Металлофиз. новейшие технол. / Metallofiz. Noveishie Tekhnol.
2013, т. 35, № 4, сс. 439–451
Оттиски доступны непосредственно от издателя
Фотокопирование разрешено только
в соответствии с лицензией

ЭЛЕКТРОННЫЕ СТРУКТУРА И СВОЙСТВА

PACS numbers: 71.45.Lr, 74.20.Fg, 74.20.Rp, 74.50.+r, 74.72.-h

Stationary Josephson Effect in Junctions Made of *d*-Wave Superconductors with Charge Density Waves

A. M. Gabovich, M. S. Li*, H. Szymczak*, and A. I. Voitenko

Institute of Physics, National Academy of Sciences of Ukraine,
46, Nauka Ave.,
03028 Kyiv, Ukraine
**Institute of Physics, Polish Academy of Sciences,*
32/46, Al. Lotników,
PL-02-668 Warsaw, Poland

Dependences of the stationary Josephson current in symmetric and non-symmetric tunnel junctions involving *d*-wave superconductors with charge density waves (CDWs) on the system parameters are calculated. Both the checkerboard and unidirectional CDW patterns are studied. The directionality of superconductive tunnelling was taken into account. As shown, CDWs can drastically influence the Josephson current, the changes being more significant in underdoped compositions.

Розраховано залежности стаціонарних джозефсоновських струмів через симетричні та несимметричні тунельні переходи, що вміщують *d*-надпровідники з хвилями зарядової густини (ХЗГ), від параметрів цих надпровідників. Розглянуто гребінчасту та шахову конфігурації ХЗГ. Прийнято до уваги спрямованість тунелювання. Показано, що ХЗГ суттєво впливають на розглядуваний джозефсоновський струм, що повинно більш сильно проявлятися для високотемпературних надпровідників з рівнем леґування нижче за оптимальний.

Рассчитаны зависимости стационарных джозефсоновских токов через симметричные и несимметричные туннельные переходы, содержащие *d*-сверхпроводники с волнами зарядовой плотности (ВЗП), от параметров этих сверхпроводников. Рассмотрены гребенчатая и шахматная конфигурации ВЗП. Принята во внимание направленность туннелирования. Показано, что ВЗП существенно влияют на рассматриваемый джозефсоновский ток, что должно сильнее проявляться для высокотемпературных сверхпроводников с уровнем легирования ниже оптимального.

Key words: high-T_c superconductors, charge density waves, checkerboard

and unidirectional CDW structures, tunnelling directionality, stationary Josephson current.

(Received December 11, 2012)

1. INTRODUCTION

Cuprates are superconductors for which the highest critical parameters in the superconducting state (the critical temperatures, T_c, the upper critical magnetic fields, H_{c2}, and the critical currents, I_c) were achieved. Therefore, they remain the most promising materials for large-scale applications [1–4]. It is natural that the properties of cuprates in the normal and superconducting states are extensively studied in order to explain both their excellent operating characteristics and specific features. The depletion of the electron density of states above T_c is one of the main enigmatic manifestations. It is observed mostly in underdoped compositions and is usually called 'pseudogap' [5–10]. It was found that the pseudogap exists below T_c as well [10–13].

There are plenty of different theories explaining this phenomenon [14–16]. In particular, our point of view presented elsewhere in more detail (see, *e.g.*, [17–19]) identifies the pseudogap in cuprates with the mean-field dielectric gap in the electron spectrum that accompanies charge density waves (CDWs) emerging above T_c and persists down to the zero temperature, $T = 0$. Such a conclusion is not at all speculative, but is based on a large body of experimental data. To be as concise as possible, the observations give rise to the following picture.

The superconducting order parameter $\bar{\Delta}(T, \theta)$ appears below T_c on the whole Fermi surface (FS). Here, θ is the angle in the momentum space. The majority of the superconducting community with a good reason considers $\bar{\Delta}(T, \theta)$ as possessing the $d_{x^2-y^2}$-wave symmetry in the conventional classification scheme [20]. Other scientists, also in accordance with certain reliable experimental data, conclude that the order parameter is an isotropic or extended s-wave one [21, 22]. A mixture of both order parameter components might occur as well, provided that the parent crystal symmetry is broken. For simplicity and definiteness, we shall hereafter assume the pure $d_{x^2-y^2}$-case.

At the same time, the dielectric order parameter (pseudogap) $\bar{\Sigma}(T, \theta)$ is an s-wave one, but, formally, its influence spans over $N = 2$ or $N = 4$, in the extended-zone scheme, FS sections (nested or dielectrized d in pairs) leaving the rest of FS in the dielectrically non-gapped (non-dielectrized, nd) state, which will be specified below. This configuration was revealed by a combination of numerous superconducting tunnelling microscopy/spectroscopy (STM/STS) [23–25], break-junction (BJ) [11, 26, 27], X-ray [28], and angle-resolved photoemission (ARPES) [29–32] measurements. In the case of four sectors, the CDW spatial pattern is a checkerboard (bidirectional) one, whereas the $N = 2$ pattern corresponds to the state of broken rotation symmetry when CDWs are unidirectional. Many experimentalists interpret their data considering CDWs as constituents of the stripe structure including charged and magnetic layers, when both the translation and rotation crystal-lattice symmetries are broken (a peculiar smectic state emerges) [33, 34]. Stripes in cuprates are considered as a manifestation of a frustrated phase separation in doped Mott insulators [35, 36].

