provides an opportunity to solve the problem of singularity at the points of «junction» of boundary streamlines and equipotential lines, contributes to increasing the accuracy of quasiconformal mappings and improving the «transparency» of solving process of the corresponding problem. Also, as expected, the «five-point» scheme for ensuring orthogonality on smooth boundary lines showed greater efficiency compared to the «two-point» one.

As a prospect for further application of the developed procedure of «fictitious orthogonalization», the mechanism of its adaptation is described on the example of electrical tomography problems.

Key words: *mathematical modeling, nonlinear problems, quasiconformal mappings, numerical methods.*

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SIMULATION OF THE CHARGE CARRIERS DISTRIBUTION IN THE ACTIVE REGION OF THE P-I-N-DIODES BY THE PERTURBATION THEORY METHODS

A mathematical model of the electron-hole plasma stationary distribution in the active region (i-region) of p-i-n-diodes in the diffusion-drift approximation is proposed. The model is represented in the form of a nonlinear singularly perturbed boundary value problem for the system of equations of the electron-hole currents continuity, the Poisson equation and the corresponding boundary conditions. The decomposition of the nonlinear boundary value problem of modeling the stationary distribution of charge carriers in the plasma of p-i-n-diodes is carried out on the basis of the solutions asymptotic representation. The model problem is reduced to a sequence of the linear boundary value problems with a characteristic separation of the main (regular) components of the asymptotics and a boundary corrections. It was found that the formulation of the problem for finding the zero term of the asymptotics regular part coincides with the classical formulation of the p-i-n-diodes characteristics modeling problem, which is carried out in the approximation of the ambipolar diffusion (approximation of a self-consistent electrostatic field). The proposed mathematical model and the method of its linearization make it possible to determing the main components in the diffusion-drift process and to study their role. For example, it becomes possible to study (including by analytical methods) the behavior of plasma in the p-i-, n-i-contacts zones. The results of the study are aimed at developing methods for designing p-i-n-diode structures, used, in particular, as active elements of the signals switches of a microwave data transmission systems and the corresponding protective devices.

Key words: *perturbation method, singular-perturbation problems, boundary layer method, diffusion-drift process, p-i-n-diode.*

1. Introduction. For switching of the microwave electromagnetic field p-i-n-diodes are used [1, 2]. The electromagnetic field is controlled by changing the concentration of the charge carriers (electrons and holes) in the active region (i-region) using a control current. The distribution of the charge carriers concentration is the basic characteristic of the microwave switches. To estimate the value of the accumulated charge in the active region, in particular, the hydrodynamic models of plasma dynamics are widely used [1-3]. The mathematical model of plasma dynamics is based on a nonlinear system of partial differential equations, for the analysis of which a number of approximations are used. In particular, the approximation of ambipolar diffusion (self-consistent field mode) [1-3] does not take into account the effects of plasma heating [1], etc. This approach allows one to obtain linearized mathematical models. However, this reduces the level of modeling adequacy.

There are a number of practical problems (for example, the problem of optimizing the characteristics of p-i-n-structures) that cannot be solved using linearized models. There is a need for the development of an appropriate mathematical modeling tools.

The aim of the work: development of the mathematical model and an method for decomposition of the problem of finding a stationary distribution of the charge carriers concentration in the semiconductor p-i-n-diodes active region within the framework of the application of the plasma dynamics diffusion-drift model and the perturbation theory methods.

2. The formulation of the problem. The p-i-n-diode active region is a limited region ($\Omega = \{(x, y, z): 0 < x < l, 0 < y < w, 0 < z < d\}$) of a semiconductor material with intrinsic conductivity, at the boundary of which ($\partial\Omega$) are p-i- and n-i-junctions intended for injection of charge carriers into the active region (holes and electrons, respectively). The cross section of the p-i-n structure is shown schematically in Figure 1. $\partial\Omega_{n,p}$ — the boundaries of injection contacts, $\partial\Omega_0$ — the isolated boundary. The basic mathematical model describing the distribution of the concentration of holes (n), electrons (p) and potential (φ) in the active region of p-i-n-diodes is based on the equation of the holes and electrons currents continuity and the Poisson's equation [1, 3]:

