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A FRACTIONAL ORDER COVID-19 EPIDEMIC MODEL WITH MITTAG-LEFFLER KERNEL

ЕПІДЕМІОЛОГІЧНА МОДЕЛЬ ДРОБОВОГО ПОРЯДКУ COVID-19 ІЗ ЯДРОМ МІТТАГ-ЛЕФФЛЕРА

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We have considered a nonlinear fractional order Covid-19 model in the Atagana – Baleanu fractional derivative sense for the analytical and computational studies. The model consists of six classes including susceptible, protected susceptible, asymptomatic infected, symptomatic infected, quarantined and the recovered individuals. The model is studied for the existence of solution with the help of the successive iterative technique with the limit point as the solution of the model and the Hyers – Ulam stability is studied with the basic results notions. A numerical scheme is produced and tested with the support of the available literature. The graphical results show prediction of the curtail of the spread in the next 5000 days. And there is a gradual increase in the population of the protected susceptible.

Розглянуто нелінійну модель дробового порядку Covid-19 у розумінні дробової похідної Атагана – Балеану для аналітичних і комп'ютерних досліджень. Модель складається з шести класів людей, які сприйнятливі, захищено сприйнятливі, безсимптомно інфіковані, симптомно інфіковані, ізольовані на карантині та одужавші. За допомогою методу послідовних наближень досліджено існування розв'язку моделі з граничною точкою, а також стійкість Хайерса – Улама. З використанням наявної літератури запропоновано й випробувано чисельну схему. Графічні результати прогнозують скорочення поширення через 5000 днів. Також поступово збільшується кількість людей, які мають захищену сприйнятливість.

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1. Introduction. Coronavirus infection 2019 (Covid-19) is a communicable respiratory disease. SARS-CoV-2 (Severe Acute Respiratory Syndrome Coronavirus) is a disease caused by a newly discovered virus strain [1]. In Wuhan, China, Covid-19 was first identified in December, 2019, and spread quickly over four months. In a short period, more than 2.9 million inhabitants in 185 nations around the world were infected and 206 thousand person died [2]. On March 11, 2020, "The World Health Organization" announced this coronavirus infections is a pandemic [3]. This disease can spread primarily from small droplets via coughing, sneezing, person-to-person or conversation. By contacting polluted surfaces, prone individuals can also be compromised. The most prevalent signs of this disease are fever, nausea, dry cough, fatigue, breath shortages. All these of signs are parts of Covid-19 [4]. Some patients can have joint pain such as nasal stuffiness, runny nose, sore throat or diarrhea. The symptoms are typically mild, but can slowly occur. In order to prevent infection, hand washing, nose covering or mouth covering while washing sneezing or coughing, avoiding nose, mouth or mouth touch and preventive steps are advised for the eyes and social distances.

Due to the seriousness of the Covid-19 pandemic, many states have done drastic decisions to curb the distribution of Covid-19 infection. In addition, they checked and covered their healthcare systems. Hence, they ruled the cancellation of public events, the closing of public events, schools, public places, borders, restrictions on travel and lockout, etc.. While those measures were helpful, that lockdown lead to socio-economic damage such as bankruptcy of many workplaces, several staff have lost their respective positions and so on. Next, the shutdown has disrupted supply chains and decreased productivity. The shutdown of China's drug-producing plants, which are the shutdown of second largest pharmaceutical product exporter, has been delayed the deliveries of generic drug processing factories [5]. The sectors of tourism, air transport, and oil were visibly influenced. It is also expected that invisible impacts are expected irrespective of the pandemic's duration. According to "The International Monetary Fund" the worldwide economy is expected to shrink by 3% in 2020 [6].

Governments are to save the failure of the economy, thinking of security measures in order to relax the lockdown. Some advanced countries intend to grant a immunity passport, which shows immunity to the illness. However, this technique has been disapproved by "The World Health Organization", since there is a lack of adequate scientific proof that reinfection is aforementioned approach is not possible. A risk balancing strategy was adopted by the South African government to lift the lockout restrictions progressively.

We refer the readers for some scientific works done on infectious diseases to [7 – 9], in particular, for developed several mathematical models related to Covid-19 to [10 – 12] and for some recent scientific works on varius fractional mathematical models [10, 13 – 34].

In this paper, we consider the following Covid-19 model for the existence, stability, and numerical simulations by using Atanga – Baleanu fractional derivative in the Caputo's sense. For the detail, the readers can get benefit from [29, 35];

$$
{}_0^{ABC}D_t^{\wp_1^*}S = \Lambda_1 + \gamma Q - \alpha_1 S - \frac{(1-\alpha_1)(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} - \mu S,
$$

$$
{}_0^{ABC}D_t^{\wp_2^*}S_P = \alpha_1 S - \mu S_P,
$$

$$
{}_0^{ABC}D_t^{\wp_3^*} I_A = \frac{(1-\alpha_1)(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} - (\alpha_2\rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu)I_A,
$$

$$
{}_0^{ABC}D_t^{\wp*_4} I_S = \alpha_2\rho I_A - (1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta)I_S,
$$

$$
{}_0^{ABC}D_t^{\wp_5^*} Q = (1-\alpha_2)I_A + (1-\alpha_3)I_S - (\gamma + r_3 + \mu + \delta)Q,
$$

$$
{}_0^{ABC}D_t^{\wp_6^*} R = \alpha_2 r_1 I_A + \alpha_3 r_2 I_S + R_3 Q - \mu R.
$$

The the population is divided into six compartments. They are: S for the susceptible, S_P are the protected susceptible class, I_A are asymptomatic infected but not quarantined, I_S are the symptomatic infected not quarantined class, the quarantined class is Q and recovered $R.$ The fractional orders $\wp_i^* \in (0,1]$. The parameters are: α_1 is fraction of protected susceptible, α_2 is fraction of unidentified asymptomatic infected, α_3 is fraction of unidentified symptomatic infected, η_1 contact rate between S and $I_A,$ η_2 is contact rate between S and $I_A,$ η_3 is contact rate between S and $Q,$ ρ is disease progression rate from I_A to $I_S,$ r_1 is the recovery rate of $I_A,$ r_2 is recovery rate of $I_S,$ r_3 is recovery rate of $Q,$ δ is the death rate due to Covid-19 disease, γ is proportion of non-effected quarantine, μ is natural mortality rate. About the ABC-fractional calculus, we highlight the following useful literature.

Definition 1. *The ABC-fractional differential operator on $\psi \in H^*(a,b),\ b > a,$ for $\wp_1^* \in \in [0,1]$ is*

$$
{}^{ABC}{}_a\mathcal{D}_\tau^{\wp_1^*}\psi(\tau) = \frac{B(\wp_1^*)}{1-\wp_1^*}\int_a^\tau \psi'(s)E_{\wp^*}\left[\frac{-\wp^*(\tau-s)^{\wp^*}}{1-\wp^*}\right]ds, \tag{1}
$$

where $B(\wp^*)$ is satisfied the property $B(0) = B(1) = 1.$

Definition 2. *For $\psi \in H^*(a,b),\ b > a,\ \wp^* \in [0,1],$ the ABR-fractional derivative is*

$$
{}^{ABR}{}_a\mathcal{D}_\tau^{\wp^*}\psi(\tau) = \frac{B(\wp^*)}{1-\wp^*}\frac{d}{d\tau}\int_a^\tau \psi(s)E_{\wp^*}\left[\frac{-\wp^*(\tau-s)^{\wp^*}}{1-\wp^*}\right]ds.
$$

Definition 3. *The AB-integral of $\psi \in H^*(a,b),\ b > a,\ 0 < \wp_1^* < 1$ is given by*

$$
{}^{AB}{}_a\mathcal{I}_\tau^{\wp_1^*}\psi(\tau) = \frac{1-\wp_1^*}{B(\wp_1^*)}\psi(\tau) + \frac{\wp_1^*}{B(\wp_1^*)\Gamma(\wp_1^*)}\int_a^\tau \psi(s)(\tau-s)^{\wp_1^*-1}ds.
$$

Lemma 1. *The AB fractional derivative and AB fractional integral of the function ψ, satisfy the Newton–Leibniz formula*

$$
{}^{AB}{}_a\mathcal{I}_\tau^{\wp_1^*}\left({}^{ABC}{}_a\mathcal{D}_\tau^{\wp_1^*}\psi(\tau)\right) = \psi(\tau) - \psi(a).
$$