The loss of rotation symmetry (corresponding to the emergence of unidirectional CDWs) may be interpreted as a particular case of the spontaneous establishing of the electronic smectic order owing to the distortion of checkerboard (double-smectic) one [33, 35, 37]. We note that, at low enough T, the unidirectional charge order pattern may emerge at least because of two possible reasons. First, it may be a result of smectic CDW appearance in the system of charge carriers, which are itinerant at high T, as a consequence of the electron-phonon Peierls [38] or excitonic [39] instability in the electron system with nested FS sections. Alternatively, the unidirectional configuration may arise as nematic or smectic electronic liquid crystals with the symmetry broken due to strong many-body correlations [35, 40]. The checkerboard CDW pattern can be dubbed a double-smectic one.

The order parameters $\bar{\Delta}$ (d-wave) and $\bar{\Sigma}$ (nonisotropic s-wave) coexist and compete leading to a peculiar phase diagram and an interesting thermodynamic phenomenon of $\bar{\Sigma}(T)$ reentrance [41]. Such interplay between superconductivity and CDWs should influence the Josephson tunnelling between CDW superconductors, which was demonstrated earlier [42]. Below, we present calculation of the *dc* Josephson current I_c in both symmetrical and non-symmetrical junctions involving those materials. We found the I_c-dependences on the system parameters and demonstrated that they are very strong. The results obtained reflect the actual influence of doping on I_c since CDWs (together with pseudogaps) gradually disappear while samples are varied from underdoped to heavily overdoped ones [43].

2. FORMULATION

Two kinds of tunnel junctions are considered. The first ones, S_{HTSC}–I–S_{HTSC}, are symmetric, with two identical CDW d-wave superconductors being separated by an insulating barrier. The junctions of the second kind, S_{HTSC}–I–S_{OS}, are non-symmetric with the counter-electrode being a conventional superconductor with the isotropic Bardeen–Cooper–Schrieffer (BCS) s-wave order parameter $\Delta_{\mathrm{BCS}}(T)$. In both cases, the normals to the layers in the HTSC are assumed to be parallel to each other and to the junction plane.

The *dc* Josephson critical current in the tunnel Hamiltonian approximation is given by the general equation [44]

$$I_c(T) = 4eT\sum_{\mathbf{pq}} |\tilde{T}_{\mathbf{pq}}|^2 \sum_{w_n} F^+(\mathbf{p}, w_n) F'(\mathbf{q}, w_n). \tag{1}$$

Here, $\tilde{T}_{\mathbf{pq}}$ are the matrix elements of tunnel Hamiltonian corresponded to various combinations of FS sections for superconductors taken on different sides of the tunnel junction, **p** and **q** are the transferred momenta, $e > 0$ is the elementary electrical charge, $F(\mathbf{p}, w_n)$ and $F'(\mathbf{q}, -w_n)$ are Gor'kov Green's functions for superconductors to the left and to the right of the tunnel barrier, respectively. The internal summation is carried out over the discrete fermionic 'frequencies' $w_n = (2n+1)\pi T$, $n = 0, \pm 1, \pm 2, \ldots$. The external summation in Eq. (1) for the Josephson tunnel current takes into account both the anisotropy of electron spectrum $\xi(\mathbf{p})$ in a superconductor, the directionality of tunnelling, and the dielectric electron-hole (CDW) gapping of the nested FS sections (if any).

Specifying the *dc* Josephson current (1), we introduce two kinds of directionality. The first one involves the factors $|\mathbf{v}_{g,nd}\times\mathbf{n}|$ and $|\mathbf{v}_{g,d}\times\mathbf{n}|$ [45], where $\mathbf{v}_{g,nd} = \nabla\xi_{nd}$ and $\mathbf{v}_{g,d} = \nabla\xi_d$ are the normal-state quasiparticle group velocities for proper FS sections. In the framework of the adopted here phenomenological approach this multiplier can be factorized into $\cos\theta$, where θ is the incidence angle at which the pair/quasiparticle transmits through the barrier, and an angle-independent coefficient, which can be in the usual way incorporated into the junction normal-state resistance RN (see below).

In addition, in agreement with previous studies [46] the tunnel matrix elements $\tilde{T}_{\mathbf{pq}}$ in Eq. (1) should also make allowance for the tunnel directionality (the angle-dependent probability of penetration through the barrier). Since we do not know the actual dependences for realistic junctions from microscopic considerations, we, hereafter, shall simulate the barrier-associated directionality as the n-th power of $\cos\theta$. In specific calculations, we put $n = 2$.

The angular dependences of both order parameters and the combined gap on the FS are anisotropic in the momentum space. The necessity of taking tunnel directionality into account demands that those orientations should be specified.