$$\Delta n = \frac{e}{kT}\left(\nabla n \cdot \nabla \varphi + n\Delta\varphi\right) + \frac{1}{D_n \tau_n^*} n \,, \tag{1}$$

$$\Delta p = \frac{e}{kT}\left(-\nabla p \cdot \nabla \varphi - p\Delta\varphi\right) + \frac{1}{D_p \tau_p^*} p \,, \tag{2}$$

$$\frac{\varepsilon\varepsilon_0}{e}\Delta\varphi = -\left(p - n + N_l\right), \tag{3}$$

where τ_p^*, τ_n^* are the characteristic relaxation lifetimes of holes and electrons in the i-region (in the general case, they depend on the local concentration of electrons and holes and are determined by a certain mechanism of the charge carriers recombination); $N_l = N_a - N_d \equiv N_l(x, y)$ — a given function of the doping profile (describes the difference between the concentrations of acceptors and donors in the active region); D_p, D_n — diffusion coefficients of holes and electrons, respectively; k — Boltzmann’s constant; T — temperature; e is the electron charge; ε, ε_0 are the relative dielectric constant of the semiconductor and the dielectric constant.

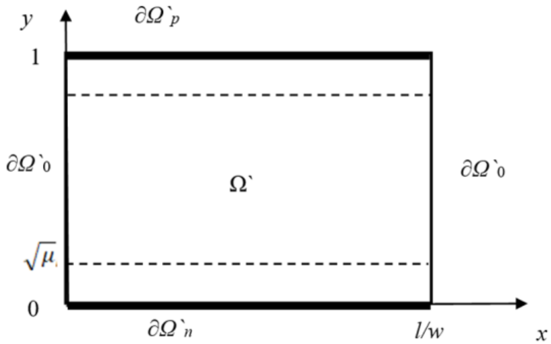

Fig. 1. *Cross-section of the p-i-n-diode`s active region*

In general, the boundary conditions are as follows:

a) on injection contacts

$$\left.\left(\left(\vec{j}_n - \vec{j}_{nr}\right)\cdot\vec{v}\right)\right|_{\partial\Omega_n} = J\,,\ \left.\left(\left(\vec{j}_p - \vec{j}_{pr}\right)\cdot\vec{v}\right)\right|_{\partial\Omega_n} = 0\,,\ \left.\varphi\right|_{\partial\Omega_n} = 0\,, \tag{4}$$

$$\left.\left(\left(\vec{j}_p - \vec{j}_{pr}\right)\cdot\vec{v}\right)\right|_{\partial\Omega_p} = J\,,\ \left.\left(\left(\vec{j}_n - \vec{j}_{nr}\right)\cdot\vec{v}\right)\right|_{\partial\Omega_p} = 0\,,\ \left.\varphi\right|_{\partial\Omega_p} = U\,, \tag{5}$$

where J is a constant that determines the injection current density (control current density); $\vec{v}$ is the normal vector to the region border; $\vec{j}_{n,p}$ are the currents density of electrons and holes; $\vec{j}_{rn,rp}$ are the density of the recombination currents. Note that boundary conditions (4)-(5) are written under the assumption that only electrons carry current through the $\partial\Omega_n$ boundary, respectively, through $\partial\Omega_p$ — holes (in the p-i-n-diodes with a wide active region the most of the injected charge carriers recombine in the i-region and not manages to get into the area of opposite contact).

Taking into account relations $\vec{j}_p = e\mu_p p\vec{E} - eD_p\nabla p$, $\vec{j}_n = e\mu_n n\vec{E} + eD_n\nabla n$, $\vec{j}_{rp} = e\alpha_p p\vec{v}$, $\vec{j}_{rn} = e\alpha_n n\vec{v}$, it can be shown that conditions (4.5) are equivalent to the following:

$$\left.\frac{\partial n}{\partial \nu} - \gamma_n n\right|_{\partial\Omega_n} = \frac{J}{eD_n},\ \left.\frac{\partial n}{\partial \nu}\right|_{\partial\Omega_p} = 0, \quad (6)$$

