the additive group* $C_4 \times C_2 \times C_2$ [16,10];

7) 11 *skew-braces with the additive group* $C_2 \times D_8$ [16,11];

8) 13 *skew-braces with the additive group* $C_2 \times Q_8$ [16,12];

9) 4 *skew-braces with the additive group* $C_2 \times C_2 \times C_2 \times C_2$ [16,14].

Proposition 15. *There exist* 79 *Miller – Moreno skew-braces with* $(A,\circ) \cong C_8 \rtimes C_2$ [16,6], *i.e.*:

1) 1 *skew-brace with the additive group* C_{16} [16,1];

2) 2 *skew-braces with the additive group* $C_4 \times C_4$ [16,2];

3) 2 *skew-braces with the additive group* $(C_4 \times C_2) \rtimes C_2$ [16,3];

4) 4 *skew-braces with the additive group* $C_4 \rtimes C_4$ [16,4];

5) 6 *skew-braces with the additive group* $C_8 \times C_4$ [16,5];

6) 6 *skew-braces with the additive group* $C_8 \rtimes C_2$ [16,6];

7) 8 *skew-braces with the additive group* D_{16} [16,7];

8) 16 *skew-braces with the additive group* QD_{16} [16,8];

9) 8 *skew-braces with the additive group* Q_{16} [16,9];

10) 1 *skew-brace with the additive group* $C_4 \times C_2 \times C_2$ [16,10];

11) 7 *skew-braces with the additive group* $C_2 \times D_8$ [16,11];

12) 7 *skew-braces with the additive group* $C_2 \times Q_8$ [16,12];

13) 8 *skew-braces with the additive group* $(C_4 \times C_2) \rtimes C_2$ [16,13];

14) 1 *skew-brace with the additive group* $C_2 \times C_2 \times C_2 \times C_2$ [16,14].

Lemma 3. *Let* $(A,+,\circ)$ *be a* p*-skew-brace with the Miller – Moreno additive* p*-group* $(A,+)$ *up to order* 16. *Then* $(A,+)$ *is isomorphic to one of the following groups*:

1) D_8 [8,3];

2) Q_8 [8,4];

3) $(C_4 \times C_2) \rtimes C_2$ [16,3];

4) $C_4 \rtimes C_4$ [16,4];

5) $C_8 \rtimes C_2$ [16,6].

Proposition 16. *There exist* 12 *skew-braces with* $(A,+) \cong D_8$ [8,3], *i.e.:*

1) 2 *skew-braces with the multiplicative group* C_8 [8,1];

2) 3 *skew-braces with the multiplicative group* $C_4 \times C_2$ [8,2];

3) 4 *skew-braces with the multiplicative group* D_8 [8,3];

4) 2 *skew-braces with the multiplicative group* Q_8 [8,4];

5) 1 *skew-brace with the multiplicative group* $C_2 \times C_2 \times C_2$ [8,5].

Proposition 17. *There exist* 8 *skew-braces with* $(A,+) \cong Q_8$ [8,4], *i.e.:*

1) 2 *skew-braces with the multiplicative group* C_8 [8,1];

2) 1 *skew-brace with the multiplicative group* $C_4 \times C_2$ [8,2];

3) 2 *skew-braces with the multiplicative group* D_8 [8,3];

4) 2 *skew-braces with the multiplicative group* Q_8 [8,4];

5) 1 *skew-brace with the multiplicative group* $C_2 \times C_2 \times C_2$ [8,5].

Proposition 18. *There exist* 191 *skew-braces with* $(A,+) \cong (C_4 \times C_2) \rtimes$ $\rtimes C_2$ [16,3], *i.e.:*

1) 14 *skew-braces with the multiplicative group* $C_4 \times C_4$ [16,2];

2) 43 *skew-braces with the multiplicative group* $(C_4 \times C_2) \rtimes C_2$ [16,3];

3) 38 *skew-braces with the multiplicative group* $C_4 \rtimes C_4$ [16,4];

4) 2 *skew-braces with the multiplicative group* $C_8 \times C_4$ [16,5];

5) 2 *skew-braces with the multiplicative group* $C_8 \rtimes C_4$ [16,6];

6) 2 *skew-braces with the multiplicative group* QD_{16} [16,8];

7) 2 *skew-braces with the multiplicative group* Q_{16} [16,9];

8) 21 *skew-braces with the multiplicative group* $C_4 \times C_2 \times C_2$ [16,10];

9) 38 *skew-braces with the multiplicative group* $C_2 \times D_8$ [16,11];

10) 7 *skew-braces with the multiplicative group* $C_2 \times Q_8$ [16,12];

11) 19 *skew-braces with the multiplicative group* $(C_4 \times C_2) \rtimes C_2$ [16,13];

12) 3 *skew-braces with the multiplicative group* $C_2 \times C_2 \times C_2 \times C_2$ [16,14].