The profiles of the order parameters $\bar{\Delta}(T, \theta)$ and $\bar{\Sigma}(T, \theta)$ in the two-dimensional momentum space can be expressed in the factorized form $\bar{\Delta}(T, \theta) = \Delta(T) f_\Delta(\theta)$ and $\bar{\Sigma}(T, \theta) = \Sigma(T) f_\Sigma(\theta)$, where the angular functions

$$f_\Delta(\theta) = \cos 2\theta, \tag{2}$$

$$f_\Sigma(\theta) = \begin{cases} 1, \text{ for } -\alpha + k\Omega < \theta < \alpha + k\Omega \text{ (FS } d-\text{sectors)} \\ 0, \text{ otherwise (FS } nd-\text{section)} \end{cases} \tag{3}$$

are shown in Fig. 1, α is the half-width of the CDW sector, k is an integer number, and

$$\Omega = \begin{cases} \pi / 2, \text{ checkerboard CDW } (N = 4) \\ \pi, \text{ unidirectional CDW } (N = 2). \end{cases} \quad (4)$$

The bisectrix of the CDW sector is directed along the basic vector $\mathbf{k}_x$ of the reciprocal crystal lattice. Equations (3) and (4) were written for layered cuprates where directions corresponding to the maxima of superconducting order parameter lobes and CDW sectors coincide (the $d_{x^2-y^2}$-wave symmetry). Note that in the case of unidirectional CDW, it is the positive superconducting order parameter lobe that overlaps the CDW sector.

The quantities N and together with the zero-T values of order parameters for 'parent' pure d-wave superconductor, Δ_0, and normal CDW metal, Σ_0, constitute a full kit of parameter describing the partially dielectrized d-wave superconductor. The solution of this self-consistent problem [18, 19, 41, 47] demonstrates a reduction of each order parameter in comparison with their corresponding 'pure' counterparts and an interplay of order parameters on the dielectrized FS sections. In particular, the total gap on those sections equals

$$\bar{D}(T, \theta) = \sqrt{\bar{\Sigma}^2(T, \theta) + \bar{\Delta}^2(T, \theta)}. \quad (5)$$

The explicit expressions for Green's functions for ordinary and partially gapped CDW d-wave superconductors are given elsewhere [18, 19, 41, 47] and are used below to obtain operational formulas.

To specify the junction setup, we have also to indicate the orientation of each electrode order parameter with respect to the junction plane. In the general case, the FS orientations on both sides are charac-

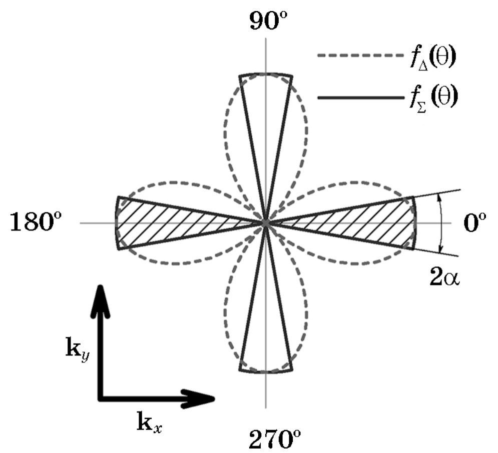

Fig. 1. Schematic diagram for the functions $f_\Delta(\theta)$ (Eq. (2)) and $f_\Sigma(\theta)$ (Eq. (3)) determining the orientation of superconducting, Δ, and CDW, Σ, order parameters in the two-dimensional reciprocal lattice of high-T_c superconductors.

terized by the angles γ and γ' between the direction of CDW sector bisectrix and the normal $\mathbf{n}$ to the junction (hereafter, all primed quantities will be associated with the right hand side electrode). The orientational dependences of *dc* Josephson current will be considered elsewhere.

Taking all the aforesaid into account and assuming the directionally coherent character of tunnelling [46], as opposed to the non-coherent approximation [44], we arrive at the following formula for the *dc* Josephson current across the tunnel junction:

$$I_c(T,\gamma,\gamma') = \frac{1}{2eR_N}\frac{1}{\pi}\int_{-\pi/2}^{\pi/2} d\theta \cos\theta W(\theta) P(T,\theta,\gamma,\gamma'), \tag{6}$$

where

$$P(T,\gamma,\gamma') = \bar{\Delta}(T,\theta,-\gamma)\bar{\Delta}'(T,\theta,-\gamma')\times$$

$$\times \int_{\min\{\bar{D}(T,\theta,-\gamma),\bar{D}'(T,\theta,-\gamma')\}}^{\max\{\bar{D}(T,\theta,-\gamma),\bar{D}'(T,\theta,-\gamma')\}} \frac{dx \tanh\dfrac{x}{2T}}{\sqrt{(x^2-[\bar{D}(T,\theta,-\gamma)]^2)([\bar{D}'(T,\theta,-\gamma')]^2-x^2)}}. \tag{7}$$

Formula (6) is applicable for both the symmetric (S_{HTSC}–I–S_{HTSC}, with an accuracy to the electrode orientation) and non-symmetric (S_{HTSC}–I–S_{OS}) junctions. The parameter R_N is the normal-state resistance of the tunnel junction determined by $|\tilde{T}_{\mathbf{pq}}|^2$ without the factorized multiplier $W(\theta)$. Integration over the angle variable θ is carried out within the interval $-\pi/2 \le \theta \le \pi/2$. The directionality coefficient $W(\theta)$ accounts for the preferred tunnelling probability perpendicular to the junction plane [46] and, as was mentioned above, will be assumed here to be $\cos^2\theta$.