2. Existence criteria. By the AB-fractional integral and Covid-19 model (1), we have

$$S(t)-S(0)=\frac{1-\wp_1^*}{\beta(\wp_1^*)}\left[\Lambda_1+\gamma Q-\alpha_1 S-\frac{(1-\alpha_1)(\alpha_2\eta_1 I_A+\alpha_3\eta_2 I_S+\eta_3 Q)S}{S+S_P+I_A+I_S+Q+R}-\mu S\right]+$$

$$+\frac{\wp_1^*}{\beta(\wp_1^*)\Gamma\wp_1^*}\int\limits_0^t(t-s)^{\wp_1^*-1}\Bigg[\Lambda_1+\gamma Q-\alpha_1 S-$$

$$-\frac{(1-\alpha_1)(\alpha_2\eta_1 I_A+\alpha_3\eta_2 I_S+\eta_3 Q)S}{S+S_P+I_A+I_S+Q+R}-\mu S\Bigg]ds,$$

$$S_p(t)-S_p(0)=\frac{1-\wp_2^*}{\beta(\wp_2^*)}[\alpha_1 S-\mu S_P]+\frac{\wp_1^*}{\beta(\wp_2^*)\Gamma\wp_2^*}\int\limits_0^t(t-s)^{\wp_2^*-1}[\alpha_1 S-\mu S_P]ds,$$

$$I_A(t)-I_A(0)=\frac{1-\wp_3^*}{\beta(\wp_3^*)}\Bigg[\frac{(1-\alpha_1)(\alpha_2\eta_1 I_A+\alpha_3\eta_2 I_S+\eta_3 Q)S}{S+S_P+I_A+I_S+Q+R}-$$

$$-(\alpha_2\rho+\alpha_2 r_1+1-\alpha_2+\mu)I_A\Bigg]+\frac{\wp_1^*}{\beta(\wp_3^*)\Gamma\wp_3^*}\int\limits_0^t(t-s)^{\wp_3^*-1}\times$$

$$\times\left[\frac{(1-\alpha_1)(\alpha_2\eta_1 I_A+\alpha_3\eta_2 I_S+\eta_3 Q)S}{S+S_P+I_A+I_S+Q+R}(\alpha_2\rho+\alpha_2 r_1+1-\alpha_2+\mu)I_A\right]ds,$$

$$I_s(t)-I_s(0)=\frac{1-\wp_4^*}{\beta(\wp_4^*)}\Big[\alpha_2\rho I_A-(1-\alpha_3+\alpha_3 r_2+\mu+\delta)I_S\alpha_2\rho I_A-$$

$$-(1-\alpha_3+\alpha_3 r_2+\mu+\delta)I_S\Big]+\frac{\wp_4^*}{\beta(\wp_4^*)(\Gamma\wp_4^*)}\times$$

$$\times\int\limits_0^t(t-s)^{\wp_4^*-1}\left[\alpha_2\rho I_A-(1-\alpha_3+\alpha_3 r_2+\mu+\delta)I_S\right]ds,$$

$$Q(t)-Q(0)=\frac{1-\wp_5^*}{\beta(\wp_5^*)}\left[(1-\alpha_2)I_A+(1-\alpha_3)I_S-(\gamma+r_3+\mu+\delta)Q\right]+$$

$$+\frac{\wp_5^*}{\beta(\wp_5^*)(\Gamma\wp_5^*)}\int\limits_0^t(t-s)^{\wp_5^*-1}\left[(1-\alpha_2)I_A+(1-\alpha_3)I_S-(\gamma+r_3+\mu+\delta)Q\right]ds,$$

$$R(t)-R(0)=\frac{1-\wp_6^*}{\beta(\wp_6^*)}\left[\alpha_2 r_1 I_A+\alpha_3 r_2 I_S+R_3 Q-\mu R\right]+$$

$$+\frac{\wp_6^*}{\beta(\wp_6^*)(\Gamma\wp_6^*)}\int\limits_0^t(t-s)^{\wp_6^*-1}\left[\alpha_2 r_1 I_A+\alpha_3 r_2 I_S+R_3 Q-\mu R\right]ds.$$

Assume the functions $Y_i,\ i = 1, \ldots, 6,$ are given below:

$$Y_1(t, S) = \Lambda_1 + \gamma Q - \alpha_1 S - \frac{(1-\alpha_1)(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} - \mu S,$$

$$Y_2(t, S_p) = \alpha_1 S - \mu S_P,$$

$$Y_3(t, I_A) = \frac{(1-\alpha_1)(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} - (\alpha_2\rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu)I_A,$$

$$Y_4(t, I_s) = \alpha_2\rho I_A - (1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta)I_S,$$

$$Y_5(t, Q) = (1-\alpha_2)I_A + (1-\alpha_3)I_S - (\gamma + r_3 + \mu + \delta)Q,$$

$$Y_6(t, R) = \alpha_2 r_1 I_A + \alpha_3 r_2 I_S + R_3 Q - \mu R,$$

$$\begin{cases} \psi_1 = \alpha_1 + k_1 + \mu, \\ \psi_2 = \mu, \\ \psi_3 = k_2 + (\alpha_2\rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu), \\ \psi_4 = \mu_c, \\ \psi_5 = 1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta, \\ \psi_6 = \mu. \end{cases}$$

Assumption ($\mathcal{B}$). *We assume that, for* $S(t),\ S^*(t),\ S_p(t),\ S_p^*(t),\ I_A(t),\ I_A^*(t),\ I_s(t),\ I_s^*(t),$ $Q(t),\ Q^*(t),\ R(t),\ R^*(t) \in L[0,1]$, *there exists constants* $\kappa_i > 0,\ i = 1, \ldots, 6,$ *such that* $\|S(t)\| \le \kappa_1,\ \|S_p(t)\| \le \kappa_2,\ \|I_A(t)\| \le \kappa_3,\ \|I_s(t)\| \le \kappa_4,\ \|Q(t)\| \le \kappa_5,\ \|R(t)\| \le \kappa_6,$ *and* $\xi_1,$ $\xi_2 > 0,$ *and*

$$\|S(t) + I_A(t) + Q(t)\| \le \xi_1,$$

$$\|I_s(t) + R(t)\| \le \xi_2.$$

Theorem 1. *The* $Y_i,\ i \in N_1^6$, *satisfy Lipschitz condition provided that Assumption* ($\mathcal{B}$) *is obeyed.*

Proof. Consider for Y_1, below

$$\big\|Y_1(t,S) - Y_1\big(t,S^*\big)\big\| = \left\| \Lambda_1 + \gamma Q - \alpha_1 S - \frac{(1-\alpha_1)(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} - \mu S - \right.$$

$$\left. - \left(\Lambda_1 + \gamma Q - \alpha_1 S^* - \frac{(1-\alpha_1)(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q)S^*}{S^* + S_P + I_A + I_S + Q + R} - \mu S^*\right) \right\| \le$$

$$\le \left\| \alpha_1 + \frac{(1-\alpha_1)\left(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q\right) S^*}{S^* + S_P + I_A + I_S + Q + R} + \mu \right\| \|S - S^*\| \le$$

$$\leq [\alpha_1 + k_1 + \mu]\|S_c - S_c^*\| = \psi_1\|S - S^*\|. \tag{2}$$

For the $Y_2(t, E_c)$, we have

$$\left\|Y_2(t, S_p) - Y_2\left(t, S_p^*\right)\right\| = \|(\alpha_1 S - \mu S_P) - (\alpha_1 S - \mu S_P^*,)\| \leq$$
$$\leq [\mu]\,\|S_c - E_c^*\| \leq \psi_2\|E_c - E_c^*\|. \tag{3}$$

The $Y_3(t, I_A^*)$ implies

$$\|Y_3(t, I_A) - Y_3\,(t, I_A^*)\| = \left\|\left(\frac{(1-\alpha_1)(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} - \right.\right.$$
$$\left. - (\alpha_2\rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu)I_A\right) -$$
$$- \left(\frac{(1-\alpha_1)(\alpha_2\eta_1 I_A^* + \alpha_3\eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A^* + I_S + Q + R} - \right.$$
$$\left.\left. - (\alpha_2\rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu)I_A^*\right)\right\| \leq$$
$$\leq \left\|\frac{(1-\alpha_1)(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} + \right.$$
$$\left. + (\alpha_2\rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu)\right\| \|I_A - I_A^*\| \leq$$
$$\leq [k_2 + (\alpha_2\rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu)]\|I_c - I_c^*\| = \psi_3\,\|I_A - I_A^*\| \tag{4}$$

for $Y_4(t, I)$.

We obtain

$$\|Y_4(t, I_s) - Y_4\,(t, I_s^*)\| = \Big\|(\alpha_2\rho I_A - (1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta)I_S) -$$
$$- (\alpha_2\rho I_A - (1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta)\,I_S^*)\Big\| \leq$$
$$\leq \|1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta\|\,\|I_s - I_s^*\| \leq$$
$$\leq \Big[1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta\Big]\|I_s - I_s^*\| \leq \psi_4\,\|I_s - I_s^*\| \tag{5}$$

for $Y_5(t, Q)$.

We get

$$\|Y_5(t, Q) - Y_5\,(t, Q^*)\| = \Big\|\,((1-\alpha_2)I_A + (1-\alpha_3)I_S - (\gamma + r_3 + \mu + \delta)Q) -$$
$$- ((1-\alpha_2)I_A + (1-\alpha_3)I_S - (\gamma + r_3 + \mu + \delta)Q^*)\,\Big\| \leq$$

$$\leq \|\gamma + r_3 + \mu + \delta\| \, \|Q - Q^*\| = \psi_5 \, \|Q - Q^*\| \tag{6}$$

for $Y_6(t, R)$.