$$\left.\frac{\partial p}{\partial \nu} - \gamma_p p\right|_{\partial\Omega_p} = -\frac{J}{eD_p},\ \left.\frac{\partial p}{\partial \nu}\right|_{\partial\Omega_n} = 0,$$

where $\gamma_n = \left(\frac{\alpha_n}{\mu_n} - \frac{\alpha_p}{\mu_p}\right)\frac{\mu_n}{D_n}$; $\gamma_p = \left(\frac{\alpha_n}{\mu_n} - \frac{\alpha_p}{\mu_p}\right)\frac{\mu_p}{D_p}$; $\alpha_{n,p}$ — the recombination coefficients of electrons and holes, respectively;

b) on the isolated borders, we obtain the similar relations:

$$\left.\frac{\partial p}{\partial \nu} - \gamma_p p\right|_{\partial\Omega_0} = 0,\ \left.\frac{\partial n}{\partial \nu} - \gamma_n n\right|_{\partial\Omega_0} = 0. \quad (7)$$

Based on the arbitrary choice of potential, we put

$$\varphi|_{\partial\Omega_n} = 0,\ \varphi|_{\partial\Omega_p} = U,\ \left.\frac{\partial\varphi}{\partial \nu}\right|_{\partial\Omega_0} = 0, \quad (8)$$

where U is a constant that determines the voltage on the p-i-n-diode contacts.

Note that in the boundary conditions, dependent characteristics are used — the density of the injected current J and the value of the potential at the p-i-contact U (in the general case, the connection between the characteristics is established by Ohm's law and depends on the conductivity of the i-region and the resistance of the external electrical circuit).

It is convenient to analyze system (1)-(3) in a normalized form. It is also proposed to consider a two-dimensional spatial model of a p-i-n-structure element (the solution is sought in the region $\Omega' = \{(x, y) : 0 < x < l, 0 < y < w\}$), since its linear dimensions along the OZ axis significantly exceed other its linear dimensions. Let us introduce into consideration the dimensionless quantities $\tilde{x} = \frac{x}{w}$ ($0 < \tilde{x} < \frac{l}{w}$), $\tilde{y} = \frac{y}{w}$ ($0 < \tilde{y} < 1$), $\tilde{\varphi} = \frac{e\varphi}{kT}$, $\tilde{U} = \frac{eU}{kT}$, $\tilde{n} = \frac{n}{N_i}$ ($0 \le \tilde{n} \le \frac{n_{\max}}{N_i}$), $\tilde{p} = \frac{p}{N_i}$

($0 \le \tilde{p} \le \frac{p_{\max}}{N_i}$), $\tilde{N}_d = \frac{N_d}{N_i}$, where N_i is a constant, determines the concentration of electrons in its own semiconductor, depends on the selected material of the semiconductor; T — constant, determines the temperature (300 ºK); k — Boltzmann's constant. We obtain a nonlinear system of partial differential equations, consisting of the Poisson's equation and the equations of continuity of diffusion-drift currents, written for the stationary case, in the following form (the ~ sign is omitted):

$$\mu\Delta\varphi = -\left(p - n + N_l\right), \tag{9}$$

$$\Delta n = \nabla n \cdot \nabla \varphi + n\Delta\varphi + A_n n \,, \tag{10}$$

$$\Delta p = -\nabla p \cdot \nabla \varphi - p\Delta\varphi + A_p p \;. \tag{11}$$

In (9)-(11) the notation is used: $\mu = \frac{\varepsilon\varepsilon_0 kT}{e^2 w^2 N_i}$, $A_n = \frac{w^2}{D_n \tau_n^*}$, $A_p = \frac{w^2}{D_p \tau_p^*}$. Estimation of the value μ is ~ 10^{-8}.