It is convenient to introduce the dimensionless superconducting, $\Delta(t) = \Delta(T)/\Delta_0$ and $\delta_{\mathrm{BCS}}(t) = \Delta_{\mathrm{BCS}}(T)/\Delta_0$, as well as dielectric, $\sigma(t) = \Sigma(T)/\Delta_0$, order parameters, the dimensional temperature, $t = T/\Delta_0$, and the dimensionless Josephson current $i_c(t) = 2eRN\Delta_0^{-2}I_c(T)$. We restrict ourselves to the zero-T case and shall consider only the simplest angular configurations to make the article as compact as possible. The omitted dependences will be studied elsewhere. Here, we are mostly interested in the amplitudes of the Josephson currents and their variations with the natural system parameters α and $\sigma_0 \equiv \Sigma_0/\Delta_0$.

3. RESULTS AND DISCUSSION

First, let us examine the situation with the preserved rotational symmetry of the underlying crystal lattice, when CDWs form the checkerboard (double-smectic) pattern ($N = 4$).

The dependences of the coherent *dc* Josephson current i_c across S_{HTSC}–I–S_{HTSC} junction on for varying σ_0 and $\gamma = \gamma' = 0$ (it means that the positive superconducting order parameter lobes in both electrodes are oriented normally to the junction plane) is shown in Fig. 2. One sees that for larger σ_0 the current i_c is rapidly reduced with α, since the CDW influence on superconductivity is destructive. At the same time, for a smaller $\sigma_0 = 0.75$ a virtual growth of the CDW sector does not lead to any result because the system remains in the pure $d_{x^2-y^2}$ BCS superconducting state (see Refs. [18, 19]). Only for large $\alpha > 35°$ the metal falls within the mixed CDW + superconducting state, so that i_c starts to decrease.

On the other hand, if one fixes α, but varies σ_0, the dependence $i_c(\sigma_0)$ is also a decaying function in the mixed-phase area of the σ_0–α phase diagram. This is demonstrated in Fig. 3 plotted for the same experimental setup as in Fig. 2.

The dependences $i_c(\alpha)$ for unidirectional (smectic) CDW structures are more interesting than their counterparts for $N = 4$. Indeed, they may exhibit varying character and even non-monotonic behaviour (if σ_0 is small enough), as is shown in Fig. 4 for different σ_0 and $\gamma = \gamma' = 0$.

These peculiarities are a consequence of the CDW-sector expansion onto the superconducting lobes free from CDW influence when the opening angle 2α is restricted to the initial two quadrants of the momentum plane. At the same time, if $\gamma' = 0$ but $\gamma = 90°$ (Fig. 5) the de-

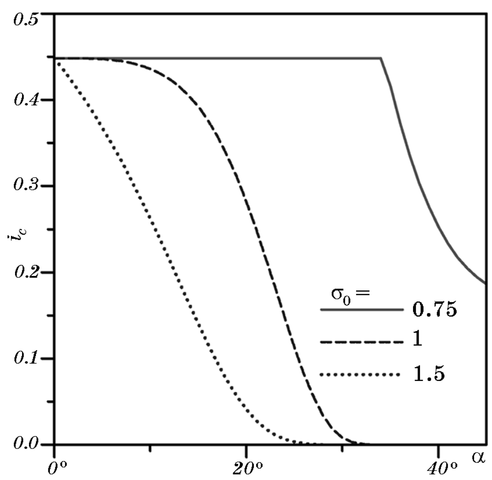

Fig. 2. Dependences of dimensionless *dc* Josephson current i_c through the symmetric S_{HTSC}–I–S_{HTSC} junction on the CDW sector opening parameter α for various ratios σ_0 between the magnitudes of parent CDW and $d_{x^2-y^2}$-wave superconducting order parameters. $N = 4$, $\gamma = \gamma' = 0$. See explanations in the text.

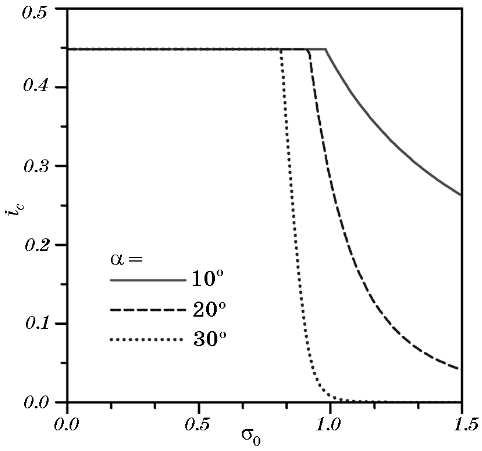

Fig. 3. The same as in Fig. 2 but on σ_0 for various α.