Further, we have

$$\|Y_6(t,R) - Y_6(t,R^*)\| = \Big\|(\alpha_2 r_1 I_A + \alpha_3 r_2 I_S + r_3 Q - \mu R,) -$$

$$- (\alpha_2 r_1 I_A + \alpha_3 r_2 I_S + r_3 Q - \mu R^*)\Big\| \leq$$

$$\leq \|\mu\| \, \|R - R^*\| = \psi_6 \, \|R - R^*\| . \tag{7}$$

Thus, from (2) to (7), we have that the $Y_i, \ i = 1, \ldots, 6,$ satisfy the Lipschitz condition. And this completes the proof.

Assuming that $S(0) = S_p(0) = I_A(0) = I_S(0) = Q(0) = R(0) = 0,$ then we have

$$S(t) = \frac{1 - \wp_1^*}{\beta(\wp_1^*)} \mathcal{Y}_1(t, S(t)) + \frac{\wp_1^*}{\beta(\wp_1^*)\Gamma(\wp_1^*)} \int_0^t (t-s)^{\wp_1^* - 1} \mathcal{Y}_1(s, S(s)) ds, \tag{8}$$

$$S_p(t) = \frac{1 - \wp_2^*}{\beta(\wp_2^*)} \mathcal{Y}_2(t, S_p(t)) + \frac{\wp_2^*}{\beta(\wp_2^*)\Gamma(\wp_2^*)} \int_0^t (t-s)^{\wp_2^* - 1} \mathcal{Y}_2(s, S_p(s)) ds, \tag{9}$$

$$I_A(t) = \frac{1 - \wp_3^*}{\beta(\wp_3^*)} \mathcal{Y}_3(t, I_A(t)) + \frac{\wp_3^*}{\beta(\wp_3^*)\Gamma(\wp_3^*)} \int_0^t (t-s)^{\wp_3^* - 1} \mathcal{Y}_3(s, I_A(s)) ds, \tag{10}$$

$$I_s(t) = \frac{1 - \wp_4^*}{\beta(\wp_4^*)} \mathcal{Y}_4(t, I_s(t)) + \frac{\wp_4^*}{\beta(\wp_4^*)\Gamma(\wp_4^*)} \int_0^t (t-s)^{\wp_4^* - 1} \mathcal{Y}_4(s, I_s(s)) ds, \tag{11}$$

$$Q(t) = \frac{1 - \wp_5^*}{\beta(\wp_5^*)} \mathcal{Y}_5(t, Q(t)) + \frac{\wp_5^*}{\beta(\wp_5^*)\Gamma(\wp_5^*)} \int_0^t (t-s)^{\wp_5^* - 1} \mathcal{Y}_5(s, Q(s)) ds, \tag{12}$$

$$R(t) = \frac{1 - \wp_6^*}{\beta(\wp_6^*)} \mathcal{Y}_6(t, R(t)) + \frac{\wp_6^*}{\beta(\wp_6^*)\Gamma(\wp_6^*)} \int_0^t (t-s)^{\wp_6^* - 1} \mathcal{Y}_6(s, R(s)) ds. \tag{13}$$

For the iterative scheme of the model (1), we define

$$S_n(t) = \frac{1 - \wp_1^*}{\beta(\wp_1^*)} \mathcal{Y}_1(t, S_{n-1}(t)) + \frac{\wp_1^*}{\beta(\wp_1^*)\Gamma(\wp_1^*)} \int_0^t (t-s)^{\wp_1^* - 1} \mathcal{Y}_1(s, S_{n-1}(s)) ds,$$

$$S_{p_n}(t) = \frac{1 - \wp_2^*}{\beta(\wp_2^*)} \mathcal{Y}_2(t, S_{p_{n-1}}(t)) + \frac{\wp_2^*}{\beta(\wp_2^*)\Gamma(\wp_2^*)} \int_0^t (t-s)^{\wp_2^* - 1} \mathcal{Y}_2(s, S_{p_{n-1}}(s)) ds,$$

$$I_{An}(t) = \frac{1 - \wp_3^*}{\beta(\wp_3^*)} \mathcal{Y}_3(t, I_{An-1}(t)) + \frac{\wp_3^*}{\beta(\wp_3^*)\Gamma(\wp_3^*)} \int_0^t (t-s)^{\wp_3^* - 1} \mathcal{Y}_3(s, I_{An-1}(s)) ds,$$

$$I_{sn}(t) = \frac{1-\wp_4^*}{\beta(\wp_4^*)}\mathcal{Y}_4(t, I_{sn-1}(t)) + \frac{\wp_4^*}{\beta(\wp_4^*)\Gamma(\wp_4^*)}\int\limits_0^t (t-s)^{\wp_4^*-1}\mathcal{Y}_4(s, I_{sn-1}(s))ds,$$

$$Q_n(t) = \frac{1-\wp_5^*}{\beta(\wp_5^*)}\mathcal{Y}_5(t, Q_{n-1}(t)) + \frac{\wp_5^*}{\beta(\wp_5^*)\Gamma(\wp_5^*)}\int\limits_0^t (t-s)^{\wp_5^*-1}\mathcal{Y}_5(s, Q_{n-1}(s))ds,$$

$$R_n(t) = \frac{1-\wp_6^*}{\beta(\wp_6^*)}\mathcal{Y}_6(t, R_{n-1}(t)) + \frac{\wp_6^*}{\beta(\wp_6^*)\Gamma(\wp_6^*)}\int\limits_0^t (t-s)^{\wp_6^*-1}\mathcal{Y}_6(s, R_{n-1}(s))ds.$$

Theorem 2. *The fractional order Covid-19 model (1) has a solution if we have*

$$\Delta = \max\{\Psi_i\} < 1, \quad i \in N_1^6.$$

Proof. We define the function

$$\mathcal{K}1_n(t) = S_{n+1}(t) - S(t), \quad \mathcal{K}2_n(t) = S_{p_{n+1}}(t) - S_p(t), \quad \mathcal{K}3_n(t) = I_{An+1}(t) - I_A(t),$$

$$\mathcal{K}4_n(t) = I_{sn+1}(t) - I_s(t), \quad \mathcal{K}5_n(t) = Q_{n+1}(t) - Q(t), \quad \mathcal{K}6_n(t) = R_{n+1}(t) - R(t).$$

Then, by using the above equations, we find that

$$\|\mathcal{K}1_n\| \le \frac{1-\wp_1^*}{\beta(\wp_1^*)}\|\mathcal{Y}_1(t, \mathcal{S}_n(t)) - \mathcal{Y}_1(t, S_{n-1}(t))\| +$$

$$+ \frac{\wp_1^*}{\beta(\wp_1^*)\Gamma(\wp_1^*)}\int\limits_0^t (t-s)^{\wp_1^*-1}\|\mathcal{Y}_1(s, S_n(s)) - \mathcal{Y}_1(t, S_{n-1}(t))\|ds \le$$

$$\le \left[\frac{1-\wp_1^*}{\beta(\wp_1^*)} + \frac{1}{\beta(\wp_1^*)\Gamma(\wp_1^*)}\right]\psi_1\|S_n - S\| \le$$

$$\le \left[\frac{1-\wp_1^*}{\beta(\wp_1^*)} + \frac{1}{\beta(\wp_1^*)\Gamma(\wp_1^*)}\right]^n \Delta^n\|S_1 - S\|$$

and

$$\|\mathcal{K}2_n\| \le \frac{1-\wp_2^*}{\beta(\wp_2^*)}\|\mathcal{Y}_2(t, S_{p_n}(t)) - \mathcal{Y}_2(t, S_{p_{n-1}}(t))\| +$$

$$+ \frac{\wp_2^*}{\beta(\wp_2^*)\Gamma(\wp_2^*)}\int\limits_0^t (t-s)^{\wp_2^*-1}\|\mathcal{Y}_2(s, S_{p_n}(s)) - \mathcal{Y}_2(t, S_{p_{n-1}}(t))\|ds \le$$

$$\le \left[\frac{1-\wp_2^*}{\beta(\wp_2^*)} + \frac{1}{\beta(\wp_2^*)\Gamma(\wp_2^*)}\right]\psi_2\|S_{p_n} - S_p\| \le$$

$$\le \left[\frac{1-\wp_2^*}{\beta(\wp_2^*)} + \frac{1}{\beta(\wp_2^*)\Gamma(\wp_2^*)}\right]^n \Delta^n\|S_{p_1} - S_p\|.$$