Proposition 19. *There exist* 190 *skew-braces with* $(A,+) \cong C_4 \rtimes C_4$ [16,4], *i.e.:*

1) 12 *skew-braces with the multiplicative group* $C_4 \times C_4$ [16,2];

2) 36 *skew-braces with the multiplicative group* $(C_4 \times C_2) \rtimes C_2$ [16,3];

3) 40 *skew-braces with the multiplicative group* $C_4 \rtimes C_4$ [16,4];

4) 4 *skew-braces with the multiplicative group* $C_8 \times C_4$ [16,5];

5) 4 *skew-braces with the multiplicative group* $C_8 \rtimes C_4$ [16,6];

6) 4 *skew-braces with the multiplicative group* D_{16} [16,7];

7) 4 *skew-braces with the multiplicative group* QD_{16} [16,8];

8) 18 *skew-braces with the multiplicative group* $C_4 \times C_2 \times C_2$ [16,10];

9) 38 *skew-braces with the multiplicative group* $C_2 \times D_8$ [16,11];

10) 8 *skew-braces with the multiplicative group* $C_2 \times Q_8$ [16,12];

11) 20 *skew-braces with the multiplicative group* $(C_4 \times C_2) \rtimes C_2$ [16,13];

12) 2 *skew-braces with the multiplicative group* $C_2 \times C_2 \times C_2 \times C_2$ [16,14].

Proposition 20. *There exist* 66 *skew-braces with* $(A,+) \cong C_8 \rtimes C_2$ [16,6], *i.e.*:

1) 2 *skew-braces with the multiplicative group* $C_4 \times C_4$ [16,2];

2) 2 *skew-braces with the multiplicative group* $(C_4 \times C_2) \rtimes C_2$ [16,3];

3) 2 *skew-braces with the multiplicative group* $C_4 \rtimes C_4$ [16,4];

4) 6 *skew-braces with the multiplicative group* $C_8 \times C_4$ [16,5];

5) 8 *skew-braces with the multiplicative group* $C_8 \rtimes C_4$ [16,6];

6) 4 *skew-braces with the multiplicative group* D_{16} [16,7];

7) 10 *skew-braces with the multiplicative group* QD_{16} [16,8];

8) 6 *skew-braces with the multiplicative group* Q_{16} [16,9];

9) 3 *skew-braces with the multiplicative group* $C_4 \times C_2 \times C_2$ [16,10];

10) 7 *skew-braces with the multiplicative group* $C_2 \times D_8$ [16,11];

11) 5 *skew-braces with the multiplicative group* $C_2 \times Q_8$ [16,12];

12) 11 *skew-braces with the multiplicative group* $(C_4 \times C_2) \rtimes C_2$ [16,13].

3. Totally split local nearrings and nearring braces from the LocalNR package. The following theorem gives examples of finite local nearrings which are totally split.

Theorem 2. *Let* R *be a finite local nearring of order* p^n. *If* $|R:L| = p$, *then* R *is totally split.*

P r o o f. If $|R:L| = p$, then $|K| = p$ and, so, K is a field. By Definition 5 and Remark 1, R is totally split.

Lemma 4 [1]. *Let* R *be a finite local nearring whose multiplicative group* R^* *is a* 2*-group. Then* R^+ *is a* 2*-group,* L *is a subgroup of index* 2 *in* R^+, *and* $R^* = 1 + L$.

As a direct consequence of Lemma 4, we obtain the following result.

Corollary 1. *Let* R *be a finite local nearring whose multiplicative group* R^* *is a* 2*-group. Then* R *is totally split.*

According to Rump [10, Definition 10], we define nearring brace.

