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# An intrinsic physical content of "single photon power" – ($h\nu\cdot\Delta\nu$)

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**Abstract.** Considered in this paper is the possibility to use information properties of photon noise inherent to thermal radiation. Using the calculations of threshold limitations for detecting the fluctuations of thermal radiation as a signal and not disturbances only, we have adduced some arguments concerning the phenomenological content of the "power of a single photon" $q_{\Delta\nu} \equiv h\nu\cdot\Delta\nu$ not defined earlier, where the frequency band $\Delta\nu$ is determined by an observation spectral slit. It has been shown that a non-traditional view of "photonic noise" determined by this factor appears in relation with the fluctuations of the photon flux at the observation spectral slit. With definite measurement parameters, this kind of noise is capable to append some essential points to classical models of photon noises and even block access to measurements of fluctuations in the power of thermal radiation. Also adduced are supplementary considerations concerning the existing models of arising shot noise at the photodetector output as a result of physical-and-statistical specificity of the electron excitation process and its kinetics. Offered is a model for a background radiator, which allows us performing the numerical calculations with the use of the specific parameters of the background structure.

**Keywords:** thermal radiation, photon noise, fluctuations, power of a single photon.

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## 1. Introduction

This work is aimed at using the fluctuations of thermal radiation (TR) as a carrier of definite physical information on the radiator. Considering the example of a small size (SR) single radiator within the model of blackbody (BB), we made an attempt to calculate threshold limitations for detecting the intrinsic photon noise of TR as a signal. This approach is justified, first, by perspectives in realization of additional physical information that is contained in the variance of TR noise inherent to SR and has been illustrated in [1-4] as based on the well-known relation between the dispersion $\langle\Delta F^2\rangle$ of a random value $F$ and its mean value $\langle F\rangle$:

$$q_F = \left\langle \Delta F^2 \right\rangle \Big/ \left\langle F \right\rangle. \quad (1.1)$$

As always, Exp. (1.1) is used in the form of the so-called relative fluctuation $q_F^* = \sqrt{\left\langle \Delta F^2 \right\rangle} \Big/ \left\langle F \right\rangle$, which, to some extent, screens the quantitative sign of its physical content $q_F$.

The simplest example concerns fluctuations of the charge $Q$ in a capacitor:

$$\left\langle \Delta Q^2 \right\rangle = e \cdot \left\langle Q \right\rangle \to q_Q = e = 1.6 \cdot 10^{-19} \quad \text{where}$$

$$q_Q^* = \frac{\sqrt{e\left\langle Q \right\rangle}}{\left\langle Q \right\rangle} = \frac{\sqrt{e}}{\sqrt{\left\langle Q \right\rangle}} \to \frac{\sqrt{1.6 \cdot 10^{-19}}}{\sqrt{\left\langle Q \right\rangle}} = \frac{4 \cdot 10^{-10}}{\sqrt{\left\langle Q \right\rangle}}.$$

Comprehension of Eq. (1.1) can be founded using the respective analysis of recognized scientific sources [5-16], etc. So, adduced in the papers [1, 2] are the examples of validity for Eq. (1.1) both for the ideal gas thermodynamic and electric current fluctuations in the cases of resistor thermal noise, current shot noise, and the generation – recombination one. It was shown that in the case of adequate statistics the value $q_F$ (i.e., the "intrinsic micro-quantity of chaos" [1] that will be renamed as "eigen-parameter"≡"e-p" in what follows) is a physical concept which is fully defined by its content and dimensionality and includes concrete characteristics of a fluctuating physical system.

Based on the analysis of the references [5-8, 13-21] and results of researches [1-4] the following improved list of "e-p"≡ $q_F$ for the BB TR is proposed:

$q_n = \left(\left\langle n \right\rangle + 1\right)$ is the "eigen-number" of photons in (BB cavity) TR field, (1.2)

$q_f = \left(\langle n\rangle + 1\right)\cdot \Delta\nu_q$ is the "eigen-flow"
BB TR photon within a band of $\Delta\nu_q$, (1.3)

$q_E = h\nu\left(\langle n\rangle + 1\right)$ is the "eigen-energy"
of TR inside (BB cavity), (1.4)

$q_\varepsilon = \left(\langle n\rangle + 1\right)\dfrac{h\nu}{V_q}$ is the "eigen-density energy"
of TR inside (BB cavity), (1.5)

$q_P = \left(\langle n\rangle + 1\right)\cdot h\nu\cdot\Delta\nu_q$ is the "eigen-power"
of BB TR within a band of $\Delta\nu_q$, (1.6)

$q_p = \left(\langle n\rangle + 1\right)\cdot\dfrac{h\nu}{V_q}\cdot\Delta\nu_q$ is the "eigen-power density"
of BB TR within a band of $\Delta\nu_q$, (1.7)

$q_{\Delta\nu} = h\nu\cdot\Delta\nu$ is "eigen-power-Δν"
of BB TR power flow which is restricted
in spectral slit with a frequency band Δν. (1.8)

Physical parameter $V_q$ in (1.5) and (1.7) is a specific volume which in the case of SR coincides with its real volume. As to the frequency band $\Delta\nu_q$ in (1.3), (1.6) and (1.7), this has to be defined in accordance with a specific physical situation (problem).

The above expressions for $q_F$ (1.3)-(1.7) are in accordance with the quantum-mechanical "TR fluctuation factor" $= \left(\langle n\rangle + 1\right)$ [8, 14-19].