Similarly,

$$\|\mathcal{K}3_n\| \leq \frac{1-\wp_3^*}{\beta(\wp_3^*)}\|\mathcal{Y}_3(t, I_{An}(t)) - \mathcal{Y}_3(t, I_{An-1}(t))\| +$$

$$+\frac{\wp_3^*}{\beta(\wp_3^*)\Gamma(\wp_3^*)}\int\limits_0^t (t-s)^{\wp_3^*-1}\|\mathcal{Y}_3(s, I_{An}(s)) - \mathcal{Y}_3(t, I_{An-1}(t))\|ds \leq$$

$$\leq \left[\frac{1-\wp_3^*}{\beta(\wp_3^*)} + \frac{1}{\beta(\wp_1^*)\Gamma(\wp_3^*)}\right]\psi_3\|I_{An} - I_A\| \leq$$

$$\leq \left[\frac{1-\wp_3^*}{\beta(\wp_3^*)} + \frac{1}{\beta(\wp_1^*)\Gamma(\wp_3^*)}\right]^n \Delta^n\|I_{A1} - I_A\|,$$

$$\|\mathcal{K}4_n\| \leq \frac{1-\wp_4^*}{\beta(\wp_4^*)}\|\mathcal{Y}_4(t, I_{sn}(t)) - \mathcal{Y}_4(t, I_{sn-1}(t))\| +$$

$$+\frac{\wp_4^*}{\beta(\wp_4^*)\Gamma(\wp_4^*)}\int\limits_0^t (t-s)^{\wp_4^*-1}\|\mathcal{Y}_4(s, I_{sn}(s)) - \mathcal{Y}_4(t, I_{sn-1}(t))\|ds \leq$$

$$\leq \left[\frac{1-\wp_4^*}{\beta(\wp_4^*)} + \frac{1}{\beta(\wp_4^*)\Gamma(\wp_4^*)}\right]\psi_4\|I_{sn} - I_s\| \leq$$

$$\leq \left[\frac{1-\wp_4^*}{\beta(\wp_4^*)} + \frac{1}{\beta(\wp_4^*)\Gamma(\wp_4^*)}\right]^n \Delta^n\|I_{s1} - I_s\|,$$

$$\|\mathcal{K}5_n\| \leq \frac{1-\wp_5^*}{\beta(\wp_5^*)}\|\mathcal{Y}_5(t, Q_n(t)) - \mathcal{Y}_5(t, Q_{n-1}(t))\| +$$

$$+\frac{\wp_5^*}{\beta(\wp_1^*)\Gamma(\wp_5^*)}\int\limits_0^t (t-s)^{\wp_5^*-1}\|\mathcal{Y}_5(s, Q_n(s)) - \mathcal{Y}_5(t, Q_{n-1}(t))\|ds \leq$$

$$\leq \left[\frac{1-\wp_5^*}{\beta(\wp_5^*)} + \frac{1}{\beta(\wp_5^*)\Gamma(\wp_5^*)}\right]\psi_5\|Q_n - Q\| \leq$$

$$\leq \left[\frac{1-\wp_5^*}{\beta(\wp_5^*)} + \frac{1}{\beta(\wp_5^*)\Gamma(\wp_5^*)}\right]^n \Delta^n\|Q_1 - Q\|,$$

$$\|\mathcal{K}6_n\| \leq \frac{1-\wp_6^*}{\beta(\wp_6^*)}\|\mathcal{Y}_6(t, R_n(t)) - \mathcal{Y}_6(t, R_{n-1}(t))\| +$$

$$+\frac{\wp_6^*}{\beta(\wp_1^*)\Gamma(\wp_6^*)}\int\limits_0^t (t-s)^{\wp_6^*-1}\|\mathcal{Y}_6(s, R_n(s)) - \mathcal{Y}_5(t, R_{n-1}(t))\|ds \leq$$

$$\leq \left[\frac{1-\wp_6^*}{\beta(\wp_6^*)} + \frac{1}{\beta(\wp_6^*)\Gamma(\wp_6^*)}\right]\psi_6\|R_n - R\| \leq$$

$$\leq \left[\frac{1-\wp_6^*}{\beta\left(\wp_6^*\right)} + \frac{1}{\beta\left(\wp_6^*\right)\Gamma\left(\wp_6^*\right)}\right]^n \Delta^n \|R_1 - R\|.$$

Thus, we have $\mathcal{K}(t)_n \to 0,\ i \in 1, \ldots, 6,$ as $n \to \infty$ for $\Delta < 1,$ which is the required proof.

3. Uniqueness solution. For our suggested model (1), we study the analysis of the uniqueness of solution.

Theorem 3. *The Covid-19 model (1) has unique solution if*

$$\left[\frac{1-\varphi_i}{\beta(\varphi_i)} + \frac{1}{\beta(\varphi_i)\Gamma(\varphi_i)}\right]\psi_i \leq 1, \quad i \in \mathcal{N}_1^6. \tag{14}$$

Proof. Let there exist another solution $\overline{S}(t),\ \overline{S_c}(t),\ \overline{I_A}(t),\ \overline{i_s}(t),\ \overline{Q}(t),\ \overline{R}(t)$ such that

$$\overline{S}(t) = \frac{1-\wp_1^*}{\beta\left(\wp_1^*\right)}\mathcal{Y}_1(t, \overline{S}(t)) + \frac{\wp_1^*}{\beta\left(\wp_1^*\right)\Gamma\left(\wp_1^*\right)}\int\limits_0^t (t-s)^{\wp_1^*-1}\mathcal{Y}_1(s, \overline{S}(s))ds,$$

$$\overline{S_p}(t) = \frac{1-\wp_2^*}{\beta\left(\wp_2^*\right)}\mathcal{Y}_2(t, \overline{S_p}(t)) + \frac{\wp_2^*}{\beta\left(\wp_2^*\right)\Gamma\left(\wp_2^*\right)}\int\limits_0^t (t-s)^{\wp_2^*-1}\mathcal{Y}_2(s, \overline{S_p}(s))ds,$$

$$\overline{I_A}(t) = \frac{1-\wp_3^*}{\beta\left(\wp_3^*\right)}\mathcal{Y}_3(t, \overline{I_A}(t)) + \frac{\wp_3^*}{\beta\left(\wp_3^*\right)\Gamma\left(\wp_3^*\right)}\int\limits_0^t (t-s)^{\wp_3^*-1}\mathcal{Y}_3(s, \overline{I_A}(s))ds,$$

$$\overline{I_s}(t) = \frac{1-\wp_4^*}{\beta\left(\wp_4^*\right)}\mathcal{Y}_4(t, \overline{I_s}(t)) + \frac{\wp_4^*}{\beta\left(\wp_4^*\right)\Gamma\left(\wp_4^*\right)}\int\limits_0^t (t-s)^{\wp_4^*-1}\mathcal{Y}_4(s, \overline{I_s}(s))ds,$$

$$\overline{Q}(t) = \frac{1-\wp_5^*}{\beta\left(\wp_5^*\right)}\mathcal{Y}_5(t, \overline{Q}(t)) + \frac{\wp_5^*}{\beta\left(\wp_5^*\right)\Gamma\left(\wp_5^*\right)}\int\limits_0^t (t-s)^{\wp_5^*-1}\mathcal{Y}_5(s, \overline{Q}(s))ds,$$

$$\overline{R}(t) = \frac{1-\wp_6^*}{\beta\left(\wp_6^*\right)}\mathcal{Y}_6(t, \overline{R}(t)) + \frac{\wp_6^*}{\beta\left(\wp_6^*\right)\Gamma\left(\wp_6^*\right)}\int\limits_0^t (t-s)^{\wp_6^*-1}\mathcal{Y}_6(s, \overline{R}(s))ds.$$

Then,

$$\|S(t) - \overline{S}(t)\| \leq \frac{1-\wp_1^*}{\beta\left(\wp_1^*\right)}\|\mathcal{Y}_1(t, S(t)) - Y_1(t, \overline{S}(t))\| +$$

$$+ \frac{\wp_1^*}{\beta\left(\wp_1^*\right)\Gamma\left(\wp_1^*\right)}\int\limits_0^t (t-s)^{\wp_1^*-1}\|Y_1(s, S(s)) - Y_1(t, \overline{S}(t))\|ds \leq$$

$$\leq \left[\frac{1-\wp_1^*}{\beta\left(\wp_1^*\right)} + \frac{1}{\beta\left(\wp_1^*\right)\Gamma\left(\wp_1^*\right)}\right]\psi_1\|S - \overline{S}\|,$$

which implies

$$\left[\frac{1-\wp_1^*}{\beta\big(\wp_1^*\big)}\psi_1+\frac{\psi_1}{\beta\big(\wp_1^*\big)\Gamma(\wp_1^*)}-1\right]\|S\_\overline{S}\|\geq 0. \tag{15}$$

