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Influence of field dependent form of collision integral on kinetic coefficients

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Abstract. The kinetic equation is turned out in the form that contains collision integral obviously dependent on the value of external electric and magnetic fields. The correspondent calculation of kinetic coefficients shows that for the case considered here they depend evidently on the ratio of average deBroigle wavelength and free-path length. Just here, the real possibility appears to reasonably separate physical kinetics by the classic and non-classic (quantum) ones.

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1. Introduction

Along the way of construction of kinetic equation from some “first principles”, the rightful place belongs to the influence of external macroscopic fields and microscopic scattering fields on movement of band charged carriers. The scattering fields give the main, principal contribution to existence and form of collision integral. Evident influence of macroscopic fields on the scattering system is not usually taken into attention, because one supposes that for the scheme of second order perturbation theory the external field can be omitted (see [1-3]). Special consideration shows that the latter approach is not universal, and in some situations the direct influence of macroscopic fields on the form of collision integral and of corresponding non-equilibrium distribution function can sufficiently change the value of kinetic coefficients.

2. One-particle density matrix of nonequilibrium many-particle system

Design by the symbols A, B etc. some set of quantum numbers (for instant, components of the wave vector) that characterizes a state of each separate band particle; farther, for simplicity, we shall say about electrons. One does not use the direct designations for spin variables and spin quantum numbers, because processes of spin overturn are not considered here. The act of averaging we designate by angle brackets; formally that procedure is performed using the non-equilibrium statistical operator of total system of electrons and external system, representing all scattering fields that interact with the electron system (see [4-7]).

Define the one-particle density matrix $\rho_{AB}(t)$ by using the following mode:

$$\rho_{AB}(t) = a_B^\dagger(t) a_A(t). \quad (1)$$

Here, t is time, a_A^\dagger and a_A are operators of generation and annihilation of electron at the state A . The averaged value of given matrix (we call it farther as the one-particle density matrix) is

$$f_{AB}(t) = \langle \rho_{AB}(t) \rangle = \langle a_B^\dagger(t) a_A(t) \rangle. \quad (2)$$

The dynamic value $C = \sum_{n=1}^N C(\vec{r}_n)$ that belongs to the additive type, in representation of secondary quantization has the form $\hat{C} = \sum_{A,B} C_{BA} a_B^\dagger a_A$, where

$C_{BA} = \int \Psi_B^*(\vec{r}) \hat{C}(\vec{r}) \Psi_A(\vec{r}) d^3\vec{r}$. In this formula, the value $\Psi_A(\vec{r})$ is the wave function of separate band particle, which belongs to the state A .

Deduce an equation, solution of which is the one-particle density matrix f_{AB} for the considered non-equilibrium system of particles. As a start point, we use the standard motion equation for operator $\rho_{AB}(t)$ at Heisenberg representation:

$$i\hbar \frac{\partial \rho_{AB}}{\partial t} = [\rho_{AB}(t), H^{tot}] \equiv \rho_{AB}(t) H^{tot} - H^{tot} \rho_{AB}(t). \quad (3)$$

3. Total Hamiltonian of band electrons and scattering system

One can represent the Hamiltonian of considered total system H^{tot} as the sum of four parts: the Hamiltonian H_e for electrons non-interacting with microscopic scattering fields, Hamiltonian H_{ee} related to inter-electron interaction, individual Hamiltonian H_S of external scattering system and Hamiltonian H_{eS} which relates to interaction of band electrons with the scattering system:

$$H^{tot} = H_e + H_S + H_{ee} + H_{eS}. \quad (4)$$

In this paper, we assume only the point charged impurities as external scattering system ($S \rightarrow I$).

At the presence of constant, uniform electrical \vec{E} and magnetic \vec{H} fields, the Hamiltonian of free charged carriers is as follows

$$H_e = \varepsilon \left(\frac{\hat{p}}{\hbar} + \frac{e}{c} \vec{A}(\vec{r}) \right) - e \vec{E} \vec{r}. \quad (5)$$

In this formula, $\hat{p} = \hbar \hat{k} = -i\hbar \frac{\partial}{\partial \vec{r}}$ is the momentum operator; $\varepsilon(\vec{p})$ – dispersion law; $\varepsilon(\vec{p})$ – operator of the kinetic energy; $\vec{A} = (1/2)[\vec{H} \times \vec{r}]$ – vector-potential of magnetic field. Further, we suppose that the dispersion law has the simple form:

$$\varepsilon(\vec{p}) = p^2/2m = \hbar^2 k^2/2m, \quad (6)$$

where m is the effective mass and $\vec{p} = \hbar \vec{k}$ is proper value of the momentum operator.

The quantum limit of strong magnetic field in this paper is not considered. Therefore, in the Hamiltonian (5) we omit the terms of the order A^2 ; the latter is acceptable under the condition $|eH|\hbar/2mc \ll \langle \varepsilon \rangle$. Let $(A_1 A_2)_+ = (1/2)(A_1 A_2 + A_2 A_1)$. Then, it follows from Eq. (5):

$$H_e = H^{(0)} + H^{(E)} + H^{(H)} = \varepsilon \left(\frac{\hat{p}}{\hbar} \right) - e \vec{E} \vec{r} + \frac{e}{mc} \left(\vec{A}(\vec{r}) \cdot \frac{\hat{p}}{\hbar} \right). \quad (7)$$

Assume the following orientation of fields:

$$\begin{aligned} \vec{E} &= (E_x, E_y, 0); \quad \vec{H} = (0, 0, H_z); \\ \vec{A} &= (-H_z y, H_z x, 0). \end{aligned} \quad (8)$$

For this case (see Eq. (6)), the separate terms of Hamiltonian H_e are

$$\begin{aligned} H^{(0)} &= \varepsilon \left(\frac{\hat{p}}{\hbar} \right) = \frac{\hbar^2 \hat{k}^2}{2m}; \\ H^{(E)} &= -e(xE_x + yE_y); \quad H^{(H)} = \frac{e\hbar H_z}{2mc} (x\hat{k}_y - y\hat{k}_x). \end{aligned} \quad (9)$$

In representation of secondary quantization, the Hamiltonian of electrons that do not interact with microscopic scattering fields is (see Eqs (7) and (9))

$$\begin{aligned} H_e &= \sum_{AB} (H_e)_{AB} a_A^\dagger a_B = \sum_{AB} (H_e)_{AB} \rho_{BA} = \\ &= \sum_{AB} \left\{ (H^{(0)})_{AB} + (H^{(E)})_{AB} + (H^{(H)})_{AB} \right\} \rho_{BA}. \end{aligned} \quad (10)$$

The Hamiltonian of interactions between band electrons and charged impurities is

$$H_{eI} = \sum_{AB} (H_{eI})_{AB} \rho_{BA}(t). \quad (11)$$

The Hamiltonian of Coulomb e - e -interaction has the following form (see [8]):

$$\begin{aligned} H_{ee} &= (1/2) \sum_{ABA'B'} V_{ABA'B'} a_A^\dagger a_{A'}^\dagger a_{B'} a_B = \\ &= (1/2) \sum_{ABA'B'} V_{ABA'B'} (\rho_{B'A'} \rho_{BA'})_+ + \sum_{ABA'} V_{ABA'A'} \rho_{BA}, \\ V_{ABA'B'} &= \frac{e^2}{\varepsilon_L} \int d^3\vec{r} \int d^3\vec{r}' \Psi_A^*(\vec{r}) \Psi_{A'}^*(\vec{r}') \frac{1}{|\vec{r} - \vec{r}'|} \Psi_{B'}(\vec{r}') \Psi_B(\vec{r}). \end{aligned} \quad (12)$$