In this case, conditions (6)-(8) take the form:

$$\left.\frac{\partial n}{\partial \nu} - \gamma_n n\right|_{\partial\Omega'_n} = \frac{J}{eD_n}\frac{w}{N_i}\,,\; \left.\frac{\partial n}{\partial \nu}\right|_{\partial\Omega'_p} = 0\,, \tag{12}$$

$$\left.\frac{\partial p}{\partial \nu} - \gamma_p p\right|_{\partial\Omega'_p} = -\frac{J}{eD_p}\frac{w}{N_i}\,,\; \left.\frac{\partial p}{\partial \nu}\right|_{\partial\Omega'_n} = 0\,,$$

$$\left.\frac{\partial p}{\partial \nu} - \gamma_p p\right|_{\partial\Omega'_0} = 0\,,\; \left.\frac{\partial n}{\partial \nu} - \gamma_n n\right|_{\partial\Omega'_0} = 0\,,$$

$$\left.\varphi\right|_{\partial\Omega'_n} = 0\,,\; \left.\varphi\right|_{\partial\Omega'_p} = U\,,\; \left.\frac{\partial \varphi}{\partial \nu}\right|_{\partial\Omega'_0} = 0\,.$$

When describing the doping profile $N_l\left(x, y\right)$, let us consider the case of a sharp boundary between the doped regions and the region of the intrinsic semiconductor:

$$N_a\left(x, y\right) = \begin{cases} 1, \left(x, y\right) \in \Omega' \\ \frac{N_{a0}}{N_i}, \left(x, y\right) \in \partial\Omega'_p \end{cases} ;\; N_d\left(x, y\right) = \begin{cases} 1, \left(x, y\right) \in \Omega' \\ \frac{N_{d0}}{N_i}, \left(x, y\right) \in \partial\Omega'_n \end{cases} . \tag{13}$$

3. The problem decomposition by the perturbation theory method. To find an approximate solution to singularly perturbed problems for differential equations (similar to system (9)-(11)), a number of asymptotic and numerical methods have been developed [4-11]. These methods, in particular, are used for mathematical modeling of processes in semiconductor devices [10-11].

Let us apply the method of boundary functions [4-9] to analyze the problem (9)-(13). A feature of this problem is the fact (in comparison with the formulations of problems [10-12]), that the proposed mathematical model (9)-(13) describes the flow of diffusion-drift currents of unbalanced charge carriers in semiconductor diodes with a wide base (there is no sharp boundary between the regions of semiconductors n — and p-types of conductivity). Thus, in the p-i-n-diode there are two contact zones (n-i and p-i) of semiconductors of different types.

Based on the statement of the problem and taking into account that the structure of the solution is mainly influenced by the conditions on the contact sections $\partial\Omega_n$ and $\partial\Omega_p$, through which the i-region is filled with minority charge carriers, we propose to find a solution in the form of the following asymptotic series:

$$\varphi = \varphi(x,y,\mu) = \Phi_{(m)}(x,y,\mu) + \underline{\Phi}_{(m)}(\underline{\xi},\mu) + \overline{\Phi}_{(m)}(\overline{\xi},\mu) + R_{\varphi(m)}(x,y,\mu) =$$

$$= \sum_{i=0}^{m} \mu^i \varphi_i(x,y) + \sum_{i=0}^{m} \mu^i \underline{\Phi}_i(\underline{\xi}) + \sum_{i=0}^{m} \mu^i \overline{\Phi}_i(\overline{\xi}) + R_{\varphi(m)}(x,y,\mu), \quad (14)$$

$$n = n(x,y,\mu) = N_{(m)}(x,y,\mu) + \underline{N}_{(m)}(\underline{\xi},\mu) + \overline{N}_{(m)}(\overline{\xi},\mu) + R_{n(m)}(x,y,\mu) =$$

$$= \sum_{i=0}^{m} \mu^i n_i(x,y) + \sum_{i=0}^{m} \mu^i \underline{N}_i(\underline{\xi}) + \sum_{i=0}^{m} \mu^i \overline{N}_i(\overline{\xi}) + R_{n(m)}(x,y,\mu),$$

$$p = p(x,y,\mu) = P_{(m)}(x,y,\mu) + \underline{P}_{(m)}(\underline{\xi},\mu) + \overline{P}_{(m)}(\overline{\xi},\mu) + R_{p(m)}(x,y,\mu) =$$

$$= \sum_{i=0}^{m} \mu^i p_i(x,y) + \sum_{i=0}^{m} \mu^i \underline{P}_i(\underline{\xi}) + \sum_{i=0}^{m} \mu^i \overline{P}_i(\overline{\xi}) + R_{p(m)}(x,y,\mu),$$