By (14), the (15) is true if $\left\|S-\overline{S}\right\|=0,$ which implies $S(t)=\overline{S}(t).$ Similarly, we have

$$\left\|S_p(t)-\overline{S_p}(t)\right\|\leq\frac{1-\wp_2^*}{\beta\big(\wp_2^*\big)}\left\|\mathcal{Y}_2(t,S_p(t))-Y_2\left(t,\overline{S_p}(t)\right)\right\|+$$

$$+\frac{\wp_2^*}{\beta\big(\wp_2^*\big)\Gamma\big(\wp_2^*\big)}\int\limits_0^t(t-s)^{\wp_2^*-1}\left\|Y_2(s,S_p(s))-Y_2(t,\overline{S_p}(t))\right\|ds\leq$$

$$\leq\left[\frac{1-\wp_2^*}{\beta\big(\wp_2^*\big)}+\frac{1}{\beta\big(\wp_2^*\big)\Gamma\big(\wp_2^*\big)}\right]\psi_2\left\|S_P-\overline{S_p}\right\|,$$

$$\left[\frac{1-\wp_2^*}{\beta\big(\wp_2^*\big)}\,\psi_2+\frac{\psi_2}{\beta\big(\wp_2^*\big)\Gamma\big(\wp_2^*\big)}-1\right]\left\|S_P-\overline{S_p}\right\|\geq 0, \tag{16}$$

which follows. By (14), the (16) is true if $\|S_p-\overline{S_p}\|=0,$ which implies $S_p(t)=\overline{S_p}(t).$ Now, for $I_A,$ we have

$$\left\|I_A(t)-\overline{I_A}(t)\right\|\leq\frac{1-\wp_3^*}{\beta\big(\wp_3^*\big)}\left\|\mathcal{Y}_3(t,I_A(t))-Y_3(t,\overline{I_A}(t))\right\|+$$

$$+\frac{\wp_3^*}{\beta\big(\wp_3^*\big)\Gamma\big(\wp_3^*\big)}\int\limits_0^t(t-s)^{\wp_3^*-1}\left\|Y_3(s,I_A(s))-Y_3(t,\overline{I_A}(t))\right\|ds\leq$$

$$\leq\left[\frac{1-\wp_3^*}{\beta\big(\wp_3^*\big)}+\frac{1}{\beta\big(\wp_3^*\big)\Gamma\big(\wp_3^*\big)}\right]\psi_3\left\|I_A-\overline{I_A}\right\|,$$

which implies

$$\left[\frac{1-\wp_3^*}{\beta\big(\wp_3^*\big)}\,\psi_3+\frac{\psi_3}{\beta\big(\wp_3^*\big)\Gamma\big(\wp_3^*\big)}-1\right]\left\|I_A-\overline{I_A}\right\|\geq 0, \tag{17}$$

which implies by (14), the (17) is true if $\|I_A-\overline{I_A}\|=0,$ which implies $I_A(t)=\overline{I_A}(t);$

$$\left\|I_s(t)-\overline{I_s}(t)\right\|\leq\frac{1-\wp_4^*}{\beta\big(\wp_4^*\big)}\left\|\mathcal{Y}_4(t,I_s(t))-Y_4(t,\overline{I_s}(t))\right\|+$$

$$+\frac{\wp_4^*}{\beta\big(\wp_4^*\big)\Gamma\big(\wp_4^*\big)}\int\limits_0^t(t-s)^{\wp_4^*-1}\left\|Y_4(s,I_s(s))-Y_4(t,\overline{I_s}(t))\right\|ds\leq$$

$$\leq \left[\frac{1-\wp_4^*}{\beta\big(\wp_4^*\big)} + \frac{1}{\beta\big(\wp_4^*\big)\Gamma\big(\wp_4^*\big)}\right]\psi_4 \left\|I_s - \overline{I_s}\right\|,$$

which implies

$$\left[\frac{1-\wp_4^*}{\beta\big(\wp_4^*\big)}\,\psi_4 + \frac{\psi_4}{\beta\big(\wp_4^*\big)\Gamma\big(\wp_4^*\big)} - 1\right]\left\|I_{s-}\overline{I_s}\right\| \geq 0. \tag{18}$$

By (14), the (18) is true if $\|I_s - \overline{I_s}\| = 0,$ which implies $I_s(t) = \overline{I_s}(t).$ Now, for $Q,$ we have

$$\left\|Q(t) - \overline{Q}(t)\right\| \leq \frac{1-\wp_5^*}{\beta\big(\wp_5^*\big)}\left\|\mathcal{Y}_5(t, Q(t)) - Y_5(t, \overline{Q}(t))\right\| +$$

$$+\frac{\wp_5^*}{\beta\big(\wp_5^*\big)\Gamma\big(\wp_5^*\big)}\int\limits_0^t (t-s)^{\wp_5^*-1}\left\|Y_5(s, Q(s)) - Y_5\left(t, \overline{Q}(t)\right)\right\| ds \leq$$

$$\leq \left[\frac{1-\wp_5^*}{\beta\big(\wp_5^*\big)} + \frac{1}{\beta\big(\wp_5^*\big)\Gamma\big(\wp_5^*\big)}\right]\psi_5 \left\|Q - \overline{Q}\right\|,$$

which implies

$$\left[\frac{1-\wp_5^*}{\beta\big(\wp_5^*\big)}\psi_5 + \frac{\psi_5}{\beta\big(\wp_5^*\big)\Gamma\big(\wp_5^*\big)} - 1\right]\left\|Q - \overline{Q}\right\| \geq 0,$$

$$\left\|R(t) - \overline{R}(t)\right\| \leq \frac{1-\wp_6^*}{\beta\big(\wp_6^*\big)}\left\|\mathcal{Y}_6(t, R(t)) - Y_6(t, \overline{R}(t))\right\| +$$

$$+\frac{\wp_6^*}{\beta\big(\wp_6^*\big)\Gamma\big(\wp_6^*\big)}\int\limits_0^t (t-s)^{\wp_6^*-1}\left\|Y_6(s, R(s)) - Y_6(t, \overline{R}(t))\right\| ds \leq$$

$$\leq \left[\frac{1-\wp_6^*}{\beta\big(\wp_6^*\big)} + \frac{1}{\beta\big(\wp_6^*\big)\Gamma\big(\wp_6^*\big)}\right]\psi_6 \left\|R - \overline{R}\right\|,$$

which implies

$$\left[\frac{1-\wp_6^*}{\beta\big(\wp_6^*\big)}\,\psi_6 + \frac{\psi_6}{\beta\big(\wp_6^*\big)\Gamma\big(\wp_6^*\big)} - 1\right]\left\|R - \overline{R}\right\| \geq 0, \tag{19}$$

which implies by (14), the (19) is true if $\|R - \overline{R}\| = 0,$ which implies $R(t) = \overline{R}(t).$ Thus the (1) has unique solution.

4. Hyers – Ulams stability.

Definition 4. *The integral system (8) – (13) is Hyers – Ulam stable if for* $\Delta_i > 0,\ i \in \mathcal{N}_1^6,$ *and* $\gamma_i > 0,\ i \in \mathcal{N}_1^6,$ *such that*

$$\left|S(t) - \frac{1-\wp_1^*}{\beta\big(\wp_1^*\big)}\,\mathcal{Y}_1(t, S(t)) - \frac{\wp_1^*}{\beta\big(\wp_1^*\big)\Gamma\big(\wp_1^*\big)}\int\limits_0^t (t-s)^{\wp_1^*-1}\mathcal{Y}_1(s, S(s))ds\right| \leq \gamma_1,$$

$$\left|S_p(t) - \frac{1-\wp_2^*}{\beta\left(\wp_2^*\right)}\mathcal{Y}_2(t, S_p(t)) - \frac{\wp_2^*}{\beta\left(\wp_2^*\right)\Gamma\left(\wp_2^*\right)}\int\limits_0^t (t-s)^{\wp_2^*-1}\mathcal{Y}_2(s, S_p(s))ds\right| \leq \gamma_2,$$

$$\left|I_A(t) - \frac{1-\wp_3^*}{\beta\left(\wp_3^*\right)}\mathcal{Y}_3(t, I_A(t)) - \frac{\wp_3^*}{\beta\left(\wp_3^*\right)\Gamma\left(\wp_3^*\right)}\int\limits_0^t (t-s)^{\wp_3^*-1}\mathcal{Y}_3(s, I_A(s))ds\right| \leq \gamma_3,$$

$$\left|I_s(t) - \frac{1-\wp_4^*}{\beta\left(\wp_4^*\right)}\mathcal{Y}_4(t, I_s(t)) - \frac{\wp_4^*}{\beta\left(\wp_4^*\right)\Gamma\left(\wp_4^*\right)}\int\limits_0^t (t-s)^{\wp_4^*-1}\mathcal{Y}_4(s, I_s(s))ds\right| \leq \gamma_4,$$

$$\left|Q(t) - \frac{1-\wp_5^*}{\beta\left(\wp_5^*\right)}\mathcal{Y}_5(t, Q(t)) - \frac{\wp_5^*}{\beta\left(\wp_5^*\right)\Gamma\left(\wp_5^*\right)}\int\limits_0^t (t-s)^{\wp_5^*-1}\mathcal{Y}_5(s, Q(s))ds\right| \leq \gamma_5,$$

$$\left|R(t) - \frac{1-\wp_6^*}{\beta\left(\wp_6^*\right)}\mathcal{Y}_6(t, Q(t)) - \frac{\wp_6^*}{\beta\left(\wp_6^*\right)\Gamma\left(\wp_6^*\right)}\int\limits_0^t (t-s)^{\wp_6^*-1}\mathcal{Y}_6(s, R(s))ds\right| \leq \gamma_5.$$