To establish accordance of Eqs. (1.5)-(1.7) to the fluctuations of TR energy density $\varepsilon(\lambda)$ in the BB cavity, one can use the classic formula for the variance $\left\langle[\Delta E(\lambda)]^2\right\rangle$ [5-7] of the total TR energy $E(\lambda) = \langle V_q\rangle\cdot\varepsilon(\lambda)$ in the BB cavity of a small constant volume[1] $\langle V_q\rangle$. In accord with the definition [22], the variance of the product of the constant $\langle V_q\rangle$ and fluctuating $\varepsilon(\lambda)$ values is $\left\langle[\Delta E(\lambda)]^2\right\rangle = \langle V_q\rangle^2\cdot\left\langle[\Delta\varepsilon(\lambda)]^2\right\rangle$. Using $q_\varepsilon$ (1.5) to determine $\left\langle[\Delta\varepsilon(\lambda)]^2\right\rangle$ in accordance with (1.1), one can obtain the following expression:

$$\left\langle[\Delta E(\lambda)]^2\right\rangle = \langle V_q\rangle^2\cdot\langle\varepsilon(\lambda)\rangle\times q_\varepsilon =$$
$$\langle V_q\rangle^2\cdot\langle\varepsilon(\lambda)\rangle\times\frac{hc}{\lambda\cdot\langle V_q\rangle}\cdot\left(1+\langle n\rangle\right) = \qquad (1.9)$$
$$= \langle E(\lambda)\rangle\times\frac{hc}{\lambda}\cdot\left(1+\langle n\rangle\right),$$

which is the Einstein formula [5-7]. Being based on (2.8) and using Eqs (1.5), (1.7), considered in the works [3, 4] was the calculation model of the principle for distant identification of both single SR and the stochastic set ("cloud") of SR [4]. It is noteworthy that while determining the thresholds for detecting the intrinsic fluctuations of TR SR BB power the "problem of the single photon power" is solved.

Thus, in the literature devoted to TR and its fluctuations (e.g. [8] and so on) one can find the understandable by its dimensionality self-sufficient expression $h\nu\cdot\Delta\nu\,[\mathrm{erg}\cdot\mathrm{s}^{-1}]$, but it, as an acting physical value, does not find its specific phenomenological interpretation. As it will be shown below, this enigmatic single-photon power $h\nu\cdot\Delta\nu$ can play a rather essential role and obtain definite phenomenological interpretation. Expediency of using the property (1.1) in the case of TR SR seems to be obvious as, in spite of wide applications of laser technologies, TR is inherent to any objects both natural and technological cloud-like SR systems.

## 2. **Fluctuations of TR in terms of the "eigen-parameter"**

*2.1. Initial conceptions.*

It is obvious that only using measurements of the TR power emitted by SR and fluctuations of this power one can obtain some information included in TR of SR. As to TR, the property (1.1) can be applied beyond controversy, and first of all, due to the fact that photons do not interact with each other (see e.g. [7, 15-17]).

In what follows, we shall consider the problem of physical informativity inherent to the TR noise variance in the most accessible form. Here, we originate from the opportunity to use the following approach: TR power variance is written in accordance with relations (1.1) and (1.5)-(1.7) when determining the thresholds for registration of the TR power variance for SR within the model of BB with the dimension corrections [23, 24].

Further, we shall use the following designations: $q_F \equiv$ "e-p" ; $c$ – light speed, ν – frequency (and the respective wavelength – $\lambda = c/\nu$) of observed TR, which corresponds to the middle of Δν (or Δλ) which is a spectral band for observations; $\langle n\rangle = \left[\left(\exp\dfrac{hc}{\lambda\cdot kT}\right) - 1\right]^{-1}$ is the Planck function, $Z(\nu)\cdot\Delta\nu = V\cdot\dfrac{8\pi\cdot\nu^2\Delta\nu}{c^3}$ is the number of TR spatial modes within the spectral frequency band Δν inside the BB cavity with the volume $V = R^3$; $\varepsilon_i(\lambda) = \dfrac{hc}{\lambda}\cdot\dfrac{8\pi}{\lambda^3}\cdot\left[1-\dfrac{\lambda^2}{4R_i^2}\right]\cdot\left(\dfrac{\Delta\lambda}{\lambda}\right)\cdot\langle n_i\rangle$ is the TR energy density inside the SR cavity of a cubic shape with the dimensional correction $\left[1-\dfrac{\lambda^2}{4R_i^2}\right]$ [23, 24] that limits the number of TR modes in the volume $V_i = R_i^3$; the

[1]Volume that did not lose its space-time relation with the number of modes of an actual frequency in its cavity yet [13].

ratio $(\Delta\lambda/\lambda)$ in the expression for $\varepsilon_i(\lambda)$ is kept constant during spectral calculations or measurements (hereinafter, the subscript "*i*" means belonging to SR) .

*2.2. The model of background TR radiator*

To calculate fluctuations of an arbitrary large background radiator, it is necessary to know the size "eigen-volume" for this background ($V_q$ – in formulas (1.5) and (1.7)), which is not described in literature accessible to the authors (for instance, [6-8, 14-21] and so on).

To solve the problem, one needs the model of a background radiator, which is capable to provide more or less adequate results for numeral calculations of the fluctuation variance for TR background $\langle \Delta P_{jB}^2 \rangle$ (hereinafter, the subscripts "*j*" and "*B*" mean belonging to background elements).

Let us assume the following simplified model: in a warm background (B) an ideal photodetector (PD) registers TR from a single SR of a cubic shape with the constant cavity volume $V_i = R_i^3$ cm$^3$; let the size of radiating area be equal $S_i \approx R_i^2$.