where $\Phi_{(m)}(x,y,\mu)$, $N_{(m)}(x,y,\mu)$, $P_{(m)}(x,y,\mu)$ — the regular part of the asymptotics; $\underline{\Phi}_{(m)}(\underline{\xi},\mu)$, $\underline{N}_{(m)}(\underline{\xi},\mu)$, $\underline{P}_{(m)}(\underline{\xi},\mu)$, $\overline{\Phi}_{(m)}(\overline{\xi},\mu)$, $\overline{N}_{(m)}(\overline{\xi},\mu)$, $\overline{P}_{(m)}(\overline{\xi},\mu)$ — the near-boundary asymptotic corrections, respectively, in the vicinity of the points $y = 0$ та $y = l$ ($\underline{\xi} = \frac{y}{\sqrt{\mu}}$, $\overline{\xi} = \frac{1-y}{\sqrt{\mu}}$ — corresponding regularizing stretches); $R_{\varphi(m)}(x,y,\mu)$, $R_{n(m)}(x,y,\mu)$, $R_{p(m)}(x,y,\mu)$ — remainders of the series.

Substituting (14) into the equation (9)-(11) and conditions (12) and using the standard procedure of «equating» form the following sequence of tasks for the area Ω'.

The main terms of regular series (14) satisfy the system of equations:

$$n_0 = p_0, \tag{15}$$

$$\Delta n_0 - \nabla \cdot \left(n_0 \nabla \varphi_0 \right) - A_n n_0 = 0,$$

$$\Delta p_0 + \nabla \cdot \left(p_0 \nabla \varphi_0 \right) - A_p p_0 = 0.$$

The boundary functions of the initial stage of the solution construction process must satisfy the systems of ordinary differential equations (16)-(17) (the variable x is included in the equations as a parameter):

$$\frac{\partial^2 \underline{\Phi}_0}{\partial \underline{\xi}^2} = -\left(\underline{P}_0 - \underline{N}_0 \right), \tag{16}$$

$$\frac{\partial^2 \underline{N}_0}{\partial \underline{\xi}^2} - \frac{\partial}{\partial \underline{\xi}} \left(\underline{N}_0 \frac{\partial \underline{\Phi}_0}{\partial \underline{\xi}} \right) - n_0 \frac{\partial^2 \underline{\Phi}_0}{\partial \underline{\xi}^2} = 0,$$

$$\frac{\partial^2 \underline{P}_0}{\partial \underline{\xi}^2} + \frac{\partial}{\partial \underline{\xi}} \left(\underline{P}_0 \frac{\partial \underline{\Phi}_0}{\partial \underline{\xi}} \right) - p_0 \frac{\partial^2 \underline{\Phi}_0}{\partial \underline{\xi}^2} = 0.$$

$$\frac{\partial^2 \overline{\Phi}_0}{\partial \overline{\xi}^2} = -\left(\overline{P}_0 - \overline{N}_0 \right), \tag{17}$$

$$\frac{\partial^2 \overline{N}_0}{\partial \overline{\xi}^2} - \frac{\partial}{\partial \overline{\xi}} \left(\overline{N}_0 \frac{\partial \overline{\Phi}_0}{\partial \overline{\xi}} \right) - n_0 \frac{\partial^2 \overline{\Phi}_0}{\partial \overline{\xi}^2} = 0,$$

$$\frac{\partial^2 \overline{P}_0}{\partial \overline{\xi}^2} + \frac{\partial}{\partial \overline{\xi}} \left(\overline{P}_0 \frac{\partial \overline{\Phi}_0}{\partial \overline{\xi}} \right) + p_0 \frac{\partial^2 \overline{\Phi}_0}{\partial \overline{\xi}^2} = 0.$$