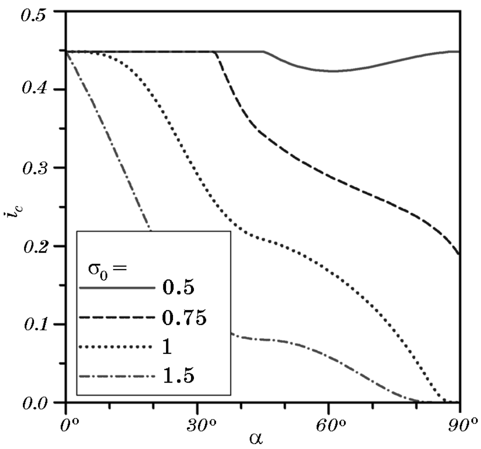

Fig. 4. The same as in Fig. 2 but for $N = 2$.

pendences $i_c(\alpha)$ differ quantitatively although their qualitative behaviour remains the same as in Fig. 4. Note that, in this case, we have a π-junction so that the current sign is negative.

For $\gamma = \gamma' = 0$ and $N = 2$, the dependences $i_c(\sigma_0)$ are also steeply falling which is depicted in Fig. 6 for different α.

The same behaviour is observed $\gamma = 90°$ (π-junction, Fig. 7) although here CDWs are less effective as a factor suppressing superconductivity.

Similar phenomena are inherent to $S_{\rm HTSC}$–I–$S_{\rm OS}$ junctions. They are analyzed below assuming $\delta_{\rm BCS}(t = 0) = 0.1$ which is appropriate, *e.g.*, for

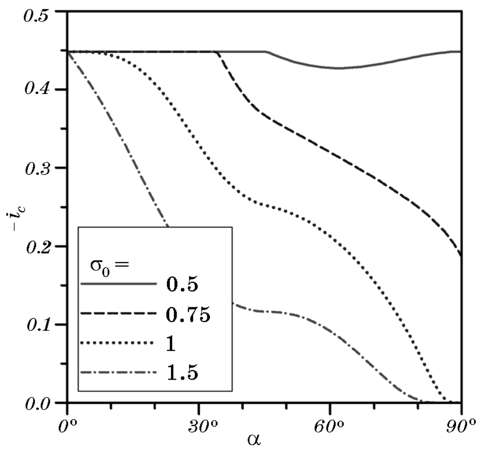

Fig. 5. The same as in Fig. 4 but for $\gamma = 90°$.

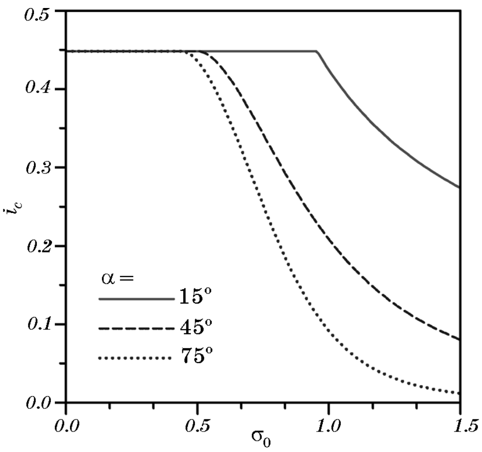

Fig. 6. The same as in Fig. 3 but for non-symmetric $S_{\rm HTSC}$–I–$S_{\rm OS}$ junction.

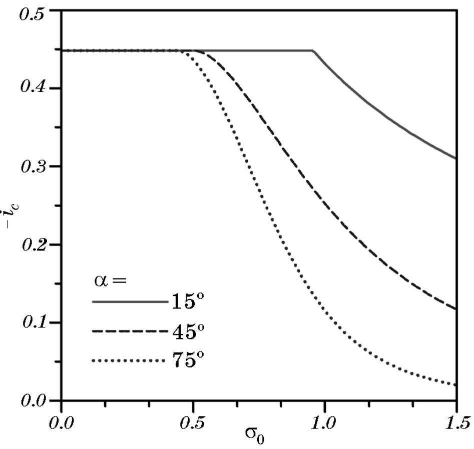

Fig. 7. The same as in Fig. 6 but for $\gamma = 90°$.

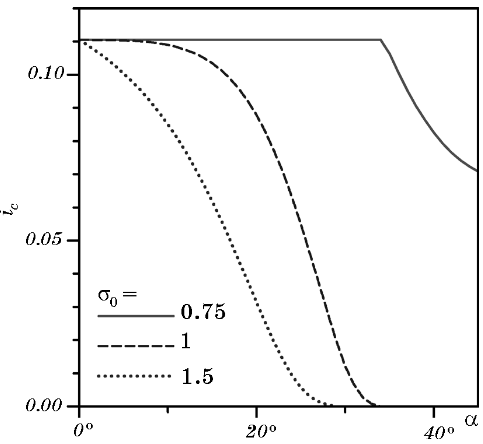

Fig. 8. The same as in Fig. 2 but for the non-symmetric S_{HTSC}–I–S_{OS} junction. $\delta_{BCS} = 0.1$, $N = 4$. See explanations in the text.

Nb as the ordinary s-wave superconductor. Dependences $i_c(\alpha)$ for the checkerboard CDW structure ($N = 4$), $\gamma = 0$ and various σ_0 are displayed in Fig. 8. Curves are similar to those from Fig. 2 with their magnitude substantially smaller due to the factor $\delta_{BCS}(t = 0)$.