Here, ε_L is the dielectric constant of the considered crystal. Excluding the term that represents a simple shift of the origin point for energy, one obtains (see [9]):

$$\begin{aligned} H_{ee} &= (1/2) \sum_{ABA'B'} V_{ABA'B'} a_A^\dagger a_{A'}^\dagger a_{B'} a_B = \\ &= -(1/2) \sum_{ABA'B'} V_{ABA'B'} (\rho_{B'A'} \rho_{BA'})_+. \end{aligned} \quad (14)$$

As a result, the total Hamiltonian presented in Eq. (3) has the form

$$H^{tot} = \sum_{AB} \{ (H_e)_{AB} + (H_{el})_{AB} \} \rho_{BA} - (1/2) \sum_{ABAB'} V_{ABAB'} (\rho_{B'A} \rho_{BA'})_+ . \quad (15)$$

The plane waves are the natural basis for spatially uniform system of electrons:

$$\Psi_A(\vec{r}) \rightarrow L^{-3/2} \exp(i\vec{k}_A \vec{r}); \quad (16)$$

then

$$V_{ABB'A'} = \frac{e^2}{2\pi^2 \epsilon_L} \int \frac{d^3 \vec{q}}{q^2} (b_{\vec{q}})_{AB} (b_{-\vec{q}})_{B'A'} , \quad (17)$$

$$(b_{\vec{q}})_{AB} = \int \Psi_A^*(\vec{r}) \exp(i\vec{q}\vec{r}) \Psi_B(\vec{r}) d^3 \vec{r} = (2\pi/L)^3 \delta(\vec{k}_A - \vec{k}_B - \vec{q}) = \delta_{\vec{k}_A, \vec{k}_B + \vec{q}} . \quad (18)$$

Substituting the expression (15) to (3) and performing necessary commutations of Fermi-operators, one obtains the following equation for the density matrix:

$$i\hbar \frac{\partial \rho_{AB}(t)}{\partial t} = \sum_{\Gamma} \{ (H_e)_{A\Gamma} \rho_{\Gamma B}(t) - (H_e)_{\Gamma B} \rho_{A\Gamma}(t) \} + \sum_{\Gamma} \{ (H(t))_{A\Gamma} \rho_{\Gamma B}(t) - (\rho_{A\Gamma}(t) (H(t))_{\Gamma B})_+ \} . \quad (19)$$

In this formula,

$$\begin{aligned} (\overline{H}(t))_{AB} &= (H_{el}(t))_{AB} + (H_{ee}(t))_{AB} = \\ &= (H_{el}(t))_{AB} + \sum_{A'B'} V_{ABB'A'} \rho_{A'B'}(t) . \end{aligned} \quad (20)$$

4. Averaged values and fluctuations

Separate the density matrix $\rho_{AB}(t)$ and Hamiltonian $\overline{H}(t)$ by averaged values and fluctuations. One assumes that the average scattering potential is zero. Therefore,

$$\rho_{AB}(t) = \langle \rho_{AB}(t) \rangle + \delta \rho_{AB}(t) = f_{AB}(t) + \delta \rho_{AB}(t) , \quad (21)$$

$$(\overline{H}(t))_{AB} = \langle (\overline{H}(t))_{AB} \rangle + \delta (\overline{H}(t))_{AB} = \delta (\overline{H}(t))_{AB} . \quad (22)$$

Independence (or very weak dependence) of electron density on spatial coordinates is provided by the following condition:

$$f_{AB}(t) = \delta_{AB} f_{AA}(t) \equiv \delta_{AB} f_A(t) . \quad (23)$$

The fluctuations $\delta \rho_{AB}(t)$ are considered as small values. Then,

$$\begin{aligned} (\delta \overline{H}(t))_{AB} &= (\delta H_{el}(t))_{AB} + \delta (H_{ee}(t))_{AB} = \\ &= (\delta H_{el}(t))_{AB} + \sum_{A'B'} V_{ABB'A'} \delta \rho_{A'B'}(t) . \end{aligned} \quad (24)$$

Accept also the following condition (See [6]):

$$\langle \partial \rho_{AB} / \partial t \rangle = \partial \langle \rho_{AB} \rangle / \partial t = \partial f_{AB} / \partial t . \quad (25)$$

Introducing (21), (24) and (25) to (19), one obtains the equation

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} \{ f_{AB}(t) + \delta \rho_{AB}(t) \} = \\ = \sum_{\Gamma} \{ (H_e)_{A\Gamma} (f_{\Gamma B}(t) + \delta \rho_{\Gamma B}(t)) - \\ - (H_e)_{\Gamma B} (f_{A\Gamma}(t) + \delta \rho_{A\Gamma}(t)) \} + \\ + \sum_{\Gamma} \{ (H(t))_{A\Gamma} + (\delta H(t))_{A\Gamma}, f_{\Gamma B}(t) + \delta \rho_{\Gamma B}(t) \}_+ - \\ - \sum_{\Gamma} \{ (H(t))_{\Gamma B} + (\delta H(t))_{\Gamma B}, f_{A\Gamma}(t) + \delta \rho_{A\Gamma}(t) \}_+ . \end{aligned} \quad (26)$$

Averaging the latter expression, we find:

$$\begin{aligned} i\hbar \frac{\partial f_{AB}(t)}{\partial t} = \\ = \sum_{\Gamma} \{ (H_e)_{A\Gamma} f_{\Gamma B}(t) - (H_e)_{\Gamma B} f_{A\Gamma}(t) \} + i\hbar \text{St} f_{AB}(t) , \end{aligned} \quad (27)$$

where

$$\begin{aligned} \text{St} f_{AB}(t) = \\ = -\frac{i}{\hbar} \sum_{\Gamma} \{ (\delta \overline{H}(t))_{A\Gamma}, \delta \rho_{\Gamma B}(t) \}_+ - (\delta \overline{H}(t))_{\Gamma B}, \delta \rho_{A\Gamma}(t) \}_+ \} . \end{aligned} \quad (28)$$

It follows from here (See. (23)):

$$i\hbar \frac{\partial f_A}{\partial t} = \sum_B \{ (H_e)_{AB} f_{BA}(t) - (H_e)_{BA} f_{AB}(t) \} + i\hbar \text{St} f_A(t) , \quad (29)$$

$$\begin{aligned} \text{St} f_A = -\frac{i}{\hbar} \sum_B \{ \{ (\delta \overline{H}(t))_{AB}, \delta \rho_{BA}(t) \}_+ \} - \\ - \{ \{ (\delta \overline{H}(t))_{BA}, \delta \rho_{AB}(t) \}_+ \} \} . \end{aligned} \quad (30)$$

One calls Eq. (29) as the kinetic equation and Eq. (30) as the collision integral.