Systems of equations (15)-(17) are supplemented with boundary conditions (12), which take the following form:

$$\left. \frac{\partial n_0}{\partial y} - \gamma_n n_0 - \gamma_n \underline{N}_0 \right|_{y=0} = 0, \quad \left. \frac{\partial \underline{N}_0}{\partial \underline{\xi}} \right|_{y=0} = \frac{J}{eD_n} \frac{w}{N_i}, \tag{18}$$

$$\left. \frac{\partial p_0}{\partial y} \right|_{y=0} = 0, \quad \left. \frac{\partial \underline{P}_0}{\partial \underline{\xi}} \right|_{y=0} = 0, \quad \left. \varphi_0 + \underline{\Phi}_0 \right|_{y=0} = 0,$$

$$\left. \frac{\partial p_0}{\partial y} - \gamma_p p_0 - \gamma_p \overline{P}_0 \right|_{y=1} = 0, \quad \left. \frac{\partial \overline{P}_0}{\partial \overline{\xi}} \right|_{y=1} = -\frac{J}{eD_n} \frac{w}{N_i},$$

$$\left. \frac{\partial n_0}{\partial y} \right|_{y=1} = 0, \quad \left. \frac{\partial \overline{N}_0}{\partial \overline{\xi}} \right|_{y=1} = 0, \quad \left. \varphi_0 + \overline{\Phi}_0 \right|_{y=1} = U,$$

$$\left.\frac{\partial n_0}{\partial x}-\gamma_n n_0\right|_{x=0,x=\frac{l}{w}}=0\,,\ \left.\frac{\partial p_0}{\partial x}-\gamma_n p_0\right|_{x=0,x=\frac{l}{w}}=0\,,\ \left.\frac{\partial \varphi_0}{\partial x}\right|_{x=0,x=\frac{l}{w}}=0\,.$$

Note, that the system of equations (15) is equivalent to the linear differential equation $\Delta n_0-\frac{1}{2}\left(A_n+A_p\right)n_0=0$, which is also obtained within the ambipolar diffusion approximation [3].

The next stages of the search for the terms of the asymptotic expansion are based on the sequential solution of the equations systems similar to (15)-(17) with the corresponding boundary conditions:

For example, at the second stage, we are looking for solutions to the following systems of equations:

$$\Delta\varphi_0=-\left(p_1-n_1\right), \tag{19}$$

$$\Delta n_1-\nabla\cdot\left(n_1\nabla\varphi_0\right)-\nabla\cdot\left(n_0\nabla\varphi_1\right)-A_n n_1=0\,,$$

$$\Delta p_1+\nabla\cdot\left(p_1\nabla\varphi_0\right)+\nabla\cdot\left(p_0\nabla\varphi_1\right)-A_p p_1=0\,;$$

$$\frac{\partial^2\underline{\Phi}_1}{\partial\underline{\xi}^2}=-\left(\underline{P}_1-\underline{N}_1\right), \tag{20}$$

$$\frac{\partial^2\underline{N}_1}{\partial\underline{\xi}^2}-\frac{\partial}{\partial\underline{\xi}}\left(\underline{N}_1\frac{\partial\underline{\Phi}_0}{\partial\underline{\xi}}\right)-\frac{\partial}{\partial\underline{\xi}}\left(\underline{N}_0\frac{\partial\underline{\Phi}_1}{\partial\underline{\xi}}\right)-$$

$$-\frac{\partial n_0}{\partial x}\frac{\partial\underline{\Phi}_0}{\partial\underline{\xi}}-n_0\frac{\partial^2\underline{\Phi}_1}{\partial\underline{\xi}^2}-n_1\frac{\partial^2\underline{\Phi}_0}{\partial\underline{\xi}^2}-\frac{\partial\underline{N}_0}{\partial\underline{\xi}}\frac{\partial\varphi_0}{\partial x}=0,$$

$$\frac{\partial^2\underline{P}_1}{\partial\underline{\xi}^2}+\frac{\partial}{\partial\underline{\xi}}\left(\underline{P}_1\frac{\partial\underline{\Phi}_0}{\partial\underline{\xi}}\right)+\frac{\partial}{\partial\underline{\xi}}\left(\underline{P}_0\frac{\partial\underline{\Phi}_1}{\partial\underline{\xi}}\right)+$$