We have $\dot{S}(t),\ \dot{S}_p(t),\ \dot{I}_A(t),\ \dot{I}_s(t),\ \dot{Q}(t),\ \dot{R}(t)$, *which implies*

$$\dot{S}(t) = \frac{1-\wp_1^*}{\beta\left(\wp_1^*\right)}\mathcal{Y}_1(t, \dot{S}(t)) + \frac{\wp_1^*}{\beta\left(\wp_1^*\right)\Gamma\left(\wp_1^*\right)}\int\limits_0^t (t-s)^{\wp_1^*-1}\mathcal{Y}_1(s, \dot{S}(s))ds,$$

$$\dot{S}_p(t) = \frac{1-\wp_2^*}{\beta\left(\wp_2^*\right)}\mathcal{Y}_2(t, \dot{S}_p(t)) + \frac{\wp_2^*}{\beta\left(\wp_2^*\right)\Gamma\left(\wp_2^*\right)}\int\limits_0^t (t-s)^{\wp_2^*-1}\mathcal{Y}_2(s, \dot{S}_p(s))ds,$$

$$\dot{I}_A(t) = \frac{1-\wp_3^*}{\beta\left(\wp_3^*\right)}\mathcal{Y}_3(t, \dot{I}_A(t)) + \frac{\wp_3^*}{\beta\left(\wp_3^*\right)\Gamma\left(\wp_3^*\right)}\int\limits_0^t (t-s)^{\wp_3^*-1}\mathcal{Y}_3\left(s, \dot{I}_A(s)\right)ds,$$

$$\dot{I}_s(t) = \frac{1-\wp_4^*}{\beta\left(\wp_4^*\right)}\mathcal{Y}_4\left(t, \dot{I}_s(t)\right) + \frac{\wp_4^*}{\beta\left(\wp_4^*\right)\Gamma\left(\wp_4^*\right)}\int\limits_0^t (t-s)^{\wp_4^*-1}\mathcal{Y}_4\left(s, \dot{I}_s(s)\right)ds,$$

$$\dot{Q}(t) = \frac{1-\wp_5^*}{\beta\left(\wp_5^*\right)}\mathcal{Y}_5(t, \dot{Q}(t)) + \frac{\wp_5^*}{\beta\left(\wp_5^*\right)\Gamma\left(\wp_5^*\right)}\int\limits_0^t (t-s)^{\wp_5^*-1}\mathcal{Y}_5\left(s, \dot{Q}(s)\right)ds,$$

$$\dot{R}(t) = \frac{1-\wp_6^*}{\beta\left(\wp_6^*\right)}\mathcal{Y}_6\left(t, \dot{R}(t)\right) + \frac{\wp_6^*}{\beta\left(\wp_6^*\right)\Gamma\left(\wp_6^*\right)}\int\limits_0^t (t-s)^{\wp_6^*-1}\mathcal{Y}_6\left(s, \dot{R}(s)\right)ds$$

such that

$$\left|S(t) - \dot{S}(t)\right| \leq \delta_1\gamma_1,$$

$$\left|S_p(t) - \dot{S}_p(t)\right| \leq \delta_2 \gamma_2,$$

$$\left|I_A(t) - \dot{I}_A(t)\right| \leq \delta_3 \gamma_3,$$

$$\left|I_s(t) - \dot{I}_s(t)\right| \leq \delta_4 \gamma_4,$$

$$\left|Q(t) - \dot{Q}(t)\right| \leq \delta_5 \gamma_5,$$

$$\left|R(t) - \dot{R}(t)\right| \leq \delta_6 \gamma_6.$$

Theorem 4. *If Assumption* $(\mathcal{B})$ *is satisfied, then (1) is Hyers – Ulam-stable.*

Proof. By Theorem 3, the Covid-19 model (1) has a unique solution, say, $S(t),\ S_p(t),\ I_A(t),\ I_s(t),\ Q(t),\ R(t)$. Let $\big(\dot{S}(t), \dot{S}_p(t), \dot{I}_A(t), \dot{I}_s(t), \dot{Q}(t), \dot{R}(t)\big)$ be an approximate solution of (1) satisfying (8) – (13). Then, we have

$$\left\|S(t) - \dot{S}(t)\right\| \leq \frac{1-\wp_1^*}{\beta\big(\wp_1^*\big)} \left\|Y_1(t, S(t)) - Y_1\left(t, \dot{S}(t)\right)\right\| +$$

$$+ \frac{\wp_1^*}{\beta\big(\wp_1^*\big)\Gamma\big(\wp_1^*\big)} \int\limits_0^t (t-s)^{\wp_1^*-1} \left\|Y_1(s, S(s)) - Y_1\left(t, \dot{S}(t)\right)\right\| ds \leq$$

$$\leq \left[\frac{1-\wp_1^*}{\beta\big(\wp_1^*\big)} + \frac{1}{\beta\big(\wp_1^*\big)\Gamma\big(\wp_1^*\big)}\right] \psi_1 \|S - \dot{S}\|.$$

Taking $\gamma_1 = \psi_1,\ \Delta = \dfrac{1-\wp_1^*}{\beta\big(\wp_1^*\big)} + \dfrac{\wp_1^*}{\beta\big(\wp_1^*\big)\Gamma\big(\wp_1^*\big)}$, this implies

$$\left\|S(t) - \dot{S}(t)\right\| \leq \gamma_1 \Delta_1.$$

Similarly, for $S_p(t),\ \dot{S}_p(t),\ I_A(t),\ \dot{I}_A(t),\ I_s(t),\ \dot{I}_s(t),\ Q(t),\ \dot{Q}(t),\ R(t),\ \dot{R}(t)$ we have

$$\left\|S_p(t) - \dot{S}_p(t)\right\| \leq \gamma_2 \Delta_2,$$

$$\left\|I_A(t) - \dot{I}_A(t)\right\| \leq \gamma_3 \Delta_3,$$

$$\left\|I_s(t) - \dot{I}_s(t)\right\| \leq \gamma_4 \Delta_4,$$

$$\left\|Q(t) - \dot{Q}(t)\right\| \leq \gamma_5 \Delta_5,$$

$$\left\|R(t) - \dot{R}(t)\right\| \leq \gamma_6 \Delta_6.$$

This implies, the system (1) is Hyers – Ulam stable which ultimately, ensures the stability of (1). This completes the proof.

5. Numerical scheme. With the help of (2) – (7), we produce the following numerical scheme:

$$\begin{cases} {}_0^{ABC}\mathcal{D}_t^{\wp_1^*}S(t) = Y_1(t,S), \\ {}_0^{ABC}\mathcal{D}_t^{\wp_2^*}S_p(t) = Y_2(t,S_p), \\ {}_0^{ABC}\mathcal{D}_t^{\wp_3^*}I_A(t) = Y_3(t,I_A), \\ {}_0^{ABC}\mathcal{D}_t^{\wp_4^*}I_s(t) = Y_4(t,I_s), \\ {}_0^{ABC}\mathcal{D}_t^{\wp_5^*}Q(t) = Y_5(t,Q), \\ {}_0^{ABC}\mathcal{D}_t^{\wp_6^*}R(t) = Y_6(t,R). \end{cases} \tag{20}$$

With the help of fractional AB-integral operator, (20) takes the following form:

$$S(t) - S(0) = \frac{1-\wp_1^*}{\beta(\wp_1^*)}\mathcal{Y}_1(t,S) + \frac{\wp_1^*}{\beta(\wp_1^*)\Gamma(\wp_1^*)}\int_0^t (t-s)^{\wp_1^*-1}\mathcal{Y}_1(s,S)ds,$$

$$S_p(t) - S_p(0) = \frac{1-\wp_2^*}{\beta(\wp_2^*)}\mathcal{Y}_2(t,I_s) + \frac{\wp_2^*}{\beta(\wp_2^*)\Gamma(\wp_2^*)}\int_0^t (t-s)^{\wp_2^*-1}\mathcal{Y}_2(s,S_p)ds,$$

$$I_A(t) - I_A(0) = \frac{1-\wp_3^*}{\beta(\wp_3^*)}\mathcal{Y}_2(t,I_A) + \frac{\wp_3^*}{\beta(\wp_3^*)\Gamma(\wp_3^*)}\int_0^t (t-s)^{\wp_3^*-1}\mathcal{Y}_3(s,I_A)ds,$$

$$I_s(t) - I_s(0) = \frac{1-\wp_4^*}{\beta(\wp_4^*)}\mathcal{Y}_4(t,I_s) + \frac{\wp_4^*}{\beta(\wp_4^*)\Gamma(\wp_4^*)}\int_0^t (t-s)^{\wp_4^*-1}\mathcal{Y}_4(s,I_s)ds,$$

$$Q(t) - Q(0) = \frac{1-\wp_5^*}{\beta(\wp_5^*)}\mathcal{Y}_5(t,Q) + \frac{\wp_5^*}{\beta(\wp_5^*)\Gamma(\wp_5^*)}\int_0^t (t-s)^{\wp_5^*-1}\mathcal{Y}_5(s,Q)ds,$$

$$R(t) - R(0) = \frac{1-\wp_6^*}{\beta(\wp_6^*)}\mathcal{Y}_6(t,R) + \frac{\wp_6^*}{\beta(\wp_6^*)\Gamma(\wp_6^*)}\int_0^t (t-s)^{\wp_6^*-1}\mathcal{Y}_6(s,R)ds.$$