Subtracting Eq. (27) from Eq. (29), we find (terms of second order of trifle are omitted here):

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} \{ \delta \rho_{AB}(t) \} = \sum_{\Gamma} \{ (H_e)_{A\Gamma} \delta \rho_{\Gamma B}(t) - (H_e)_{\Gamma B} \delta \rho_{A\Gamma}(t) \} + \\ + [f_B(t) - f_A(t)] (\delta \overline{H}(t))_{AB} . \end{aligned} \quad (31)$$

Farther in Eq. (31), we consider the average distribution function to be smooth in comparison with fluctuating values. Using the Laplas transformation [10]

$$\begin{aligned} \xi(\omega) &= \int_0^{\infty} \xi(t) \exp(i\omega t) dt , \\ \xi(t) &= \frac{1}{2\pi} \int_{-\infty+i0}^{\infty+i0} \xi(\omega) \exp(-i\omega t) d\omega , \end{aligned} \quad (32)$$

one obtains:

$$-i\hbar\delta\rho_{AB}(t=0) + \hbar\omega\delta\rho_{AB}(\omega) = \sum_{\Gamma} \left\{ (H_e)_{A\Gamma} \delta\rho_{\Gamma B}(\omega) - (H_e)_{\Gamma B} \delta\rho_{A\Gamma}(\omega) \right\} + (f_B - f_A) (\delta\bar{H}(\omega))_{AB}, \quad (33)$$

$$\text{St } f_A = -\frac{1}{2\pi^2\hbar} \text{Im} \left\{ \int d\omega \int d\omega' \exp[-i(\omega + \omega')] \times \sum_B \left\langle \left(\delta\rho_{AB}(\omega), (\delta\bar{H}_{BA}(\omega'))_+ \right) \right\rangle \right\}. \quad (34)$$

5. Correlation of scattering potential for charged impurities

Designate the non-screened electrical potential created by a charge disposed in the point $\vec{r}=0$ by the symbol $\varphi_I(\vec{r}) = H_e(\vec{r})/e$. The total potential created by all centers is

$$\varphi^{(I)}(\vec{r}) = \sum_{j=1}^N \varphi_I(\vec{r} - \vec{r}_j), \quad (35)$$

$$\varphi^{(I)}(\omega, \vec{q}) = C(\omega, 0) \varphi_I(\vec{q}) \sum_{j=1}^N \exp(-i\vec{q}\vec{r}_j) \rightarrow 2\pi\delta(\omega) \varphi_I(\vec{q}) \sum_{j=1}^N \exp(-i\vec{q}\vec{r}_j). \quad (36)$$

Here, \vec{r}_j is the radius-vector of j -th impurity, $\varphi_I(\vec{q}) = 4\pi e/q^2 \varepsilon_L$, N – total number of impurities in crystal.

Calculating correlations over positions of impurities, we have (See [9] and [11])

$$\langle \delta\varphi^{(I)}(\omega, \vec{q}) \delta\varphi^{(I)}(\omega', \vec{q}') \rangle = +4\pi^4 \delta(\omega + \omega') \delta(\vec{q} + \vec{q}') \langle \delta\varphi_I^2 \rangle_{\vec{q}, \omega}. \quad (37)$$

Here,

$$\begin{aligned} \langle \delta\varphi_I^2 \rangle_{\vec{q}, \omega} &= \langle \delta\varphi_I^2 \rangle_{\vec{q}} \cdot \delta(\omega); \\ \langle \delta\varphi_I^2 \rangle_{\vec{q}} &= 2\pi n_I [\varphi_I(\vec{q})]^2 = 32\pi^3 e^2 n_I / \varepsilon_L^2 q^4. \end{aligned} \quad (38)$$

Turn out to the equation for fluctuation of the density matrix in the form (31) and use the following approximation:

$$(H_e)_{A\Gamma} = \delta_{A\Gamma} \hbar\bar{\omega}_A = \delta_{A\Gamma} \bar{\varepsilon}_A \quad (39)$$

The standard approach (See [1-3]) is related with neglecting the field terms in collision integral; in particular one uses the form (7), where

$$H_e \rightarrow H^{(0)}. \quad (40)$$

Then

$$(H_e)_{A\Gamma} = (H^0)_{A\Gamma} = \delta_{A\Gamma} \hbar\omega_A = \delta_{A\Gamma} \varepsilon_A, \quad (41)$$

and equation for the fluctuation of density matrix (33) accepts the simplified form

$$-i\hbar\delta\rho_{AB}(t=0) + (\hbar\omega - \varepsilon_A + \varepsilon_B) \delta\rho_{AB}(\omega) + (f_A - f_B) (\delta H(\omega))_{AB} = 0. \quad (42)$$

If field terms in the Hamiltonian H_e are maintained (this is non-standard or “field” approach), using the form (39) gives such equation:

$$-i\hbar\delta\rho_{AB}(t=0) + (\hbar\omega - \bar{\varepsilon}_A + \bar{\varepsilon}_B) \delta\rho_{AB}(\omega) + (f_A - f_B) (\delta H(\omega))_{AB} = 0. \quad (43)$$

Then after formal transition $i/(x+i0) \rightarrow 2\pi\delta(x)$ and designation $\bar{\varepsilon}_{AB} = \bar{\varepsilon}_A - \bar{\varepsilon}_B = \hbar\bar{\omega}_{AB}$ we obtain

$$\begin{aligned} \delta\rho_{AB}(\omega) &= 2\pi \hbar \delta(\hbar\omega - \bar{\varepsilon}_{AB}) \times \\ &\times \left[\delta\rho_{AB}(t=0) - (i/\hbar) (f_B - f_A) \delta H_{AB}(\omega) \right]. \end{aligned} \quad (44)$$

The standard approach can be considered as a limited case of the non-standard approach. For this approach, we apply the following approximation:

$$\bar{\varepsilon}_A \rightarrow \varepsilon_A. \quad (45)$$

The distinction between Hamiltonians H_e and $H^{(0)}$ (See (7)) is the principal one, and as result the essential difference can appear between coefficients calculated by these two ways. Farther, it is convenient to use the numerical factor χ :

$$\chi = 0 \text{ for the case of standard variant (a)}, \quad (46a)$$

$$\chi = 1 \text{ for the case of non-standard (field) variant (b)}. \quad (46b)$$

Below, when calculating kinetic coefficients, we will see at which condition both variants give practically the same and at which the opposite condition when a substantial difference appears.

As the initial form of density matrix $\rho_{AB}(t)$, we use

$$\rho_{AB}(t=0) = a_B^+ a_A. \quad (47)$$

Here and farther, we don't show the argument $t=0$ for Fermi-operators a_A and a_A^+ .

