



**MICHAL FEČKAN**  
**(on his 60th birthday)**

At the beginning of December, 2020, the outstanding Slovak mathematician with world significance, Prof. RNDr. Michal Fečkan, DrSc., celebrated his 60th anniversary. Prof. Fečkan was born in Nové Zámky, Slovakia. After finishing his high school education in 1980, he enrolled at the Faculty of Mathematics and Physics of the Comenius University, Bratislava, where he read mathematical analysis. After finishing his university studies in 1985, he joined the Mathematical Institute of the Slovak Academy of Sciences. Here he started his PhD studies under the supervision of Prof. M. Medveď and in 1993 he defended his PhD thesis “Qualitative Properties of Diffeomorphisms.”

In 1995, he started to work at the Faculty of Mathematics and Physics, where in 2000 he became associate professor.

In 2001, he defended his DrSc. thesis “Chaos in nonlinear dynamical systems-analytical methods” to obtain DrSc. — the highest scientific degree in Slovakia, and in 2006 he was

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appointed full professor by the President of Slovakia. He also has a permanent contact and collaboration with the Mathematical Institute of the Slovak Academy of Sciences.

The scientific career of Prof. Fečkan started with his research on dynamical systems and differential equations in 90-th years of the last century.

Topological bifurcation theory is one of the most essential topics in mathematics. Prof. Fečkan got original bifurcation results for the existence of oscillations and chaotic behavior of differential equations and discrete dynamical systems under variation of involved parameters. Using topological degree theory and a perturbation approach in dynamical systems, a broad variety of nonlinear problems are studied, including: non-smooth mechanical systems with dry frictions; weakly coupled oscillators; systems with relay hysteresis; differential equations on infinite lattices of Frenkel–Kontorova and discretized Klein–Gordon types; blue sky catastrophes for reversible dynamical systems; buckling of beams; and discontinuous wave equations. The first his monograph [1] is devoted to these actual problems.

The second his book “Bifurcation and Chaos in Discontinuous and Continuous Systems” [2] provides rigorous mathematical functional-analytical tools for handling chaotic bifurcations along with precise and complete proofs together with concrete applications presented by many stimulating and illustrating examples. A broad variety of nonlinear problems are studied involving difference equations, ordinary and partial differential equations, differential equations with impulses, piecewise smooth differential equations, differential and difference inclusions, and differential equations on infinite lattices as well.

The third monograph “Poincaré–Andronov–Melnikov Analysis for Non-Smooth Systems” [3] is devoted to the study of bifurcations of periodic solutions for general  $n$ -dimensional discontinuous systems. The authors study these systems under assumptions of transversal intersections with discontinuity-switching boundaries. Furthermore, bifurcations of periodic sliding solutions are studied from sliding periodic solutions of unperturbed discontinuous equations, and bifurcations of forced periodic solutions are also investigated for impact systems from single periodic solutions of unperturbed impact equations. In addition, the book presents studies for weakly coupled discontinuous systems, and also the local asymptotic properties of derived perturbed periodic solutions.

The relationship between non-smooth systems and their continuous approximations is investigated as well. Examples of two-, three- and four-dimensional discontinuous ordinary differential equations and impact systems are given to illustrate the theoretical results. The authors use the so-called discontinuous Poincaré mapping which maps a point to its position after one period of the periodic solution. This approach is rather technical, but it does produce results for general dimensions of spatial variables and parameters as well as the asymptotical results such as stability, instability, and hyperbolicity (see [3]).

Recently, Prof. Fečkan has been concentrating on fractional differential equations, which is currently a hot topic. The solutions of these problems, as well as the main part of his works, are applied to many problems of physics, chemistry and economy, as for example: chaotic membrane oscillating, Stieltjes string, Duffing’s equation of string oscillation or McMillan mappings.

The fourth monograph “Fractional-Order Equations and Inclusion” [4] presents fractional difference, integral, differential, evolution equations and inclusions, and discusses existence and asymptotic behavior of their solutions. Controllability and relaxed control results are obtained.