For unidirectional CDWs ($N = 2$) $i_c(\alpha)$ are depicted in Fig. 9 ($\gamma = 0$) and in Fig. 10 ($\gamma = 90°$, π-junction).

One can see here a curious phenomenon. Namely, in certain ranges of α the Josephson current may even slightly grow with the increas-

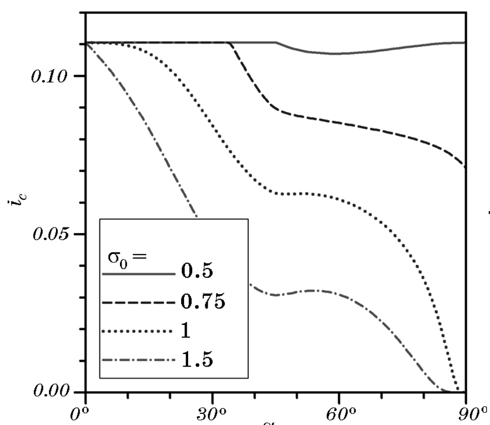

Fig. 9. The same as in Fig. 8 but for $N = 2$.

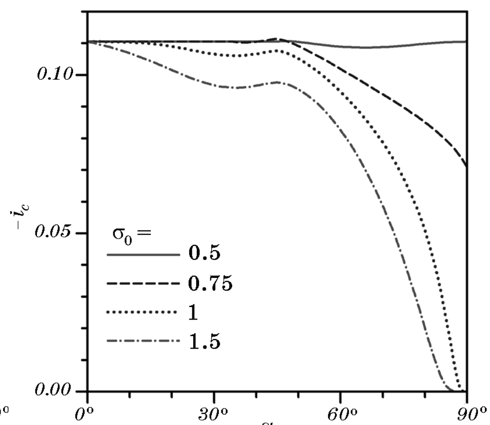

Fig. 10. The same as in Fig. 9 but for $\gamma = 90°$.

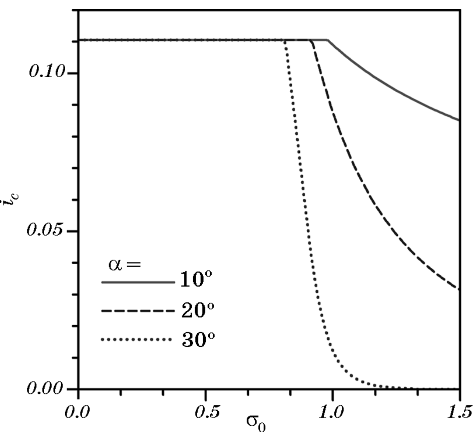

Fig. 11. The same as in Fig. 3 but for the S_{HTSC}–I–S_{OS} junctions.

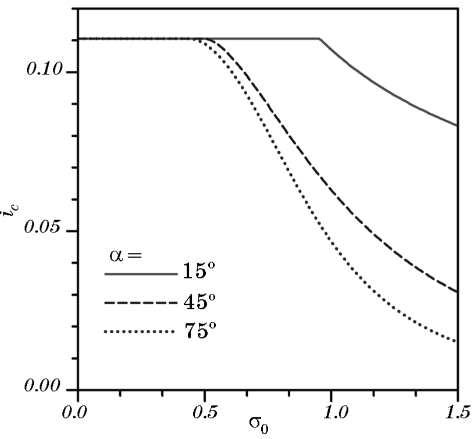

Fig. 12. The same as in Fig. 11 but for $N = 2$.

ing sector width. The reason consists in the alternating signs of the neighbouring superconducting lobes. Therefore, at $\alpha = 45°$ one can see a metamorphosis when CDW sectors begin to suppress the lobes with the opposite sign. Moreover, when $\gamma = 90°$, the curves $i_c(\alpha)$ are rather flat with minor maxima at small α's but become very abrupt at $\alpha > 45°$. Such dependences may be used as an indicator of CDWs in cuprates.

The dependences $i_c(\sigma_0)$ are intuitively more clear. They are demonstrated in Fig. 11 for $N = 4$, in Fig. 12 for $N = 2$ and $\gamma = 0$, and in Fig. 13 for $N = 2$ and $\gamma = 90°$. It is remarkable that for $\alpha = 45°$ (see Fig. 13) the curve $i_c(\sigma_0)$ exhibits a weak growth with σ_0 near $\sigma_0 \cong 0.75$ before a stan-

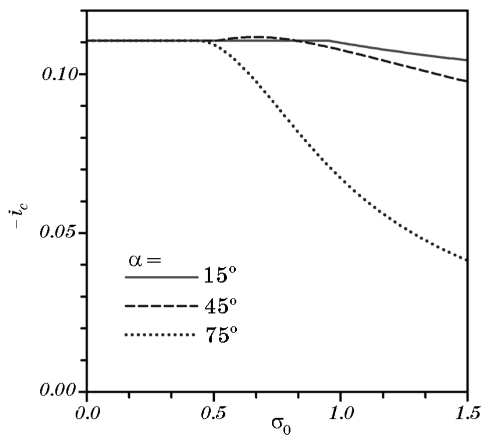

Fig. 13. The same as in Fig. 12 but for $\gamma = 90°$.

dard fall at larger σ_0.