Construct the correlator for $\delta\rho_{AB}(t=0) = a_B^+ a_A - \langle a_B^+ a_A \rangle$ and $\delta\rho_{A'B'}(t=0) = a_{B'}^+ a_{A'} - \langle a_{B'}^+ a_{A'} \rangle$:

$$\begin{aligned} \langle \delta\rho_{AB}(t=0) \delta\rho_{A'B'}(t=0) \rangle &= \\ \langle a_B^+ a_A a_{B'}^+ a_{A'} \rangle - \langle a_B^+ a_A \rangle \langle a_{B'}^+ a_{A'} \rangle. \end{aligned} \quad (48)$$

Using Bogolubov's principle of weakening of correlations and performing Week-coupling for two-particle correlator $\langle a_B^+ a_A a_{B'}^+ a_{A'} \rangle$, one obtains the following expression:

$$\begin{aligned} & \left\langle \left(\delta \rho_{AB}^{(0)}(\omega), \delta \rho_{A'B'}^{(0)}(\omega') \right)_+ \right\rangle = \\ & 2\pi^2 \delta(\omega + \omega') \delta(\omega - \bar{\omega}_{AB}) \delta_{A'B} \delta_{AB'} \times \\ & \times [f_A(1-f_B) + f_B(1-f_A)]. \end{aligned} \quad (49)$$

Completing the calculation of corresponding correlators, represent collision integral in the form

$$\text{St } f_A = \text{St}_{el} f_A + \text{St}_{ee} f_A, \quad (50)$$

where

$$\begin{aligned} \text{St}_{ee} f_A &= \frac{4e^4}{\hbar^2 L^3} \sum_{BA'B'} \int \frac{d^3 \bar{q}}{q^4 |\varepsilon(\bar{\omega}_{AB}, \bar{q})|^2} \times \\ & \times |(b_{\bar{q}})_{AB}|^2 |(b_{\bar{q}})_{A'B'}|^2 \delta(\bar{\omega}_{AB} - \bar{\omega}_{A'B'}) \times \\ & \times [f_B(1-f_A) f_{A'}(1-f_{B'}) - f_A(1-f_B) f_{B'}(1-f_{A'})], \end{aligned} \quad (51)$$

$$\begin{aligned} \text{St}_{el} f_A &= \frac{1}{(2\pi)^3 \hbar^2} \sum_B \int d^3 \bar{q} \delta(\bar{\omega}_{AB}) |(b_{\bar{q}})_{AB}|^2 \times \\ & \times (f_B - f_A) \langle \delta H_{\bar{q}}^2 \rangle_{\bar{q}}. \end{aligned} \quad (52)$$

Introducing the designations $A \rightarrow \bar{k}$ and $B \rightarrow \bar{k}'$ in Eq. (52), we obtain (here $f_{\bar{k}} \equiv f(\bar{k})$)

$$\begin{aligned} \text{St}_{el} f(\bar{k}) &= \frac{1}{(2\pi)^3 \hbar} \times \\ & \times \int d^3 \bar{q} \delta(\bar{\varepsilon}_{\bar{k}} - \bar{\varepsilon}_{\bar{k}-\bar{q}}) (f_{\bar{k}-\bar{q}} - f_{\bar{k}}) \langle \delta H_{\bar{q}}^2 \rangle_{\bar{q}}. \end{aligned} \quad (53)$$

In practical calculations, we shall use in future the following approximation for dielectric function (here $1/q_0$ is screening length, \mathcal{G} is step-function):

$$1/\varepsilon(\omega, \bar{q}) 1/\varepsilon(\omega, \bar{q}) \rightarrow (1/\varepsilon_L) \mathcal{G}(q - q_0). \quad (54)$$

6. Calculation of the energies $\bar{\varepsilon}_A$

Accept the components of wave vectors \bar{k} as quantum numbers:

$$A \rightarrow \bar{k}_A = (k_{Ax}, k_{Ay}, k_{Az}). \quad (55)$$

The set of matrix elements of Hamiltonian H_e is (See (7))

$$(H_e)_{A\Gamma} = (H^{(0)})_{A\Gamma} + (H^{(E)})_{A\Gamma} + (H^{(H)})_{A\Gamma}. \quad (56)$$

Here,

$$\begin{aligned} (H^{(0)})_{A\Gamma} &= \left(\hbar^2 / 2m \right) \left(\hat{k}^2 \right)_{A\Gamma}, \\ (H^{(E)})_{A\Gamma} &= -e [(x)_{A\Gamma} E_x + (y)_{A\Gamma} E_y], \\ (H^{(H)})_{A\Gamma} &= \frac{e\hbar H_z}{2mc} (x\hat{k}_y - y\hat{k}_x)_{A\Gamma}. \end{aligned} \quad (57)$$

Note that the Hamiltonian H_e containing field-dependent terms is not arbitrary invariant in space. The problem disappears when using the standard approach.

Usually, this approach is applied without sufficient basis (see, for instants, [1] and [2]). The most convenient for calculations are the following wave functions

$$\Psi_A(w) \equiv \Psi(k_{aw}w) = L^{-1/2} \exp(ik_{aw}w). \quad (58)$$

Here and farther, $w = x, y, z$ and $-L/2 < w < L/2$. The linear dimension L of the system exceeds utmost an every characteristic length. These functions are proper functions for the operator of momentum $\hat{\hbar}\hat{k}$ (and for the operator of kinetic energy):

$$-i\nabla_w \Psi(k_{Aw}; w) = k_{Aw} \Psi(k_{Aw}; w),$$

$$\nabla_x^2 \Psi(k_{Aw}; w) = -k_{Aw}^2 \Psi(k_{Aw}; w).$$

When the parabolic law of dispersion takes place, $\hat{\varepsilon} \Psi(\bar{k}_A; \bar{r}) = \varepsilon(\hbar k_A) \Psi(\bar{k}_A; \bar{r}) = (\hbar^2 k_A^2 / 2m) \Psi(\bar{k}_A; \bar{r})$.

Write this way:

$$\begin{aligned} (1)_{A\Gamma} &= \delta_{A\Gamma}, \quad (\hat{k}_w)_{A\Gamma} = k_{Aw} \delta_{A\Gamma}, \\ (\hat{\varepsilon})_{A\Gamma} &= (\hbar^2 k_A^2 / 2m) \delta_{A\Gamma}. \end{aligned} \quad (59)$$

Now consider the non-standard (field) variant. In consequence of (46), the matrix elements of Hamiltonian H_e can be presented by the form

$$(H_e)_{A\Gamma} = (H^{(0)})_{A\Gamma} + \chi [(H^{(E)})_{A\Gamma} + (H^{(H)})_{A\Gamma}]. \quad (60)$$

When one uses the field variant, the Hamiltonian H_e evidently depends on spatial coordinates. But at the same time, all points of \bar{r} -space are equivalent. Note that wave functions are invariant to the shift of argument w on the length proportional to the deBroigle wavelength. For a minimal shift

$$\begin{aligned} \Psi_A(w + \lambda_{aw}) &= L^{-1/2} \exp[ik_{aw}(w + \lambda_{aw})] = \\ & = \Psi_A(w) = \Psi(k_{aw}w), \end{aligned} \quad (61)$$

where

$$\lambda_{aw} = 2\pi/k_{aw}. \quad (62)$$

Using (61) and shifting the area of integration to $-L/2 + \lambda_{aw} < w < L/2 + \lambda_{aw}$, calculate the matrix-components of radius-vector:

$$(w)_{AB} = \int_{L/(-)}^{L/(+)} w \Psi^*(k_{Aw}; w) \Psi(k_{Bw}; w) dw = \quad (63)$$

$$= \int_{L/(-)}^{L/(+)} w \Psi^*(k_{Aw}; w + \lambda_{aw}) \Psi(k_{Bw}; w + \lambda_{bw}) dw.$$

The diagonal element ($B \rightarrow A$) is

$$(w)_{AA} = \int_{L/(-)}^{L/(+)} w \Psi^*(k_{Aw}; w) \Psi(k_{Aw}; w) dw = L^{-1} \int_{L/(-)}^{L/(+)} w dw. \quad (64)$$