The model of such a background[2] (Fig. 1) that provides the possibility for a well grounded calculations of the TR background power seems as follows. Within the boundaries of the observed background area $S_{obs}$, the area $S_B = \Delta P_{jB}^2 \cdot N_{jB}$, serves as the emitting one (see the subscript "*B*") that consist of $N_{jB}$ identical and small in their

sizes stationary radiators (with their non-fluctuating volume $R_{jB}^3$). When using the dimensional corrections [23, 24] to $N_{jB}$ small radiators, we consider their size in calculations as close to $(R_{jB})^2 \approx (2R_i)^2$.

In calculations below, we shall use the following numeric parameters for the model of the background (Fig. 1):

$$S_{jB} = (\text{Spacing} + R_{jB} + \text{Spacing})^2\,\text{cm}^2 = $$
$$= (0.002 + 0.004 + 0.002 -)^2 = 0.000064\,\text{cm}^2.$$

Density $N_{jB}$ inside $S_{obs}$ is

$$\Gamma_{jB} = \frac{1}{S_{jB}} = \frac{1}{(0.008\,\text{cm})^2} = 15625\,\text{cm}^{-2},\; N_{jB} = \Gamma_{jB} \cdot S_{obs}.$$

To simplify the formulas, we use the "lensless[3]" choice of the system "radiators (SR + B) - photodetector (PD)". The values $l_{iD}$ and $L_{BD}$ correspond to distances from SR and its background $S_B$ to PD, respectively. The input PD hole can be defined via the radius $r_D$; the value $2\pi \cdot r_D^2$ will serve as an area of the PD photosensitive element. Joint action of the TR system (SR + B) on PD will depend on spatial apertures in accordance with the above approximations that do not contradict to the conventional ones [25]: the aperture, within the boundaries of which PD registers TR of a single SR is $A_{iD} \cong \left(S_i \cdot \frac{r_D^2}{2 l_{iD}^2}\right)$, and the background aperture given the number of elements $N_{jB}$ is $A_{BD} = \left(S_{jB} \cdot N_{jB} \cdot \frac{r_D^2}{2 \cdot L_{BD}^2}\right)$. In the offered model, the spectrum of TR power that is emitted by SR into semi-

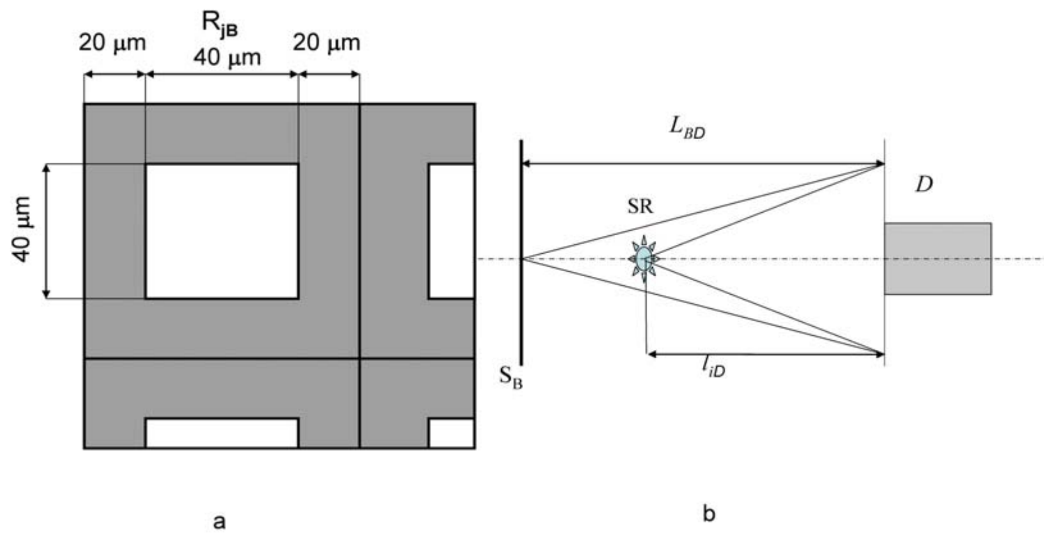


**Fig. 1.** The model of a background (a), and a geometry of the system (b).

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[2] Any other model suitable for numeral calculations of the background TR dispersion (except the offered one) is not known for the authors; therefore they did not make respective references.

[3] As the considered version of the problem has an illustrative character of calculations, small numeral errors will not impact on results and conclusions of principle.

space can be defined as $(P_i)_{2\pi} = \varepsilon_i(\lambda)\cdot\frac{c}{4}\cdot R_i^2$ (see [8], p. 5, (1.9)), and its fraction that excites PD can be written as:

$$P_{iD}(\lambda) = \varepsilon_i(\lambda)\cdot\frac{c}{4}\cdot R_i^2\cdot\frac{r_D^2}{2\,l_{iD}^2}. \qquad (2.1)$$

The variances of the TR power both from SR and background single element $N_{jB}$ are defined in accordance with [22] as variances of the product of fluctuating ($\varepsilon_i(\lambda)$ and $\varepsilon_{j\Phi}(\lambda)$) and constant ($A_{iD}$ and $A_{BD}$) values. The spectrum of the signal, i.e., the variance of the SR TR power (2.1) with account of (1.1) and (1.7) becomes (when $V_q = R_i^3$):

$$\left\langle \Delta P_{iD}^2 \right\rangle = \left\langle \text{ŕ}\Gamma_i^2 \right\rangle \cdot \left(\frac{c}{4}\cdot R_i^2\cdot\frac{r_D^2}{2\,l_{iD}^2}\right)^2 =$$

$$\left\langle \text{ŕ}\Gamma_i \right\rangle \cdot \left(\frac{c}{4}\cdot R_i^2\cdot\frac{r_D^2}{2\,l_{iD}^2}\right)\cdot\frac{hc}{\lambda\cdot R_i^3}\cdot\left(1+\langle n_i\rangle\right)\cdot\left(\frac{c}{4}\cdot R_i^2\cdot\frac{r_D^2}{2\,l_{iD}^2}\right). \qquad (2.2)$$