$$+\frac{\partial p_0}{\partial x}\frac{\partial\underline{\Phi}_0}{\partial\underline{\xi}}+p_0\frac{\partial^2\underline{\Phi}_1}{\partial\underline{\xi}^2}+p_1\frac{\partial^2\underline{\Phi}_0}{\partial\underline{\xi}^2}+\frac{\partial\underline{P}_0}{\partial\underline{\xi}}\frac{\partial\varphi_0}{\partial x}=0;$$

$$\frac{\partial^2\overline{\Phi}_1}{\partial\overline{\xi}^2}=-\left(\overline{P}_1-\overline{N}_1\right), \tag{21}$$

$$\frac{\partial^2\overline{N}_1}{\partial\overline{\xi}^2}-\frac{\partial}{\partial\overline{\xi}}\left(\overline{N}_1\frac{\partial\overline{\Phi}_0}{\partial\overline{\xi}}\right)-\frac{\partial}{\partial\overline{\xi}}\left(\overline{N}_0\frac{\partial\overline{\Phi}_1}{\partial\overline{\xi}}\right)-$$

$$-\frac{\partial n_0}{\partial x}\frac{\partial\overline{\Phi}_0}{\partial\overline{\xi}}-n_0\frac{\partial^2\overline{\Phi}_1}{\partial\overline{\xi}^2}-n_1\frac{\partial^2\overline{\Phi}_0}{\partial\overline{\xi}^2}-\frac{\partial\overline{N}_0}{\partial\overline{\xi}}\frac{\partial\varphi_0}{\partial x}=0,$$

$$\frac{\partial^2 \overline{P}_0}{\partial \overline{\xi}^2}+\frac{\partial}{\partial \overline{\xi}}\left(\overline{P}_1\frac{\partial \overline{\Phi}_0}{\partial \overline{\xi}}\right)+\frac{\partial}{\partial \overline{\xi}}\left(\overline{P}_0\frac{\partial \overline{\Phi}_1}{\partial \overline{\xi}}\right)+$$

$$+\frac{\partial p_0}{\partial x}\frac{\partial \overline{\Phi}_0}{\partial \overline{\xi}}+p_0\frac{\partial^2 \overline{\Phi}_1}{\partial \overline{\xi}^2}+p_1\frac{\partial^2 \overline{\Phi}_0}{\partial \overline{\xi}^2}+\frac{\partial \overline{P}_0}{\partial \overline{\xi}}\frac{\partial \varphi_0}{\partial x}=0.$$

Systems of equations (19)-(21) are supplemented by the following boundary conditions (similar to (18)):

$$\left.\frac{\partial n_1}{\partial y}-\gamma_n n_1-\gamma_n \underline{N}_1\right|_{y=0}=0\,,\ \left.\frac{\partial \underline{N}_1}{\partial \underline{\xi}}\right|_{y=0}=0\,,$$

$$\left.\frac{\partial p_1}{\partial y}\right|_{y=0}=0\,,\ \left.\frac{\partial \underline{P}_1}{\partial \underline{\xi}}\right|_{y=0}=0\,,\ \left.\varphi_1+\underline{\Phi}_1\right|_{y=0}=0\,,$$

$$\left.\frac{\partial p_1}{\partial y}-\gamma_p p_1-\gamma_p \overline{P}_1\right|_{y=1}=0\,,\ \left.\frac{\partial \overline{P}_1}{\partial \overline{\xi}}\right|_{y=1}=0\,,$$

$$\left.\frac{\partial n_1}{\partial y}\right|_{y=1}=0\,,\ \left.\frac{\partial \overline{N}_1}{\partial \overline{\xi}}\right|_{y=1}=0\,,\ \left.\varphi_1+\overline{\Phi}_1\right|_{y=1}=0\,,$$

$$\left.\frac{\partial n_1}{\partial x}-\gamma_n n_1\right|_{x=0,x=\frac{l}{w}}=0\,,\ \left.\frac{\partial p_1}{\partial x}-\gamma_n p_1\right|_{x=0,x=\frac{l}{w}}=0\,,\ \left.\frac{\partial \varphi_1}{\partial x}\right|_{x=0,x=\frac{l}{w}}=0\,.$$