Here $L(\pm) = \pm L/2 + \lambda_{aw}$. As it follows from (64):

$$(w)_{AA} = \lambda_{aw} = 2\pi/k_{aw}. \quad (65)$$

Further, we assume the following relation to be valid:

$$L/|\lambda_{aw}| = L|k_{aw}|/2\pi \rightarrow \infty \quad (66)$$

and accept

$$\begin{aligned} (w)_{A\Gamma} &\approx \text{Re}(w)_{A\Gamma}, \\ (w)_{A\Gamma} &\rightarrow \delta_{A\Gamma}(w)_{AA} = (2\pi/k_{aw})\delta_{A\Gamma}. \end{aligned} \quad (67)$$

As a result of the limit $L \rightarrow \infty$, one obtains the formula (39), where

$$\bar{\varepsilon}_A = \frac{\hbar^2 \bar{k}_A^2}{2m} - \frac{2\pi e E_x}{k_{Ax}} - \frac{2\pi e E_y}{k_{Ay}} + \hbar \Omega \left(\frac{k_{Ay}}{k_{Ax}} - \frac{k_{Ax}}{k_{Ay}} \right), \quad (68)$$

$$\Omega = \pi e H_z / mc. \quad (69)$$

The transformation (63) is not possible only one. We accept the shown form because just that gives expected physical result (See below (124) and (125)).

Using the designations $\bar{k}_A \rightarrow \bar{k}$ and $\bar{k}_B \rightarrow \bar{k} - \bar{q}$, one obtains the form

$$\begin{aligned} \bar{\varepsilon}_{AB} = \bar{\varepsilon}_A - \bar{\varepsilon}_B &\rightarrow \bar{\varepsilon}_{\bar{k}} - \bar{\varepsilon}_{\bar{k}-\bar{q}} = \frac{\hbar^2}{2m} (2\bar{k}\bar{q} - q^2) + \\ &+ \frac{2\pi e E_x q_x}{k_x(k_x - q_x)} + \frac{2\pi e E_y q_y}{k_y(k_y - q_y)} + \hbar \Omega (k_x q_y - k_y q_x) \times \\ &\times \left[\frac{1}{k_x(k_x - q_x)} + \frac{1}{k_y(k_y - q_y)} \right]. \end{aligned} \quad (70)$$

Simplify the calculations with the help of the approximation $|q_w| \ll |k_w|$ and such changes:

$$\begin{aligned} k_x^2 \rightarrow \langle k_x^2 \rangle &= (1/3) \langle k^2 \rangle = 2m \langle \varepsilon \rangle / 3\hbar^2, \\ k_y^2 \rightarrow \langle k_y^2 \rangle &= (1/3) \langle k^2 \rangle = 2m \langle \varepsilon \rangle / 3\hbar^2. \end{aligned} \quad (71)$$

Here, $\langle \varepsilon \rangle = 3k_B T F_{3/2}(\eta) / 2F_{1/2}(\eta)$ is average energy;

$$F_r(\eta) = \frac{1}{\Gamma(r+1)} \int_0^\infty \frac{w^r dw}{1 + \exp(w - \eta)}; \quad \eta = \varepsilon_F / k_B T. \quad (72)$$

As a result,

$$\begin{aligned} \bar{\varepsilon}_{\bar{k}} \equiv \bar{\varepsilon}(\bar{k}) &= \frac{\hbar^2 \bar{k}^2}{2m} - \frac{2\pi e E_x}{k_x} - \frac{2\pi e E_y}{k_y} + \hbar \Omega \left(\frac{k_y}{k_x} - \frac{k_x}{k_y} \right), \\ \varepsilon_{\bar{k}} &= \frac{\hbar^2 \bar{k}^2}{2m}, \end{aligned} \quad (73)$$

$$\begin{aligned} \bar{\varepsilon}_{\bar{k}} - \bar{\varepsilon}_{\bar{k}-\bar{q}} &= \frac{\hbar^2}{2m} (2\bar{k}\bar{q} - q^2) + \\ &+ \frac{3\pi e \hbar^2}{m \langle \varepsilon \rangle} (E_x q_x + E_y q_y) + \frac{3\hbar^3 \Omega (k_x q_y - k_y q_x)}{m \langle \varepsilon \rangle}. \end{aligned} \quad (73a)$$

We don't consider here quantized magnetic field (that is $b \ll 1$); therefore values of the order b^2 will be every case omitted. Under the designations

$$3\pi e \bar{E} / \langle \varepsilon \rangle = \bar{k}^{(E)}, \quad 3\hbar \Omega / \langle \varepsilon \rangle = b \quad (74)$$

the expression (73a) can be written as

$$\begin{aligned} \bar{\varepsilon}_{\bar{k}} - \bar{\varepsilon}_{\bar{k}-\bar{q}} &= \frac{\hbar^2}{m} \left\{ \left(k_x^{(E)} + k_x - b k_y \right) q_x + \right. \\ &\left. + \left(k_y^{(E)} + k_y + b k_x \right) q_y + k_z q_z - \frac{q^2}{2} \right\}. \end{aligned} \quad (75)$$

The latter expression prompts to introduce the new vector $\bar{\kappa}(\bar{k})$:

$$\bar{\kappa}(\bar{k}) = (\kappa_x(\bar{k}), \kappa_y(\bar{k}), \kappa_z(\bar{k})). \quad (76)$$

Here,

$$\begin{aligned} \kappa_x(\bar{k}) &= k_x + k_x^{(E)} - b k_y; \quad \kappa_y(\bar{k}) = k_y + k_y^{(E)} + b k_x; \\ \kappa_z(\bar{k}) &= k_z. \end{aligned} \quad (77)$$

The reverse transformation (if using the inequality $b^2 \ll 1$) is

$$\begin{aligned} k_x &= \kappa_x + b \kappa_y - k_x^{(E)} - b k_y^{(E)}, \\ k_y &= \kappa_y - b \kappa_x - k_y^{(E)} + b k_x^{(E)}. \end{aligned} \quad (78)$$

Then (for approximations shown before) one obtains from (75)

$$\bar{\varepsilon}_{\bar{k}} - \bar{\varepsilon}_{\bar{k}-\bar{q}} = \hbar (\bar{\omega}_{\bar{k}} - \bar{\omega}_{\bar{k}-\bar{q}}) = \frac{\hbar^2}{m} \left\{ \bar{\kappa}\bar{q} - \frac{q^2}{2} \right\}. \quad (79)$$

Introducing the mechanical momentum $\bar{p} = \hbar \bar{k}$, one can see that at (73a) two latter terms are quantum amendment to classical part. Therefore, the retained field terms in collision integral give the reason to name the considered kinetic equation as the quantum kinetic one.