By dividing the assumed interval $[0,t]$ into subintervals by the help of point t_{m+1}, for $m = 0,1,2\ldots,$ we obtain

$$S(t_{m+1}) - S(0) = \frac{1-\wp_1^*}{\beta(\wp_1^*)}\mathcal{Y}_1(t_m,S) + \frac{\wp_1^*}{\beta(\wp_1^*)\Gamma(\wp_1^*)}\sum_{k=0}^{n}\int_{t_k}^{t_{k+1}} (t_{m+1}-s)^{\wp_1^*-1}\mathcal{Y}_1(s,S)ds,$$

$$S_p(t_{m+1}) - S_p(0) = \frac{1-\wp_2^*}{\beta(\wp_2^*)}\mathcal{Y}_2(t_m,S_p) + \frac{\wp_2^*}{\beta(\wp_2^*)\Gamma(\wp_2^*)}\sum_{k=0}^{n}\int_{t_k}^{t_{k+1}} (t_{m+1}-s)^{\wp_2^*-1}\mathcal{Y}_2(s,S_p)ds,$$

$$I_A(t_{m+1}) - I_c(0) = \frac{1-\wp_3^*}{\beta(\wp_3^*)}\mathcal{Y}_3(t_m, I_A) + \frac{\wp_3^*}{\beta(\wp_3^*)\Gamma(\wp_3^*)}\sum_{k=0}^{n}\int\limits_{t_k}^{t_{k+1}}(t_{m+1}-s)^{\wp_3^*-1}\mathcal{Y}_3(s, I_A)ds,$$

$$I_s(t_{m+1}) - I_s(0) = \frac{1-\wp_4^*}{\beta(\wp_4^*)}\mathcal{Y}_4(t_m, I_s) + \frac{\wp_4^*}{\beta(\wp_4^*)\Gamma(\wp_4^*)}\sum_{k=0}^{n}\int\limits_{t_k}^{t_{k+1}}(t_{m+1}-s)^{\wp_4^*-1}\mathcal{Y}_4(s, I_s)ds,$$

$$Q(t_{m+1}) - Q(0) = \frac{1-\wp_5^*}{\beta(\wp_5^*)}\mathcal{Y}_5(t_m, Q) + \frac{\wp_5^*}{\beta(\wp_5^*)\Gamma(\wp_5^*)}\sum_{k=0}^{n}\int\limits_{t_k}^{t_{k+1}}(t_{m+1}-s)^{\wp_5^*-1}\mathcal{Y}_5(s, Q)ds,$$

$$R(t_{m+1}) - R(0) = \frac{1-\wp_6^*}{\beta(\wp_6^*)}\mathcal{Y}_6(t_m, R) + \frac{\wp_6^*}{\beta(\wp_6^*)\Gamma(\wp_6^*)}\sum_{k=0}^{n}\int\limits_{t_k}^{t_{k+1}}(t_{m+1}-s)^{\wp_6^*-1}\mathcal{Y}_6(s, R)ds. \quad (21)$$

Now, by using Lagrange's interpolation, we get

$$S(t_{m+1}) = S(0) + \frac{1-\wp_1^*}{\beta(\wp_1^*)}\mathcal{Y}_1(t_k, S) + \frac{\wp_1^*}{\mathcal{B}(\wp_1^*)}\sum_{k=0}^{n}\Bigg[\frac{h^{\wp_1^*}\mathcal{Y}_1(t_k, S)}{\Gamma(\wp_1^*+2)}\times$$
$$\times\Big((m+1-k)^{\wp_1^*}(m-k+2+\wp_1^*) - (m-k)^{\wp_1^*}(m-k+2+2\wp_1^*)\Big) -$$
$$-\frac{h^{\wp_1^*}\mathcal{Y}_1(t_{k-1}, S)}{\Gamma(\wp_1^*+2)}\Big((m+1-k)^{\wp_1^*} - (m-k)^{\wp_1^*}(m+1-k+\wp_1^*)\Big)\Bigg],$$

$$S_p(t_{m+1}) = S_p(0) + \frac{1-\wp_2^*}{\beta(\wp_2^*)}\mathcal{Y}_2(t_k, S_p) + \frac{\wp_2^*}{\mathcal{B}(\wp_2^*)}\sum_{k=0}^{n}\Bigg[\frac{h^{\wp_2^*}\mathcal{Y}_2(t_k, S_p)}{\Gamma(\wp_2^*+2)}\times$$
$$\times\Big((m+1-k)^{\wp_2^*}(m-k+2+\wp_2^*) - (m-k)^{\wp_2^*}(m-k+2+2\wp_2^*)\Big) -$$
$$-\frac{h^{\wp_2^*}\mathcal{Y}_2(t_{k-1}, S_p)}{\Gamma(\wp_2^*+2)}\Big((m+1-k)^{\wp_2^*} - (m-k)^{\wp_2^*}(m+1-k+\wp_2^*)\Big)\Bigg],$$

$$I_A(t_{m+1}) = I_A(0) + \frac{1-\wp_3^*}{\beta(\wp_3^*)}\mathcal{Y}_3(t_k, I_A) + \frac{\wp_3^*}{\mathcal{B}(\wp_3^*)}\sum_{k=0}^{n}\Bigg[\frac{h^{\wp_3^*}\mathcal{Y}_3(t_k, I_A)}{\Gamma(\wp_3^*+2)}\times$$
$$\times\Big((m+1-k)^{\wp_3^*}(m-k+2+\wp_3^*) - (m-k)^{\wp_3^*}(m-k+2+2\wp_3^*)\Big) -$$
$$-\frac{h^{\wp_3^*}\mathcal{Y}_3(t_{k-1}, I_A)}{\Gamma(\wp_3^*+2)}\Big((m+1-k)^{\wp_3^*} - (m-k)^{\wp_3^*}(m+1-k+\wp_3^*)\Big)\Bigg],$$

$$I_s(t_{m+1}) = I_s(0) + \frac{1-\wp_4^*}{\beta(\wp_4^*)}\mathcal{Y}_4(t_k, I_s) + \frac{\wp_4^*}{\mathcal{B}(\wp_4^*)}\sum_{k=0}^{n}\Bigg[\frac{h^{\wp_4^*}\mathcal{Y}_4(t_k, I_s)}{\Gamma(\wp_4^*+2)}\times$$

$$\times \Big((m+1-k)^{\wp_4^*} (m-k+2+\wp_4^*) (m-k)^{\wp_4^*} (m-k+2+2\wp_4^*) \Big) -$$

$$- \frac{h^{\wp_4^*} \mathcal{Y}_4(t_{k-1}, I_s)}{\Gamma(\wp_4^*+2)} \Big((m+1-k)^{\wp_4^*} - (m-k)^{\wp_4^*} (m+1-k+\wp_4^*) \Big) \Bigg],$$

$$Q(t_{m+1}) = Q(0) + \frac{1-\wp_5^*}{\beta(\wp_5^*)} \mathcal{Y}_5(t_k, Q) + \frac{\wp_5^*}{\mathcal{B}(\wp_5^*)} \sum_{k=0}^{n} \Bigg[\frac{h^{\wp_5^*} \mathcal{Y}_5(t_k, Q)}{\Gamma(\wp_5^*+2)} \times$$

$$\times \Big((m+1-k)^{\wp_5^*} (m-k+2+\wp_5^*) (m-k)^{\wp_5^*} (m-k+2+2\wp_5^*) \Big) -$$

$$- \frac{h^{\wp_5^*} \mathcal{Y}_5(t_{k-1}, Q)}{\Gamma(\wp_5^*+2)} \Big((m+1-k)^{\wp_5^*} - (m-k)^{\wp_5^*} (m+1-k+\wp_5^*) \Big) \Bigg],$$

$$R(t_{m+1}) = R(0) + \frac{1-\wp_6^*}{\beta(\wp_6^*)} \mathcal{Y}_6(t_k, R) + \frac{\wp_6^*}{\mathcal{B}(\wp_6^*)} \sum_{k=0}^{n} \Bigg[\frac{h^{\wp_6^*} \mathcal{Y}_6(t_k, R)}{\Gamma(\wp_6^*+2)} \times$$

$$\times \Big((m+1-k)^{\wp_6^*} (m-k+2+\wp_6^*) (m-k)^{\wp_6^*} (m-k+2+2\wp_6^*) \Big) -$$

$$- \frac{h^{\wp_6^*} \mathcal{Y}_6(t_{k-1}, R)}{\Gamma(\wp_6^*+2)} \Big((m+1-k)^{\wp_6^*} - (m-k)^{\wp_6^*} (m+1-k+\wp_6^*) \Big) \Bigg].$$

This numerical scheme will help us to predict the role of protected susceptible which has been practically exercised in various nations as a control strategy. Although, this strategy has a worst effect on the economy of a nation but it is essential to curtail the spread of the infection of lethal Covid-19. The sensitivity analysis has been given in [36] which shows that the role of such a strategy is very much effective in the curtail of the spread.