By its analogy with (2.1) and (2.2), let us define the power $(P_{jB})_{2\pi}$ and the variance of TR emitted by small background elements $N_{jB}$. In the case of statistical independency inherent to TR processes from background elements, a constant value of their number $N_{jB}$, sizes $R_{jB}$ and location within the boundaries of $S_{obs}$, the total power of the background TR at the PD input can be written as

$$P_{BD} = N_{jB}\cdot\varepsilon_{jB}\cdot\left(\frac{c}{4}\cdot R_{jB}^2\right)_{2\pi}\cdot\left(\frac{r_D^2}{2\,L_{BD}^2}\right). \qquad (2.3)$$

The dispersion of the sum (2.3) defined as the sum of the dispersions for separate background elements $\left\langle \Delta P_{jB}^2 \right\rangle$ using the Burgess[4] theorem for $N_{jB} = \text{const}$ in accord with (1.1) and (1.7) acquires the following form

$$\left\langle \Delta P_{BD}^2 \right\rangle = \left\langle \text{ŕ}\Gamma_{jB} \right\rangle \cdot \left(\frac{c}{4}\cdot N_{jB}\cdot R_{jB}^2\right)\cdot\left(\frac{r_D^2}{2\,L_{iD}^2}\right)\times$$
$$\times\frac{hc}{\lambda\cdot R_{jB}^3}\cdot\left(1+\langle n_{jB}\rangle\right)\cdot\left(\frac{c}{4}\cdot R_{jB}^2\right)\cdot\left(\frac{r_D^2}{2\,L_{iD}^2}\right). \qquad (2.4)$$

## 3. Photocurrents at the output of an ideal PD

Descending to the photoelectric currents corresponding to the SR TR powers (2.1), (2.2) and $S_B$ TR (2.3), (2.4), let us remind you about the important property of the principle adduced in [26]:

"… the spectrum of photocurrent power reconstitutes the optical spectrum independently of the optical field statistics". It is this condition that allows us prolonging our consideration of the problem formulated in the Introduction. In the model adopted above for the background and the scheme of joint exciting the photodetector by the system (SR + B), acting at the PD output are the currents (3.1)-(3.6) defined in accordance to the conventional rules [8].

Originating from relations (2.1)-(2.4), at the output of ideal PD we have ($e$ is the electron charge, $\eta$ - quantum efficiency of PD):

– the stationary current caused by SR TR power (2.1)

$$\left\langle I_{iD} \right\rangle = \left(\frac{\eta\cdot e}{h\nu}\right)\cdot\left\langle P_{iD} \right\rangle; \qquad (3.1)$$

– the respective $\langle I_{iD}\rangle$ (3.1) shot noise

$$\sqrt{\left\langle \Delta I_{isn}^2 \right\rangle} = \sqrt{2\cdot e\cdot\Delta f\cdot\left\langle I_{iD}\right\rangle}; \qquad (3.2)$$

– the signal corresponding to the SR TR variance (2.2)

$$\sqrt{\left\langle \Delta I_{iD}^2 \right\rangle} = \sqrt{\left(\frac{\eta\cdot e}{h\nu}\right)^2\cdot\left\langle \Delta P_{iD}^2 \right\rangle\cdot\left(2\cdot\frac{\Delta f}{\Delta\nu}\right)}; \qquad (3.3)$$

– the current corresponding to the background TR power (2.3)

$$\left\langle I_{BD} \right\rangle = \left(\frac{\text{ŕE}\cdot e}{h\text{ŕЛ}}\right)\cdot\left\langle P_{BD} \right\rangle; \qquad (3.4)$$

– the corresponding $\langle I_{BD}\rangle$ (3.4) shot noise

$$\sqrt{\left\langle \Delta I_{Bsn}^2 \right\rangle} = \sqrt{2\cdot e\cdot\Delta f\cdot\left\langle I_{BD}\right\rangle}; \qquad (3.5)$$

– the current corresponding to the background TR dispersion (2.4)

$$\sqrt{\left\langle \Delta I_{BD}^2 \right\rangle} = \sqrt{\left(\frac{\eta\cdot e}{h\nu}\right)^2\cdot\left\langle \Delta P_{BD}^2 \right\rangle\cdot\left(2\cdot\frac{\Delta f}{\Delta\nu}\right)}. \qquad (3.6)$$

The coefficient $\left(2\cdot\frac{\Delta f}{\Delta\nu}\right)$ [8] in (3.3) and (3.6) defines, to some extent, the fraction of chaotic fluctuations inherent to TR within the band $\Delta\nu$ at the PD input, which is really registered as a chaotic in time current at the PD output within the frames of its electron band of frequencies $\Delta f$. When calculating and measuring the current and power spectra corresponding to TR, the values $\left(2\cdot\frac{\Delta f}{\Delta\nu}\right)$ in formulas (3.3), (3.6) can be kept constants, by changing the electron band of frequencies (e.g., at $\left(\frac{\Delta f}{\Delta\nu}\right) = 10^{-2}$ and $\left(\frac{\Delta\lambda}{\lambda}\right) = 10^{-2}$ one has $\Delta f = \frac{3\cdot10^6}{\lambda\cdot10^{-4}}$).