7. Balance of forces

For stationary spatially uniform system kinetic equation (29) has the form

$$\frac{e}{\hbar} \left\{ \bar{E} + \frac{1}{c} [\bar{v}(\bar{k}) \times \bar{H}] \right\} \frac{\partial f_{\bar{k}}}{\partial \bar{k}} = \text{St} f_{\bar{k}}. \quad (80)$$

Construct the first moment of the equation (80), applying there to both sides the operator

$$2(2\pi)^{-3} \int \bar{k} d^3 \bar{k}. \quad (81)$$

Then, we obtain a vector equation, having the sense of balance of dynamical and statistical fields forced in all the system of band carriers:

$$e \left[\vec{E} + (1/c)(\vec{H} \times \vec{u}) \right] + \frac{2\hbar}{(2\pi)^3 n} \int \vec{k} \text{St}_{e-S} f_{\vec{k}} d^3 \vec{k} = \quad (82)$$

$$= e \left[\vec{E} + (1/c)(\vec{H} \times \vec{u}) \right] + \vec{F}_{el} = 0.$$

Here, the value \vec{F}_{el} is resistant force acting from the side of charged impurities (something as “friction force”). The values

$$n = \frac{2}{(2\pi)^3} \int f_{\vec{k}} d^3 \vec{k}, \quad (83)$$

$$\vec{u} = \frac{\int \vec{v}(\vec{k}) f(\vec{k}) d^3 \vec{k}}{\int f(\vec{k}) d^3 \vec{k}} = \frac{2}{(2\pi)^3 n} \int \vec{v}(\vec{k}) f_{\vec{k}} d^3 \vec{k}$$

are density of electrons and drift velocity of whole band electrons.

After non-complicated transformations of the formula (53), we obtain the following expression:

$$\vec{F}_{el} = -\frac{e^4 n_I m}{\pi^3 \hbar^2 n \varepsilon_L^2} \int f(\vec{k}) d^3 \vec{k} \int \vec{q} d^3 \vec{q} \delta(\vec{k} \vec{q} - q^2/2) q^{-4}. \quad (84)$$

Performing here integration over components of vector \vec{q} (see (78)) we find

$$\vec{F}_{el} = -\frac{2me^4 n_I}{\pi^3 \hbar^2 n \varepsilon_L^2} \ln\left(\frac{q_M}{q_0}\right) \int f(\vec{k}(\vec{\kappa})) \kappa^{-3}(\vec{k}) \vec{\kappa} d^3 \vec{\kappa}. \quad (85)$$

For standard variant $\chi = 0$ (See (46)), the expression (85) transforms to

$$\vec{F}_{el} = -\frac{2me^4 n_I}{\pi^3 \hbar^2 n \varepsilon_L^2} \ln\left(\frac{q_M}{q_0}\right) \int f(\vec{k}) k^{-3} \vec{k} d^3 \vec{k}. \quad (86)$$

8. The model of non-equilibrium distribution function

As one can see, the friction forces (85) and (86) are linear integral functionals of the non-equilibrium distribution function $f(\vec{k})$. As the sufficiently simple model of $f(\vec{k})$, we accept here Fermi-function with a shifted argument:

$$f(\vec{k}) = f_0(\vec{k} - \vec{k}_u) = \left[1 + \exp\left(\frac{\hbar^2(\vec{k} - m\vec{u}/\hbar^2)/2m - \varepsilon_F}{k_B T}\right) \right]^{-1}. \quad (87)$$

Introduce three-dimensional vectors $\vec{K}, \vec{K}^{(u)}$ and several dimensionless values:

$$\vec{K}^{(u)} = (K_x^{(u)}, K_y^{(u)}, 0), \quad K_x^{(u)} = k_x^{(E)} + bk_y^{(E)} + mu_x/\hbar, \quad (88)$$

$$K_y^{(u)} = k_y^{(E)} - bk_x^{(E)} + mu_y/\hbar,$$

$$\vec{K} = (K_x, K_y, K_z) = (\kappa_x + b\kappa_y, \kappa_y - b\kappa_x, \kappa_z). \quad (89)$$

$$\hbar \vec{K} / \sqrt{2mk_B T} = \vec{t},$$

$$\hbar \vec{K}^{(u)} / \sqrt{2mk_B T} = \vec{Y}, \quad \varepsilon_F / k_B T = \eta. \quad (90)$$

Then, the “friction” force (85) takes the form

$$\vec{F}_{el} = -\frac{2^{13/2} m^{3/2} (k_B T)^{1/2} e^4 n_I}{\pi^3 \hbar^3 n \varepsilon_L^2} \ln\left(\frac{q_M}{q_0}\right) \times \int \frac{t^{1-p} [\vec{t} + \chi b(\vec{e}_z \times \vec{t})]}{1 + \exp\left(\left(\vec{t} - \vec{Y}\right)^2 - \eta\right)} d^3 \vec{t}. \quad (91)$$

Introduce the dimensionless electric field and current density:

$$\vec{W} = \vec{E} / E_T, \quad \vec{J} = \vec{j} / j_0 = \sqrt{m/k_B T} \vec{u}. \quad (92)$$

Here,

$$E_T = \langle \varepsilon \rangle \sqrt{2mk_B T} / 3\pi e \hbar, \quad j_0 = en \sqrt{k_B T} / \sqrt{m}. \quad (93)$$

It follows from (90), (92) and (46) that

$$\vec{Y} = \vec{J} + \chi [\vec{W} + b(\vec{W} \times \vec{e}_z)]. \quad (94)$$

Then the balance equation accepts the form

$$\vec{W} + b(\vec{e}_z \times \vec{J}) = \frac{\Theta_{(I)}}{F_{3/2}(\eta)} \times \int \frac{t^{1-p} [\vec{t} - \chi b(\vec{t} \times \vec{e}_z)] d^3 \vec{t}}{1 + \exp\left\{ \left[\vec{t} - \chi \vec{W} - \chi b(\vec{W} \times \vec{e}_z) - \vec{J} \right]^2 - \eta \right\}}, \quad (95)$$

where

$$\Theta_{(I)} = \frac{2^{5/2} \pi^3 e^4 \hbar n_I}{\varepsilon_L^2 m^{1/2} (k_B T)^{5/2}} \ln\left(\frac{q_M}{q_0}\right). \quad (96)$$

For $q_M \gg q_0$

$$\ln(q_M/q_0) \approx \frac{1}{2} \ln \left\{ \frac{4\varepsilon_L \sqrt{8k_B T} \hbar F_1^2(\eta)}{e^2 \sqrt{m\pi} F_{1/2}^2(\eta) F_{-1/2}(\eta)} \right\}. \quad (97)$$

If external magnetic field is absent, the equation (95) is converted to the following equation:

$$\vec{W} = \frac{\Theta_{(I)}}{F_{3/2}(\eta)} \int \frac{\vec{t} t^{1-p} d^3 \vec{t}}{1 + \exp\left\{ \left[\vec{t} - \chi \vec{W} - \vec{J} \right]^2 - \eta \right\}}. \quad (98)$$

Designate a mobility tensor by the symbol $\hat{\mu}^{(x)}$ and write here:

$$\vec{j} = en \hat{\mu}^{(x)} \vec{E}, \quad \vec{J} = (\hat{\mu}^{(x)} / \mu_{\min}) \vec{W}, \quad (99)$$

$$\begin{aligned}\mu_{\min} &= \frac{j_0}{enE_T} = \frac{3\pi e\hbar}{\sqrt{2m}\langle\varepsilon\rangle} = \\ &= \frac{\sqrt{2}\pi e\hbar}{mk_B T} \frac{F_{1/2}(\eta)}{F_{3/2}(\eta)} = M \frac{F_{1/2}(\eta)}{F_{3/2}(\eta)}.\end{aligned}\quad (100)$$

Dimensionless magnetic field $b = \mu_{\min} H_z / c$. In absence of magnetic field $\mu_{ij}^{(x)} = \mu^{(x)} \delta_{i,j}$. In the system CGSE and for $m = 2 \cdot 10^{-28}$, $T = 100\text{K}$, we find: $M = 8.1 \cdot 10^5$ CGSE.