Note, that additional conditions are imposed on the near-boundary functions

$$\lim_{\underline{\xi}\to\infty}\underline{N}_i\left(\underline{\xi}\right)=\lim_{\underline{\xi}\to\infty}\underline{P}_i\left(\underline{\xi}\right)=\lim_{\overline{\xi}\to\infty}\overline{N}_i\left(\overline{\xi}\right)=\lim_{\overline{\xi}\to\infty}\overline{P}_i\left(\overline{\xi}\right)=0\,,$$

$$\lim_{\underline{\xi}\to\infty}\underline{\Phi}_i\left(\underline{\xi}\right)=\lim_{\overline{\xi}\to\infty}\overline{\Phi}_i\left(\overline{\xi}\right).$$

Using considerations similar to [4-9], we find at estimates of the remainder terms of the asymptotic series:

$$R_{\varphi(m)}\left(x,y,\mu\right)=O\left(\mu^{m+1}\right),\ R_{n(m)}\left(x,y,\mu\right)=O\left(\mu^{m+1}\right),$$

$$R_{p(m)}\left(x,y,\mu\right)=O\left(\mu^{m+1}\right).$$

4. Conclusion. The mathematical model of the charge carriers stationary distribution in the electron-hole plasma of semiconductor p-i-n diodes and the methodology for the decomposition of the corresponding nonlinear model problem is proposed. The mathematical model is based on the use of a hydrodynamic model of the dynamics of a two-component plasma and the asymptotic representation of solutions of the corresponding singularly perturbed model problems for nonlinear systems of differential equations. A preliminary analysis of the results obtained indicates that the proposed approach is promising. In addition to the fact that the classical formulations of the problems of modeling the characteristics of p-i-n-structures are automatically included in the framework of the proposed scheme for finding solutions to the original problem, the presented method allows us to make significant amendments to the solution. This not only increases the level of modeling adequacy, but also provides an understanding of the features of a number of physical processes (diffusion-drift, recombinant) in the near-contact regions of the active region of p-i-n-diodes.

The proposed approach can become a basic tool for studying nonlinear thermal, diffusion-drift, generation-recombination stationary and non-stationary processes occurring in bulk and integral p-i-n-structures under the action of external microwave radiation, and predicting new physical effects in the systems under study.

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МОДЕЛЮВАННЯ РОЗПОДІЛУ НОСІЇВ ЗАРЯДУ В АКТИВНІЙ ОБЛАСТІ P-I-N-ДІОДІВ МЕТОДАМИ ТЕОРІЇ ЗБУРЕНЬ

Запропоновано математичну модель стаціонарного розподілу електронно-діркової плазми в активній області (i-області) p-i-n-діодів у дифузійно-дрейфовому наближенні. Модель подається у вигляді нелінійної сингулярно збуреної крайової задачі для системи рівнянь неперервності електронно-діркових струмів і Пуассона з відповідними граничними умовами. Проведено декомпозицію нелінійної крайової задачі моделювання стаціонарного розподілу носіїв заряду в плазмі p-i-n-діодів на основі асимптотичного представлення розв'язків. Модельна задача приведена до послідовності лінійних крайових задач із характерним виділенням основних (регулярних) складових асимптотик і примежових поправок. Встановлено, що постановка задачі для знаходження нульового члена регулярної частини асимптотик співпадає із класичною постановкою задачі моделювання характеристик p-i-n-діодів, яка здійснюється в наближенні амбіполярної дифузії (наближення самоузгодженого поля плазми). Запропонована математична модель і метод її лінеаризації дозволяють виділити у дифузійно-дрейфовому процесі головні складові і дослідити їх роль. Наприклад, з'являється можливість вивчення (у тому числі аналітичними методами) поведінки плазми в зонах p-i-, n-i-контактів. Результати дослідження спрямовані на розвиток методів проектування p-i-n-діодних структур, які використовуються, зокрема, в якості активних елементів комутаторів сигналів надвисокочастотних систем передачі інформації і відповідних захисних пристроях.

Ключові слова: *метод збурень, сингулярно збурені задачі, метод примежового шару, дифузійно-дрейфовий процес, p-i-n-діоди.*

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