[4]Burgess R.E. Faraday, Soc.Discussions, 1959, v.28, p.151; reference cited in [20].

## 4. Thresholds for detection of single SR TR fluctuations by an ideal PD

### *4.1. Detection thresholds $\left\langle \Delta P_{iD}^2 \right\rangle$ on the background of intrinsic noises (single-noise-limited detection – SNLD mode [8])*

Before starting our consideration of the main task (as to the threshold values (3.3)) formulated above, let us remind you about the problems that contain some conceptions poorly ascertained from the physical viewpoint.

Let us define the threshold values $\left\langle P_{iD} \right\rangle_{th}$ (2.9) and $\left\langle \Delta P_{iD}^2 \right\rangle_{th}$ (2.2) via the signal-to-noise ratio for respective "measured" currents revealed with the background of intrinsic noises. Extrinsic noises are absent here.

1) As always, the threshold $\left\langle P_{iD} \right\rangle_{th}$ is defined by the following TR power $\left\langle P_{iD} \right\rangle_{th} = \dfrac{2 \cdot h\nu \cdot \Delta f}{\eta}$ [8] by detection of the signal $\left\langle I_{iD} \right\rangle_{th}$ (3.1) with the background $\left\langle \Delta I_{isn}^2 \right\rangle$ (3.2). In this case, there are no explanations why the noises $\left\langle \Delta P_{iD}^2 \right\rangle$ related to the intrinsic fluctuations of TR in a BB cavity[5] of the radiator are not taken into account. At the same time, the very combination $2 \cdot h\nu \cdot \Delta f$ can be hardly explained in a phenomenological aspect. But taking $\left\langle \Delta I_{iD}^2 \right\rangle$ into account together with $\left\langle \Delta I_{isn}^2 \right\rangle$ results in a more informative expression (the essence of SNLD condition [8] remains valid), namely:

$$\left\langle P_{iD} \right\rangle_{th} = 2 \cdot \frac{h\nu \cdot \Delta f}{\eta} \cdot \left( 1 + \eta \cdot \frac{\lambda}{4 \cdot R_i \cdot \left( \dfrac{\Delta\lambda}{\lambda} \right)} \cdot \left(1 + \left\langle n_i \right\rangle\right) \cdot \left( \frac{r_D^2}{2 \cdot l_{iD}^2} \right) \right), \tag{4.1}$$

which contains definite information upon this radiator and its aperture (Fig. 2).

2) After respective simple algebraic operations with account of expressions (2.1), (2.2) and (3.1)-(3.3), the threshold values for the signal of our interest $\left\langle \Delta I_{iD}^2 \right\rangle$ (3.3) against the background of the intrinsic shot noise $\left\langle \Delta I_{isn}^2 \right\rangle$ (3.2) can be reduced to the following expression

$$\frac{hc}{\lambda \cdot R_i^3} \cdot \left(1 + \left\langle n_i \right\rangle\right) \cdot \left( \frac{c}{4} \cdot R_i^2 \cdot \frac{r_D^2}{l_i^2} \right) = \left\{ \frac{h\nu \cdot \Delta\nu}{\eta} \right\}. \tag{4.2}$$

[5] Definition and measurement of the noise-coefficient for semiconductor photodiodes takes into account the photon-noise stated in the paper [27] but doesn't consider the specific parameters of the radiator (except for (⟨n⟩+1) [8, 14-19]) as well as the aperture features.

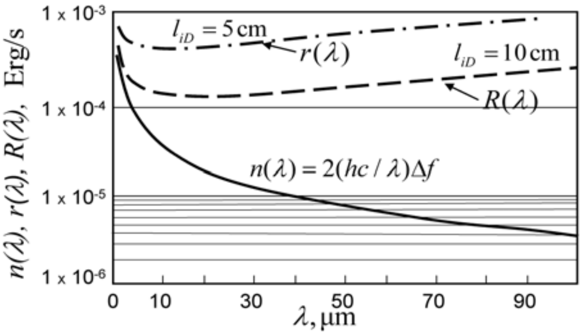


**Fig. 2.** Spectral dependences of $\left\langle P_{iD} \right\rangle_{th} = \dfrac{2 \cdot h\nu \cdot \Delta f}{\eta}$ (curve $n(\lambda)$) and $\left\langle P_{iD} \right\rangle_{th}$ (4.1) at two distances to the radiator (size $R_i = 20$ μm): $l_{iD} = 10$ cm (curve $R(\lambda)$), and $l_{iD} = 5$ cm (curve $r(\lambda)$).

It is obvious that (4.2) describes equality of two "intrinsic powers": on the left – "e-p" (1.7), ("intrinsic micro-quantity of chaos" in terms [1, 2]) that corresponds to the frequency band $\Delta\nu_q$ defined via the aperture factor and the SR intrinsic volume $V_i = R_i^3 \equiv V_q$ (1.5), (1.7), i.e. $\Delta\nu_q \equiv \Delta\nu_i = \left( \dfrac{c}{2 \cdot R_i} \cdot \dfrac{r_D^2}{2 \cdot l_i^2} \right)$; on the right – "single photon power " defined through the optical observation band divided by the value of PD quantum efficiency:

$$q_{\Delta\nu} = \left\{ \frac{h\nu \cdot \Delta\nu}{\eta} \right\}. \tag{4.3}$$

This "single photon power" occurs in [8] (e.g. page 87) also as the factor defining power within a single mode of TR, namely: "…the power per mode is $\overline{P}_k = \varepsilon\, f_k\, h\nu\, \Delta\nu$ …"; here, $\varepsilon$ is emissivity and $f_k$ – Bose-Einstein occupancy probability. It seems very doubtful that to determine the single mode TR power one use the spectral frequency band $\Delta\nu$. As it follows from the commonly known literature, the TR power within the band $\Delta\nu$ quantifies the number of modes $Z(z) \cdot \Delta\nu$ (see the above item 1.2) in the range of which it is observed (measured) or calculated.