Define the conditional free-path length \bar{L} and average length of the deBroigle wave $\bar{\lambda}$ by the relations:

$$\bar{L}^{(x)} = |\hat{\mu}^{(x)}| \sqrt{2m\langle\varepsilon\rangle} / e, \quad \bar{\lambda} = \hbar / \sqrt{2m\langle\varepsilon\rangle}.\quad (101)$$

Then the equality

$$\bar{L}^{(x)} \gg \bar{\lambda}\quad (102)$$

can be written as $J \gg W$ or $|\hat{\mu}^{(x)}| \gg \mu_{\min}$. Under the condition $\bar{L}^{(x)} \leq \bar{\lambda}$, or $|\bar{J}| \leq |\bar{W}|$, the concept ‘‘mobility’’, how one will see below, loses its usual meaning, and description of macroscopic movement of band carriers requires other ways.

If electrical field and current density are weak, that is

$$|\bar{W}| + |\bar{J}| \ll 1,\quad (103)$$

one can linearise the model non-equilibrium distribution $f(\vec{k})$. In this case, the relation of dimensionless electrical field \bar{W} with dimensionless density of current \bar{J} becomes the linear equation

$$\begin{aligned}\bar{W} + b(\bar{e}_z \times \bar{J}) &= \frac{2\Theta_{(I)}}{F_{3/2}(\eta)} \int \frac{[\bar{t} + \chi b(\bar{t} \times \bar{e}_z)]}{t^3 [1 + \exp(t^2 - \eta)]^2} \times \\ &\times \{ \bar{t} \cdot [\bar{J} + \chi \bar{W} + \chi b(\bar{W} \times \bar{e}_z)] \} \exp(t^2 - \eta) d^3 \bar{t}.\end{aligned}\quad (104)$$

For the case $\chi = 0$, one obtains after performing the integration over angles in the latter formula:

$$\bar{W} + b(\bar{e}_z \times \bar{J}) = \frac{4\pi \Theta_{(I)}}{3F_{3/2}(\eta)} \bar{J} \int \frac{t^{5-p} \exp(t^2 - \eta)}{[1 + \exp(t^2 - \eta)]^2} dt.\quad (105)$$

9. Current-voltage characteristics in absence of magnetic fields

For $b = 0$ (that is at $H = 0$), the linear equation (105) has the form (external macroscopic electrical field is directed along the x -axis):

$$W_x = (J_x + \chi W_x) Q_{(I)}(\eta) = \Theta_{(I)} \cdot \alpha(\eta) (J_x + \chi W_x),\quad (106)$$

$$\begin{aligned}\alpha(\eta) &= \frac{8\pi}{3F_{3/2}(\eta)} \int_0^\infty \frac{t \exp(t^2 - \eta)}{[1 + \exp(t^2 - \eta)]^2} dt = \\ &= \frac{4\pi}{3} \frac{F_{-1}(\eta)}{F_{3/2}(\eta)} = \frac{4\pi}{3[1 + \exp(-\eta)] F_{3/2}(\eta)}.\end{aligned}\quad (107)$$

Accordingly to (99) and (106), the current-voltage characteristic has the form

$$J_x = \frac{1}{\mu_{\min}} \mu^{(x)} W_x = W_x (1/Q_{(I)}(\eta) - \chi).\quad (108)$$

It follows from here:

$$\mu^{(0)} = \mu_{\min} / Q_{(I)}, \quad \mu^{(1)} = \mu_{\min} (1 - Q_{(I)}) / Q_{(I)};\quad (109)$$

$$\mu^{(1)} / \mu^{(0)} = 1 - Q_{(I)}.\quad (110)$$

One can see from the formula (123) that for field variant ($\chi = 1$) the concept ‘‘mobility’’ has a meaning only under the condition:

$$Q_{(I)} < 1.\quad (111)$$

In this case, $0 < \mu^{(1)} < \mu^{(0)}$. It also follows from (109) that distinction between results of calculations for standard and field variants disappears under the condition $Q_{(I)} \ll 1$ or $\mu^{(0)} \gg \mu_{\min}$.

10. Galvanomagnetic kinetic effects

10.1. Kinetic characteristics calculated for standard linear equation of forces balance

Supposing $\chi = 0$, we write the vector equation (105) as the system

$$W_x - bJ_y = Q_{(I)}(\eta) J_x;\quad (112a)$$

$$W_y + bJ_x = Q_{(I)}(\eta) J_y.\quad (112b)$$

Here, we accept $b^2 \ll 1$. One writes the solution of the system (112) in the form (99). Components of mobility tensor $\hat{\mu}^{(0)}$ are

$$\begin{aligned}\mu_{xx}^{(0)}(H) = \mu_{yy}^{(0)}(H) &= \frac{\mu_{\min} Q_{(I)}}{Q_{(I)}^2 + b^2(H)}, \\ \mu_{xy}^{(0)}(H) = -\mu_{yx}^{(0)}(H) &= -b(H) \frac{\mu_{\min}}{Q_{(I)}^2 + b^2(H)}.\end{aligned}\quad (113)$$

At presence of magnetic field $\vec{H} = H_z \bar{e}_z$ and at current $\vec{j} = (j_x, 0)$, the longitudinal component j_x , transverse component of electrical field E_y and Hall constant $R_H^{(0)}$ are

$$j_x(H) = en \left[\mu_{xx}^{(0)}(H) + \frac{(\mu_{xy}^{(0)}(H))^2}{\mu_{xx}^{(0)}(H)} \right] E_x = \quad (114)$$

$$= en \mu_{\parallel}^{(0)}(H) E_x,$$

$$\mu_{\parallel}^{(0)}(H) = \frac{\mu_{\min}}{Q_{(I)}},$$

$$E_y(H) = \mathfrak{G}^{(0)}(H) E_x = \frac{\mu_{yx}^{(0)}(H)}{\mu_{xx}^{(0)}(H)} E_x = \frac{b(H)}{Q_{(I)}} E_x, \quad (115)$$

$$R_H^{(0)}(H) = \left| \frac{E_y(H)}{H_z j_x(H)} \right| = \left| \frac{1}{enc} \right|. \quad (116)$$

As one can see, in the standard variant the longitudinal conductivity $\sigma_{\parallel}^{(0)} = en\mu_{\parallel}^{(0)}$ and Hall constant $R_H^{(0)}$ do not depend on the intensity of magnetic field.