Combining rigorous deduction with abundant examples, it is of interest to nonlinear science researchers using fractional equations as a tool, and physicists, mechanics researchers and engineers studying relevant topics (see [4]).

The next results of Prof. Fečkan from [5] are aimed primarily towards physicists and mechanical engineers specializing in modeling, analysis, and control of discontinuous systems with friction and impacts. The book fills a gap in the existing literature by offering an original contribution to the field of discontinuous mechanical systems based on mathematical and numerical modeling as well as the control of such systems. Each chapter provides the reader with both the theoretical background and results of verified and useful computations, including solutions of the problems of modeling and application of friction laws in numerical computations, results from finding and analyzing impact solutions, the analysis and control of dynamical systems with discontinuities, etc. The content offers a smooth correspondence between science and engineering and will allow the reader to discover new ideas. Also there is emphasized the unity of diverse branches of physics and mathematics towards understanding complex piecewise-smooth dynamical systems. Mathematical models presented here will be important in numerical experiments, experimental measurements, and optimization problems and they will be found in applied mechanics.

Prof. Fečkan with co-authors extend in [6] classical Hermite – Hadamard type inequalities to the fractional case via establishing fractional integral identities, and discusses Riemann – Liouville and Hadamard integrals, respectively, by various convex functions. Illustrating theoretical results via applications in special means of real numbers, it is an essential reference for applied mathematicians and engineers working with fractional calculus.

Non-instantaneous impulsive differential equations are widely used in physics, biology, dynamics and ecology and have a wide-ranging scope within the scientific industry. Prof. Fečkan with co-authors of [7] pave the way for a better fundamental understanding of the mathematical models and how they can be implemented.

His research project was supported by the National Science Foundation, USA, in collaboration with University of the Greensboro in Northern Caroline. He had many invitations to the University of Ancona, Italy, as well as to universities in Hungary, Greece and Australia. Prof. Fečkan has an extensive cooperation with scientists from the whole world with more than 80 co-authors.

His name shines in 314 scientific papers including 7 monographs (the list of his monographs is in the references below [1 – 7]). In 2008, 2011, 2017, he received the prizes of the Literary Foundation of Slovak Republic.

His scientific results met with a great interest of the scientific community — his papers were cited almost 3000 times. In 2019 his results achieved such a high number of citations that he was ranked among 1% of the most cited scientists in his field in the whole world being the only representative of Slovakia in the prestigious ranking Highly Cited Researchers.

Prof. Fečkan is an editor of 10 mathematical journals, including *Mathematica Slovaca* (Slovakia), *Miskolc Mathematical Notes* (Hungary), *Dynamics of Partial Differential Equations* (USA), *Electronic Journal of Qualitative Theory of Differential Equations*, (Hungary), *Aplimat — Journal of Applied Mathematics and Engineering* (Slovakia), *Communications in Mathematical Analysis* (USA), *Journal of Applied Mathematics, Statistics and Informatics* (Slovakia),

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The complete list of Prof. M. Fečkan's publications can be found at: <http://www.mat.savba.sk/~feckan> and <https://sluzby.fmph.uniba.sk/ludia/feckan1>.

Differential Equations and Applications (Croatia), Discontinuity, Nonlinearity and Complexity (USA), Journal of Modeling, Simulation, Identification, and Control (USA).

In 2018, he was awarded the title of Scholar of the Year 2017 of the Slovak Republic, he is a member of the Learned Society of the Slovak Academy of Sciences.

In 2019, his University decorated him with the Golden medal.

Important parts of his scientific life are also pedagogical activities at the Faculty of Mathematics, Physics and Informatics of the Comenius University — teaching his master students, supervising his 5 PhD students and PostDocs, as well as, at the Mathematical Institute of the Slovak Academy of Sciences.

*On the occasion of your 60th birthday, we, all your colleagues and friends wish you first of all good health, a lot of satisfaction and joy in your family and also many productive years in finding and solving open problems. We are looking forward to your new interesting results and books.*

*Zhivio, mnoga leta, Michal!*

*N. Dilna, A. Dvurečenskij*

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