The equality $q_p = q_{\Delta\nu}$ (4.2) puts more definite phenomenological content into the value $q_{\Delta\nu}$ (4.3), since the equality (4.2) also defines the value $q_{\Delta\nu}$ as "e-p" in the list (1.2)-(1.8), taking into account the physical sense of $q_p$ (1.7). In what follows (see the item 4.2 below), our reasoning in regard to the physical meaning of the "power = $h\nu\,\Delta\nu$" confirms the validity of consideration of the detection thresholds $\left\langle \Delta I_{iD}^2 \right\rangle$ (3.3) with the total noise background: (3.2), (3.5), and (3.6).

### *4.2. Detection thresholds $\left\langle \Delta P_{iD}^2 \right\rangle$ in presence of external thermal noise*

When calculating the thresholds of the signal $\left\langle \Delta I_{iD}^2 \right\rangle$, the total current that takes into account external noises will be defined using the following logic.

The fluctuating total virtual current $\left\langle \Delta I_{\Sigma}^2 \right\rangle$ measured by a PD in view of statistical independency for the currents (3.2), (3.5), (3.6) and containing only averaged values will be determined via the sum of variances in the accepted approximation

$$\left\langle \Delta I_{\Sigma}^2 \right\rangle = 2e\Delta f \cdot \left(\frac{\eta \cdot e}{hc/\lambda}\right) \cdot \left\langle P_{iD} \right\rangle + \left(\frac{\eta \cdot e}{hc/\lambda}\right)^2 \cdot \left\langle \Delta P_{iD}^2 \right\rangle \cdot 2\frac{\Delta f}{\Delta \nu} + 2e\Delta f \cdot \left(\frac{\eta \cdot e}{hc/\lambda}\right) \cdot \left\langle P_{BD} \right\rangle + \left(\frac{\eta \cdot e}{hc/\lambda}\right)^2 \cdot \left(q_{jp}\right) \cdot \left\langle P_{BD} \right\rangle \cdot 2\frac{\Delta f}{\Delta \nu} . \tag{4.4}$$

Having used (4.4) and (3.1)-(3.6) to write the signal-to-noise ratio via PD output currents expressed through the corresponding TR power at the PD input one has

$$\frac{\left\langle \Delta I_{iD}^2 \right\rangle}{\left\langle \Delta I_{\Sigma}^2 \right\rangle - \left\langle \Delta I_{iD}^2 \right\rangle} = \frac{\left(\frac{\eta \cdot e}{hc/\lambda}\right)^2 \cdot \left\langle \Delta P_{iD}^2 \right\rangle \cdot 2\frac{\Delta f}{\Delta \nu}}{\left[2e\Delta f \cdot \left(\frac{\eta \cdot e}{hc/\lambda}\right) \cdot \left\langle P_{iD} \right\rangle + 2e\Delta f \cdot \left(\frac{\eta \cdot e}{hc/\lambda}\right) \cdot \left\langle P_{BD} \right\rangle + \left(\frac{\eta \cdot e}{hc/\lambda}\right)^2 \cdot \left(q_{jp}\right) \cdot \left\langle P_{BD} \right\rangle \cdot 2\frac{\Delta f}{\Delta \nu}\right]} . \tag{4.4*}$$

Let us cancel (like in Eq. (4.1)) all the reducible terms in the numerator and denominator. After this conventional procedure [8] for conversion from PD currents to TR powers, the threshold condition for $\left\langle \Delta P_{iD}^2 \right\rangle_{th}$ (4.4*) can be reduced to the following form

$$\eta \cdot \frac{\left\langle \Delta P_{iD}^2 \right\rangle_{th}}{\left\langle P_{iD} \right\rangle \cdot h\nu \cdot \Delta\nu + \left\langle P_{BD} \right\rangle \cdot h\nu \cdot \Delta\nu + \eta \cdot \left(q_{jp}\right) \cdot \left\langle P_{BD} \right\rangle} = 1 . \tag{4.5}$$

The formula (4.5) considered definition (1.1) and physical meaning of the equality (4.2) allows to consider the expression $(h\nu \cdot \Delta\nu) \cdot \left\langle P_{iD} \right\rangle$ in the denominator (4.5) as the variance of $P_{iD}$ (2.1) related to the "photon noise at the spectral slit". Let us designate it as

$$\left\langle \Delta P_{\Delta\nu}^2 \right\rangle = (h\nu \cdot \Delta\nu) \cdot \left\langle P_{iD} \right\rangle . \tag{4.6}$$

The same reasoning concerns the expression $(h\nu \cdot \Delta\nu) \cdot \left\langle P_{BD} \right\rangle$, too.

The factor $q_{\Delta\nu} = h\nu\,\Delta\nu$ is physically defined as "e-p" (1.8) for TR power flowing through an observation spectral slit $\Delta\nu$ ("e-p" for slit-fluctuation). This "Δ-noise" (4.6), created artificially to some extent, is defined via the above list of "e-p" (1.2)-(1.8).