10.2. Kinetic characteristics calculated for non-standard linear equation of forces balance

Write the components of linear vector equation for $\chi = 1$:

$$W_x - bJ_y = Q_{(I)}(\eta)(J_x + W_x - bJ_y); \quad (117a)$$

$$W_y + bJ_x = Q_{(I)}(\eta)(J_y + W_y + bJ_x). \quad (117b)$$

Solving this system of equations, represent the solution in the form (99). One applies the magnetic field to be not quantized, that is $|b| = \mu^{(0)}H/c \ll 1$; but the value $|b|$ can be comparable with $Q_{(I)}$ and even exceeds it. As a result,

$$\mu_{xx}^{(1)} = \mu_{yy}^{(1)} = \mu_{\min} \frac{(1-Q_{(I)})Q_{(I)}}{Q_{(I)}^2 + b^2(1-Q_{(I)})^2},$$

$$\mu_{xy}^{(1)} = -\mu_{yx}^{(1)} = -\mu_{\min} b \frac{(1-Q_{(I)})^2}{Q_{(I)}^2 + b^2(1-Q_{(I)})^2}. \quad (118)$$

Consider the case $\vec{j} = (j_x, 0)$. Then,

$$j_x = en \left[\mu_{xx}^{(1)} + \frac{(\mu_{xy}^{(1)})^2}{\mu_{xx}^{(1)}} \right] E_x = en\mu_{\parallel}^{(1)} E_x,$$

$$\mu_{\parallel}^{(1)}(b) = \mu_{\min} \frac{(1-Q_{(I)})}{Q_{(I)}}, \quad \mu_{\parallel}^{(1)}(b) = \mu_{\parallel}^{(0)}(b) - \mu_{\min}, \quad (119)$$

$$\mathfrak{G}^{(1)}(b) = \frac{E_y(b)}{E_x} = \frac{\mu_{yx}^{(1)}(b)}{\mu_{xx}^{(1)}(b)} = b \frac{1-Q_{(I)}}{Q_{(I)}} = \mathfrak{G}^{(0)}(b)(1-Q_{(I)}),$$

$$R_H^{(1)}(b) = \left| \frac{E_x \mathfrak{G}^{(1)}(b)}{H_z j_x(b)} \right| = \left| \frac{1}{enc} \right|. \quad (120)$$

Comparing the results of standard and field variants, we find:

$$\mu_{\parallel}^{(1)} = \mu_{\parallel}^{(0)}(1-Q_{(I)}), \quad \mathfrak{G}^{(1)} = \mathfrak{G}^{(0)}(1-Q_{(I)}),$$

$$R_H^{(1)}(b) = R_H^{(0)}(b) = R_H. \quad (121)$$

It follows that formulae (119), (120) and the meanings of mobility and Hall-angle have a sense at the following condition only:

$$Q_{(I)} < 1. \quad (122)$$

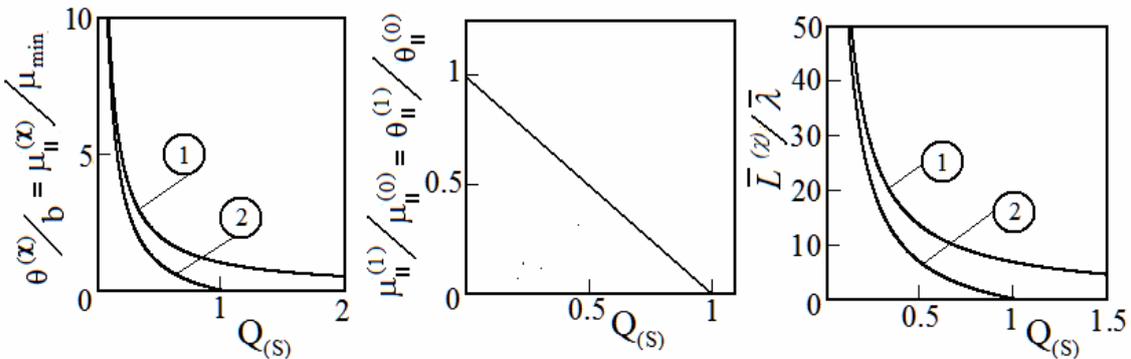
The free-path distance (see (101)) is

$$\bar{L}^{(1)} = \bar{L}^{(0)}(1-Q_{(I)}). \quad (123)$$

If for the inequality (122) the value $Q_{(I)}$ is sufficiently close to unity, one can say about small mobility or about definite “demobilization” of band electrons due to extremely high intensity of scattering. The limit of mobility $\mu_{\parallel}^{(1)} = 0$ achieves at $Q_{(I)} = 1$.

One obtains from (101) and (121):

$$\frac{\bar{L}^{(x)}}{\bar{\lambda}} = \frac{3\pi}{\sqrt{2} Q_{(S)}} (1-Q_{(I)})^x; \quad Q_{(I)} = 3\pi\bar{\lambda}/\sqrt{2}\bar{L}^{(0)}. \quad (124)$$



It follows from here that retention of field terms in collision integral is the reason of appearance of quantum amendment to kinetic coefficients, for instance:

$$\mu_{\parallel}^{(1)} = \mu_{\parallel}^{(0)} \left(1 - \frac{3\pi}{\sqrt{2}} \frac{\bar{\lambda}}{L^{(0)}} \right). \quad (125)$$

As a result, we obtain the important conclusion: a quantum kinetic equation distinguishes from classical kinetic equation by retention of field terms in collision integral.

Therefore, we can use the expression “classical approach” instead of “standard variant” and “quantum approach” instead of “field variant” (See the forms (46)).

Below in Fig. 1 there are presented several plots, drawn using the formulae (115) and (119). One can see that, under the field approach to collision integral typical, the kinetic characteristics substantially differ by the values corresponding to standard variant even at $Q_{(I)} > 0.1$.

11. Discussion

One can make the conclusion that regard for the field terms in collision integral results there in appearance of deBroigle wavelength λ and comparability of that with the free-path distance L . Taking into account the finite ratio of λ and L , we can say about quantum kinetic equation in total. If a consideration does not use directly the field terms in collision integral, the kinetic equation leaves to be the classic one.

References

1. A.I. Anselm, *Introduction to the Theory of Semiconductors*. Nauka, Moscow, 1978 (in Russian).
2. E.M. Lifshits and L.P. Pitaevskiy, *Physical Kinetics*. Nauka, Moscow, 1984 (in Russian).
3. V.F. Gantmaher and I.B. Levinson, *Scattering of Current Carriers in Metals and Semiconductors*. Nauka, Moscow, 1984 (in Russian).
4. N.N. Bogolubov, *Lectons for Quantum Statistics*. Radianska Shkola, 1949 (in Ukrainian).
5. N.N. Bogolubov, *Collected works in 12 volumes, v. 5: Non-equilibrium Statistical Mechanics*. Nauka, Moscow, 2006 (in Russian).
6. Yu.L. Klimontovich, *Statistical Physics*. Nauka, Moscow, 1978 (in Russian).
7. D.N. Zubarev, *Non-equilibrium Statistical Thermodynamics*. Nauka, Moscow, 1971 (in Russian).
8. L.D. Landau and E.M. Lifshits, *Quantum Mechanics*. Nauka, Moscow, 1963 (in Russian).
9. I.I. Boiko, *Kinetics of Electron Gas Interacting with Fluctuating Potential*. Naukova dumka, Kyiv, 1993 (in Russian).
10. M.A. Lavrentiev and B.W. Shabat, *Methods of Functions of Complex Variables*. Moscow, 1958 (in Russian).
11. I.I. Boiko, *Impurity Scattering of Band Carriers // Semiconductor Physics, Quantum Electronics & Optoelectronics*, **13**(2), p. 214-220 (2010).
12. I.I. Boiko, *Dependence of the Collision Integral on Electric Field // Semiconductor Physics, Quantum Electronics & Optoelectronics*, **18**(2), p. 138-143 (2015).