4. CONCLUSIONS

The presented theory shows that one can successfully study stationary Josephson current i_c in S_{HTSC}–I–S_{HTSC} or S_{HTSC}–I–S_{OS} junctions in order to uncover the existence and the role of CDWs in d-wave superconductors. Our calculations were carried out for the set of parameters modelling high-T_c oxides. Two kinds of dependences were studied: $i_c(\alpha)$ and $i_c(\sigma_0)$ where both arguments decrease with oxygen doping. The CDW sector opening 2α can be directly measured by photoemission studies. Therefore, it is possible to qualitatively check the theory. It would be especially challenging to find the predicted non-monotonic behaviour of $i_c(\alpha)$ inherent to certain situations predicted by the calculations. Note that the results obtain differ from those for CDW s-wave superconductors obtained earlier [48].

ACKNOWLEDGEMENTS

The authors are grateful to Sergey Borisenko (Dresden) and Alexander Kordyuk (Kyiv) for useful discussions. The work was partially supported by the Project No. 8 of the 2012-2014 Scientific Cooperation Agreement between Poland and Ukraine.

REFERENCES

1. J. R. Hull and M. Murakami, *Proc. IEEE*, **92**: 1705 (2004).
2. R. M. Scanlan, A. P. Malozemoff, and D. C. Larbalestier, *Proc. IEEE*, **92**: 1639 (2004).
3. S. A. Gourlay, G. Sabbi, F. Kircher, N. Martovetsky, and D. Ketchen, *Proc. IEEE*, **92**: 1675 (2004).
4. W. V. Hassenzahl, D. W. Hazelton, B. K. Johnson, P. Komarek, M. Noe, and C. T. Reis, *Proc. IEEE*, **92**: 1655 (2004).
5. K. Fujita, A. R. Schmidt, E-A. Kim, M. J. Lawler, D. H. Lee, J. C. Davis, H. Eisaki, and S-i. Uchida, *J. Phys. Soc. Jpn.*, **81**: 011005 (2012).
6. H. Mukuda, S. Shimizu, A. Iyo, and Y. Kitaoka, *J. Phys. Soc. Jpn.*, **81**: 011008 (2012).
7. T. Yoshida, M. Hashimoto, I. M. Vishik, Z-X. Shen, and A. Fujimori, *J. Phys. Soc. Jpn.*, **81**: 011006 (2012).
8. S. E. Sebastian, N. Harrison, and G. G. Lonzarich, *Rep. Prog. Phys.*, **75**: 102501 (2012).
9. S. I. Vedeneev, *Usp. Fiz. Nauk*, **182**: 669 (2012).
10. A. Dubroka, L. Yu, D. Munzar, K. W. Kim, M. Rössle, V. K. Malik, C. T. Lin, B. Keimer, Th. Wolf, and C. Bernhard, *Eur. Phys. J. Special Topics*, **188**: 73 (2010).
11. T. Ekino, Y. Sezaki, and H. Fujii, *Phys. Rev. B*, **60**: 6916 (1999).