To put more definite phenomenological meaning into the values $q_{\Delta\nu}$ and $\left\langle \Delta P_{\Delta\nu}^2 \right\rangle$, let us consider spectral and quantitative (calculated) features of the signal threshold values $\sqrt{\left\langle \Delta P_{iD}^2 \right\rangle_{th}}$ as compared with the spectra of $\sqrt{\left\langle \Delta P_{iD}^2 \right\rangle_{\Delta\nu}}$ and the signal $\sqrt{\left\langle \Delta P_{iD}^2 \right\rangle}$. In the above accepted model for the background (§1, item 1.3) given the definitions (3.1) to (3.6) as well as expressions (4.5), (4.6), the spectral dependence of the threshold power $\left\langle \Delta P_{iD}^2 \right\rangle$ for the signal defined through the respective photocurrent at the PD output $\sqrt{\left\langle \Delta I_{iD}^2 \right\rangle_{th}}$ in terms of wavelengths λ and for η = 1 can be expressed as follows

$$\sqrt{\left\langle \Delta P_{iD}^2 \right\rangle_{th}} = \sqrt{\frac{hc}{\lambda} \cdot \frac{c}{\lambda} \cdot \left(\frac{\Delta\lambda}{\lambda}\right) \cdot \left\langle P_{iD} \right\rangle + \frac{hc}{\lambda} \cdot \left[\frac{c}{\lambda} \cdot \left(\frac{\Delta\lambda}{\lambda}\right) + \frac{1}{R_{jB}^3} \cdot \left(1 + \left\langle n_{jB} \right\rangle\right) \cdot \left(\frac{c}{4} \cdot R_{jB}^2 \cdot \frac{r_D^2}{2 \cdot L_{jB}^2}\right)\right] \cdot \left\langle P_{BD} \right\rangle} . \tag{4.7}$$

Being targeted to compare the spectra of the noise signal (2.10) and "Δ-noise" (4.6) with those of the signal threshold (4.7) for various temperatures, we have depicted them in Figs 3 and 4. The signal (2.2) defined via the current (3.3) takes the detailed look:

$$\sqrt{\left\langle \Delta P_{iD}^2 \right\rangle} = \frac{hc}{\lambda}\cdot\sqrt{\frac{8\pi}{\lambda^3}\cdot\left(\frac{\Delta\lambda}{\lambda}\right)\cdot\left[1-\left(\frac{\lambda}{2R_i}\right)^2\right]\cdot\langle n_i\rangle\cdot\frac{1}{R_i^3}\cdot\left(1+\langle n_i\rangle\right)\cdot\left(\frac{c}{4}\cdot R_i^2\cdot\frac{r_D^2}{2\cdot l_{iD}^2}\right)^2}\ . \tag{4.8}$$

"Δ-noise" (4.6), also in the detailed notation, is as follows:

$$\sqrt{\left\langle \Delta P_{\Delta\nu}^2 \right\rangle} = \frac{hc}{\lambda}\cdot\sqrt{\left(\frac{c}{\lambda}\cdot\left(\frac{\Delta\lambda}{\lambda}\right)\right)\cdot\left\langle\frac{8\pi}{\lambda^3}\cdot\left[1-\frac{\lambda^2}{4\cdot R_i^2}\right]\cdot\left(\frac{\Delta\lambda}{\lambda}\right)\cdot\langle n_i\rangle\cdot\left(\frac{c}{4}\cdot R_i^2\cdot\frac{r_D^2}{2\cdot L_{jB}^2}\right)\right\rangle} \tag{4.9}$$

The parameters of the SR and the respective background are chosen first of all considering the necessity to simplify our phenomenological analysis. The numeral values of threshold powers $\sqrt{\left\langle \Delta P_{iD}^2 \right\rangle_{th}}$ for the chosen parameters are determined for the wavelength (shown with arrows ↓ in Fig. 3) of the intersection points between the signal spectrum (4.8) and those of its threshold values (4.7) for various temperatures.

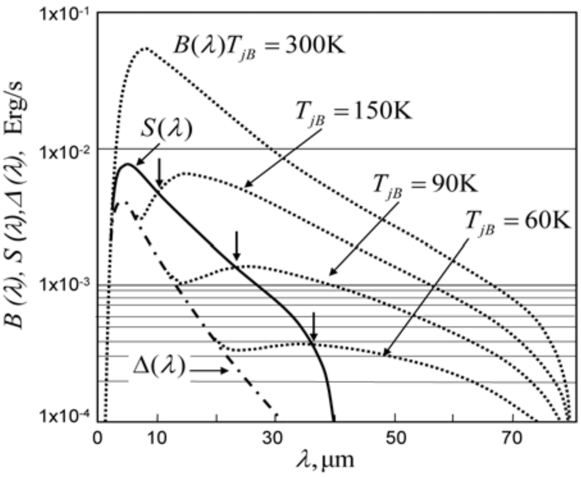


**Fig. 3.** Illustration of spectral distributions of TR powers (4.8), (4.9) and signal thresholds (4.7) $\sqrt{\left\langle \Delta P_{iD}^2 \right\rangle_{th}} = B(\lambda)$ for three temperatures ($T_{jB}$) of background radiators. The designations and numeral parameters: $B(\lambda) = \sqrt{\left\langle \Delta P_{iD}^2 \right\rangle_{th}}\Big|_{T_{iB}}$;

$S(\lambda) = \sqrt{\left\langle \Delta I_{iD}^2 \right\rangle_{th}}\Big|_{600\,\mathrm{K}} \equiv \mathrm{SIGNAL}$;

$\Delta(\lambda) = \sqrt{\left\langle \Delta I_{i\Delta}^2 \right\rangle_{th}}\Big|_{600\,\mathrm{K}} \equiv$ "Δ – noise";

$A_{iD} = R_i^2\cdot\left(r_D^2/2l_{iD}^2\right) = (0.002)^2\cdot\left(2^2/2.8^2\right)$; $T_{SR} = 600$ K;

$\Delta\lambda/\lambda = 10^{-3}$;

$A_{iB} = \left[(0.004)^2\cdot\left(1.563\cdot10^4\right)\cdot30^2\cdot\left(2^2/2.600^2\right)\right]$;

$\Delta\lambda/\lambda = 10^{-3}$; $T_{jB}$ = 300, 150, 90, 60 K.