Note that braces of order 64 are not implemented in the *AllBraces* library. From the package LocalNR [5, 6] we can extract local nearrings of order 128 via $LocalNearRing\,(k,\ell,m,n,w)$ where the arguments k, ℓ, m, n are from IdGroup of the additive group and the multiplicative group, respectively, w is the position in the list.

As an example, we choose the following nearring

$$R := LocalNearRing\,(128,160,64,183,2)$$

of order 128 with $|R:L| = 2$. Obviously, $R^* = 1 + L$, and so we can extract the nearring ideal $I = (L,+,\cdot)$ and the multiplicative and additive groups of a nearring brace $B = (L, 1+L)$ of order 64. Therefore, $L \cong \mathrm{C}_{32} \times \mathrm{C}_2$ and $1 + L \cong \mathrm{C}_{16} \times \mathrm{C}_2 \times \mathrm{C}_2$.

Acknowledgement. The authors are grateful to IIE-SRF for the support of their fellowships at the University of Warsaw.

1. *Раєвська М. Ю., Сисак Я. П.* Про локальні майже-кільця з мультиплікативною групою Міллера – Морено // Укр. мат. журн. – 2012. – **64**, № 6. – С. 811–818.
   Те саме: *Raievska M., Sysak Ya. P.* On local near-rings with Miller – Moreno multiplicative group // Ukr. Math. J. – 2012. – **64**, No. 6. – P. 930–937.
   – https://doi.org/10.1007/s11253-012-0688-z.
2. *GAP* – Groups, Algorithms, Programming, Version 4.12.0. – 2022.
   – https://www.gap-system.org.
3. *Maxson C. J.* On local near-rings // Math. Z. – 1968. – **106**, No. 3. – P. 197–205.
   – https://doi.org/10.1007/BF01110133.
4. *Miller G. A., Moreno H. C.* Non-abelian groups in which every subgroup is abelian // Trans. Am. Math. Soc. – 1903. – **4**, No. 4. – P. 398–404.
5. *Raievska I., Raievska M., Sysak Y.* DatabaseEndom128. Version v0.2 (2022) [Data set]. – Zenodo. – https://zenodo.org/records/7225377.
6. *Raievska I., Raievska M., Sysak Y.* LocalNR. Package of local nearrings. Version 1.0.4. – GAP package. – 2024. – https://gap-packages.github.io/LocalNR.
7. *Rédei L.* Das "schiefe Produkt" in der Gruppentheorie mit Anwendung auf die endlichen nichtkommutativen Gruppen mit lauter kommutativen echten Untergruppen und die Ordnungszahlen, zu denen nur kommutative Gruppen gehören // Comment. Math. Helv. – 1947. – **20**. – P. 225–264.
8. *Rump W.* Classification of cyclic braces // J. Pure Appl. Algebra. – 2007. – **209**, No. 3. – P. 671–685. – https://doi.org/10.1016/j.jpaa.2006.07.001.
9. *Rump W.* Classification of the affine structures of a generalized quaternion group of order ≥ 32 // J. Group Theory. – 2020. – **23**, No. 5. – P. 847–869.
   – https://doi.org/10.1515/jgth-2019-0174.
10. *Rump W.* Set-theoretic solutions to the Yang – Baxter equation, skew-braces, and related near-rings // J. Algebra Appl. – 2019. – **18**, No. 8. – Article ID 1950145. – 22 p. – https://doi.org/10.1142/S0219498819501457.
11. *Vendramin L., Konovalov O.* YangBaxter. Combinatorial Solutions for the Yang – Baxter equation. Version 0.10.1 – GAP package. – 2022.
    – https://gap-packages.github.io/YangBaxter/

БРЕЙСИ ТА КОСІ БРЕЙСИ І GAP: ПАКЕТИ YANGBAXTER TA LOCALNR

З використанням пакету YangBaxter виконано класифікацію брейсів (до порядку 32) і косих брейсів (до порядку 16) відповідно до адитивних або мультиплікативних груп, які є p-групами Міллера – Морено. Крім того, наведено деяку інформацію щодо майже кільцевих брейсів з пакету LocalNR.

Key words: *брейс, косий брейс, майже кільце, локальне майже кільце, група Міллера – Морено.*

[1] University of Warsaw, Warsaw, Poland,
[2] Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine

Одержано 17.10.23