12. V. M. Krasnov, A. Yurgens, D. Winkler, P. Delsing, and T. Claeson, *Phys. Rev. Lett.*, **84**: 5860 (2000).
13. A. A. Kordyuk, S. V. Borisenko, V. B. Zabolotnyy, R. Schuster, D. S. Inosov, D. V. Evtushinsky, A. I. Plyushchay, R. Follath, A. Varykhalov, L. Patthey, and H. Berger, *Phys. Rev. B*, **79**: 020504 (2009).
14. N. Kristoffel, *Mod. Phys. Lett. B*, **17**: 451 (2003).
15. M. Norman, D. Pines, and C. Kallin, *Adv. Phys.*, **54**: 715 (2005).
16. T. M. Rice, K-Y. Yang, and F. C. Zhang, *Rep. Prog. Phys.*, **75**: 016502 (2012).
17. A. M. Gabovich and A. I. Voitenko, *J. Phys.: Condens. Matter*, **9**: 3901 (1997).
18. T. Ekino, A. M. Gabovich, M. S. Li, M. Pękała, H. Szymczak, and A. I. Voitenko, *J. Phys.: Condens. Matter*, **23**: 385701 (2011).
19. T. Ekino, A. M. Gabovich, M. S. Li, M. Pękała, H. Szymczak, and A. I. Voitenko, *Symmetry*, **3**: 699 (2011).
20. V. P. Mineev and K. V. Samokhin, *Introduction to Unconventional Superconductivity* (Amsterdam: Gordon and Breach Science Publishers: 1999).
21. R. A. Klemm, *Philos. Mag.*, **85**: 801 (2005).
22. G. M. Zhao, *Phys. Scr.*, **83**: 038302 (2011).
23. T. Kasai, S. Yamashita, H. Nakajima, T. Fujii, I. Terasaki, T. Watanabe, H. Shibata, and A. Matsuda, *Physica C*, **470**: S173 (2010).
24. A. R. Schmidt, K. Fujita, E-A. Kim, M. J. Lawler, H. Eisaki, S. Uchida, D-H. Lee, and J. C. Davis, *New J. Phys.*, **13**: 065014 (2011).
25. I. Zeljkovic, E. J. Main, T. L. Williams, M. C. Boyer, K. Chatterjee, W. D. Wise, Y. Yin, M. Zech, A. Pivonka, T. Kondo, T. Takeuchi, H. Ikuta, J. Wen, Z. Xu, G. D. Gu, E. W. Hudson, and J. E. Hoffman, *Nature Mater.*, **11**: 585 (2012).
26. T. Ekino, S. Hashimoto, T. Takasaki, and H. Fujii, *Phys. Rev. B*, **64**: 092510 (2001).
27. T. Ekino, A. M. Gabovich, M. S. Li, M. Pękała, H. Szymczak, and A. I. Voitenko, *Phys. Rev. B*, **76**: 180503 (2007).
28. J. Chang, E. Blackburn, A. T. Holmes, N. B. Christensen, J. Larsen, J. Mesot, R. Liang, D. A. Bonn, W. N. Hardy, A. Watenphul, M. V. Zimmermann, E. M. Forgan, and S. M. Hayden, *Nature Phys.*, **8**: 871 (2012).
29. A. A. Kordyuk, V. B. Zabolotnyy, D. V. Evtushinsky, B. Büchner, and S. V. Borisenko, *J. Electron Spectrosc. Relat. Phenom.*, **181**: 44 (2010).
30. A. A. Kordyuk, V. B. Zabolotnyy, D. V. Evtushinsky, D. S. Inosov, T. K. Kim, B. Büchner, and S. V. Borisenko, *Eur. Phys. J. Special Topics*, **188**: 153 (2010).
31. S. V. Borisenko, V. B. Zabolotnyy, A. A. Kordyuk, D. V. Evtushinsky, T. K. Kim, E. Carleschi, B. P. Doyle, R. Fittipaldi, M. Cuoco, A. Vecchione, and H. Berger, *J. Vis. Exp.*, **68**: e50129 (2012).
32. I. M. Vishik, M. Hashimoto, R-H. Hed, W-S. Lee, F. Schmitt, D. Lu, R. G. Moore, C. Zhang, W. Meevasana, T. Sasagawa, S. Uchida, K. Fujita, S. Ishida, M. Ishikado, Y. Yoshida, H. Eisaki, Z. Hussain, T. P. Devereauxa, and Z-X. Shen, *Proc. Nat. Acad. Sci. USA*, **109**: 18332 (2012).
33. S. A. Kivelson, I. P. Bindloss, E. Fradkin, V. Oganesyan, J. M. Tranquada, A. Kapitulnik, and C. Howald, *Rev. Mod. Phys.*, **75**: 1201 (2003).
34. G. Ghiringhelli, M. Le Tacon, M. M. S. Blanco-Canosa, C. Mazzoli, N. B. Brookes, G. M. De Luca, A. Frano, D. G. Hawthorn, F. He, T. Loew, M. M. Sala, D. C. Peets, M. Salluzzo, E. Schierle, R. Sutarto, G. A. Sawatzky, E. Weschke, B. Keimer, and L. Braicovich, *Science*, **337**: 821 (2012).
35. M. Vojta, *Adv. Phys.*, **58**: 699 (2009).
36. E. Dagotto, *Science*, **309**: 257 (2005).

37. E. Fradkin, S. A. Kivelson, M. J. Lawler, J. P. Eisenstein, and A. P. Mackenzie, *Annu. Rev. Condens. Matter Phys.*, **1**: 153 (2010).
38. P. Monceau, *Adv. Phys.*, **61**: 325 (2012).
39. Yu. V. Kopaev, *Trudy Fiz. Inst. Akad. Nauk SSSR*, **86**: 3 (1975).
40. K. Sun, B. M. Fregoso, M. J. Lawler, and E. Fradkin, *Phys. Rev. B*, **78**: 085124 (2008).
41. A. M. Gabovich and A. I. Voitenko, *Phys. Rev. B*, **80**: 224501 (2009).
42. A. M. Gabovich and A. I. Voitenko, *Fiz. Nizk. Temp.*, **38**: 414 (2012).
43. A. Damascelli, Z. Hussain, and Z-X. Shen, *Rev. Mod. Phys.*, **75**: 473 (2003).
44. I. O. Kulik and I. K. Yanson, *Josephson Effect in Superconducting Tunnel Structures* (Jerusalem: Israel Program for Scientific Translation: 1972).
45. Yu. S. Barash, A. V. Galaktionov, and A. D. Zaikin, *Phys. Rev. B*, **52**: 665 (1995).
46. M. Ledvij and R. A. Klemm, *Phys. Rev. B*, **51**: 3269 (1995).
47. A. M. Gabovich, A. I. Voitenko, T. Ekino, M. S. Li, H. Szymczak, and M. Pęka ła, *Adv. Condens. Matter Phys.*, 2010; Article ID 681070 (2010).
48. A. M. Gabovich, D. P. Moiseev, A. S. Shpigel, and A. I. Voitenko, *Phys. Status Solidi B*, **161**: 293 (1990).