Further comparison of these spectra shows that measuring the signal thresholds $\sqrt{\left\langle \Delta P_{iD}^2 \right\rangle_{th}}$ is not always possible.

If the observation parameters are chosen differently, "Δ-noise" (4.9), (designated as Δ(λ) in Fig. 3), when $\Delta(\lambda) > Q(\lambda)$, the signal spectra are "blocked" by photon flux fluctuations at the spectral slit, that is by the "Δ-noise" (4.9).

As it follows from Figs 2 and 3, there are specific circumstances (e.g., mutual combinations of numeral characteristics inherent to a radiating system SR+BG as well as parameters of measuring process) when fluctuation limitations (like the usual shot noise) for measurements of TR signal threshold characteristics arise in the course of TR flux passing within an optical band Δν inside a detecting apparatus before reaching the sensitive element of PD. In the literature (see, for instance, [6-8, 14-21]) TR fluctuations of this kind were not considered. However, as it follows from Figs 3

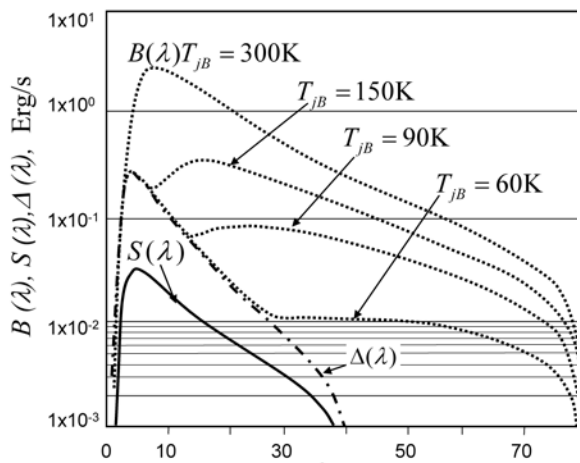


**Fig. 4.** Illustration of "blocked" spectral distributions of TR powers (4.8), (4.9) and signal thresholds (4.7) $\sqrt{\left\langle \Delta P_{iD}^2 \right\rangle_{th}} = T(\lambda)$ for three temperatures ($T_{jB}$) of background radiators. The designations and numeral parameters are same as for Fig. 3.

and 4, one should not neglect the intrinsic TR noises (4.5) and (4.8) within the spectral band. Thus, for example in [8] the shot noise related to a photocurrent is calculated in the traditional way when originating only from statistical (or in [17] from quantum-and-statistical) notions as to appearance of electrons in the PD active area when these are excited by photons. In relation with the considered problem, there arises the following question: is it possible that the measured photocurrent shot-noise is always determined by random electrons appearing statistically inside the photosensitive area of PD?

### 5. Conclusions

1. Conversion of the common threshold expression in current terms at the PD output to that (4.5) for the respective TR powers at the PD input results in the expression for an additional photonic noise that arises due to limitations of spatial-and-time conditions for TR flux passing through the observation slit $\Delta\nu$. As a result, the process of detection is limited by the "photonic $\Delta\nu$-noise" $\left\langle \Delta P_{\Delta\nu}^2 \right\rangle = \left( h\nu\, \Delta\nu \right) \cdot \left\langle P_{iD} \right\rangle$, (4.6), along with the traditional "electron shot noise".

2. As an "eigen-parameter" (in terms of [1, 2] it is the "intrinsic micro-quantity of chaos") in this case we deal with well-known but a phenomenologically unascertained up to date "single photon power" $q_{\Delta\nu} = h\nu\, \Delta\nu$.

3. Inclusion of the "power" $q_{\Delta\nu}$ (4.3) in the list (1.2)-(1.8) as an "intrinsic power of TR flux" (1.8) seems to be adequate. From a phenomenological viewpoint, the "power" $q_{\Delta\nu}$ defines the dispersion of additional photonic fluctuations in the TR flux through an observation slit $\Delta\nu$.

4. In the case of photoelectronic detection the TR signals at the PD output, along with traditional photonic (3.3) and shot noises [8, 10], it seems reasonable to take into account the "photonic $\Delta\nu$-noises" (4.6) considered above (Figs 3 and 4).

5. Used in this work a rather risky approximation (first of all, ignoring the correlation processes, moments of higher orders, etc. [17]) does not reduce to zero the above considered physical concepts as well as their role and numeric values.

Thus, the following result is non-trivial: it is possible to measure a numeral meaning of the physical value "power of a single photon" $q_{\Delta\nu} = h\nu\, \Delta\nu$ under specific physical conditions (with the cold enough background: e.g. in Fig. 4 it can be realised at $T_{jB} < 120$ K and high enough ratio $\frac{\Delta\nu}{\nu} \sim 0.1$) only by measuring (with an ideal PD) the TR flux fluctuations at the spectral slit $\Delta$ŕJ.

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