5.1. Numerical results. In this section, we are providing a detail of numerical results related to the model with the available data in literature. The parameters and initial data was carried out from the available literature. The initial values are: $S(0) = 59300000,\ S_P(0) = 0,\ I_S(0) = 0,$ $Q(0) = 0,\ I_A(0) = 2079,\ R(0) = 903,$ and the parametric values are $\alpha_1 = 0.0008,\ \alpha_2 = 0.1,$ $eta_1 = 0.25,\ \rho = 0.0001,\ \eta_2 = 0,\ \eta_3 = 0.385,\ r_1 = 0.2976,\ r_2 = 0,\ \gamma = 0,\ r_3 = 0.2976,$ $\mu = 0.00236/90,\ \delta = 0.017/90,\ \alpha_3 = 1,\ \Lambda_1 = 296425.875/90$ [36].

In Fig. 1, we have a joint comparative simulation for the two classes $S(t)$ and $S_P(t)$ for the orders $1.0,\ 0.99,\ 0.98,\ 0.97.$ The second figure that is Fig. 2, represents a graphical study of the $S(t)$ class for various orders $1.0,\ 0.99,\ 0.98,\ 0.97$ for a long time of 5000 days. There is a decrease in the population of the class. Also, as much the order is decreasing a comparative large decrease is observed in the population while the behavior of the class remains similar. Figure 3 shows a comparative analysis of the $S_P(t)$ for the mentioned orders and a gradual increase can be seen in the graph.

The Fig. 4 is for the infection population which shows an increase up to the 300 days while a decrease is observed after 300 to 600. Figure 5 shows a numerical representation of the $I_S(t)$ class for the various orders and the Fig. 6 is for the $R(t)$ class.

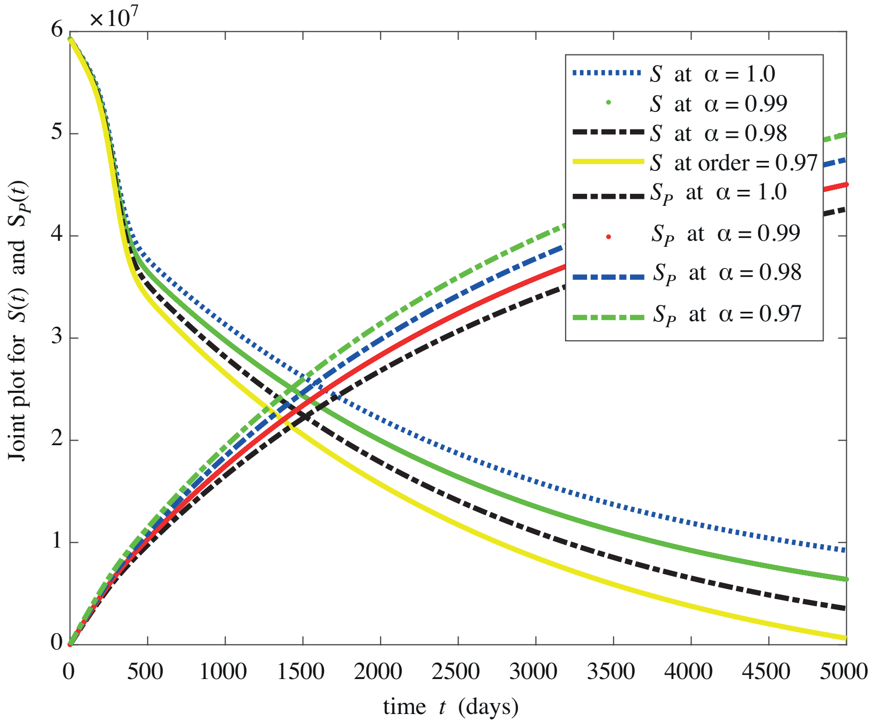

Fig. 1. Comparative analysis for the $S(t)$ and $S_P(t)$ for the orders 1.0, 0.99, 0.98, 0.97.

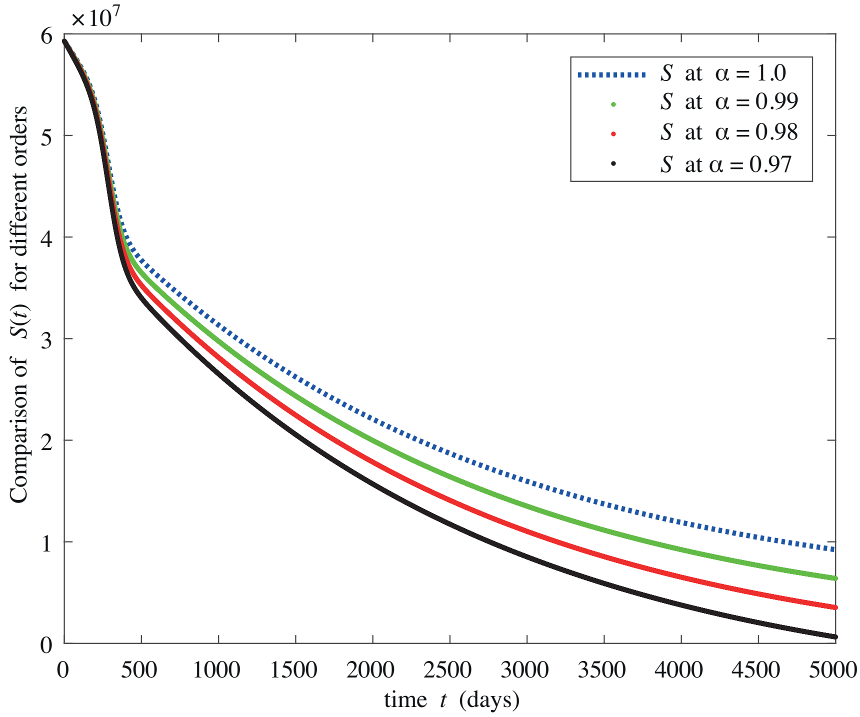

Fig. 2. Comparative analysis for the $S(t)$ for the orders 1.0, 0.99, 0.98, 0.97.

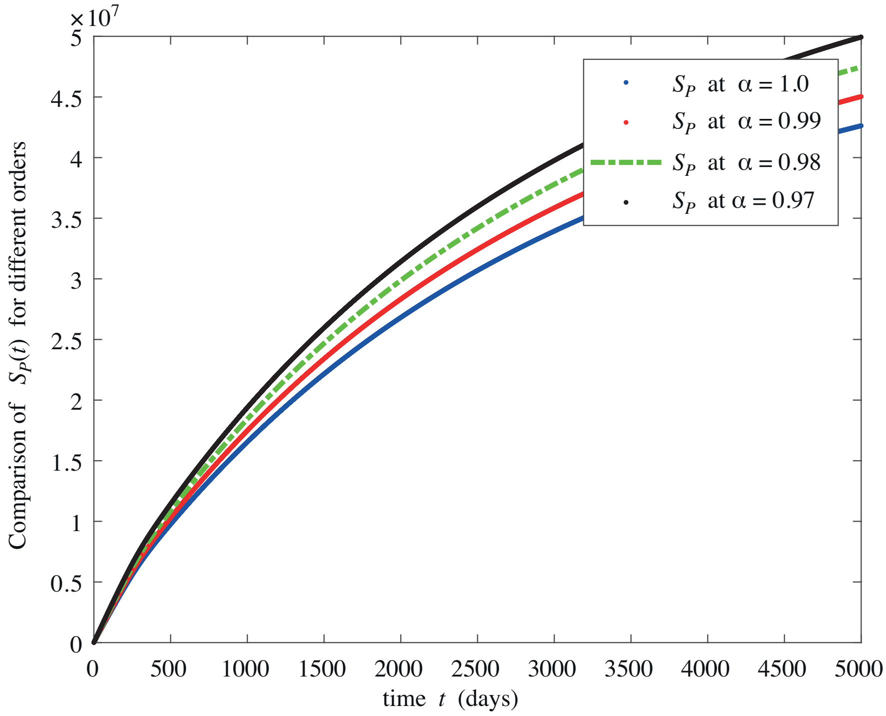

Fig. 3. Comparative analysis for the $S_P(t)$ for the orders 1.0, 0.99, 0.98, 0.97.

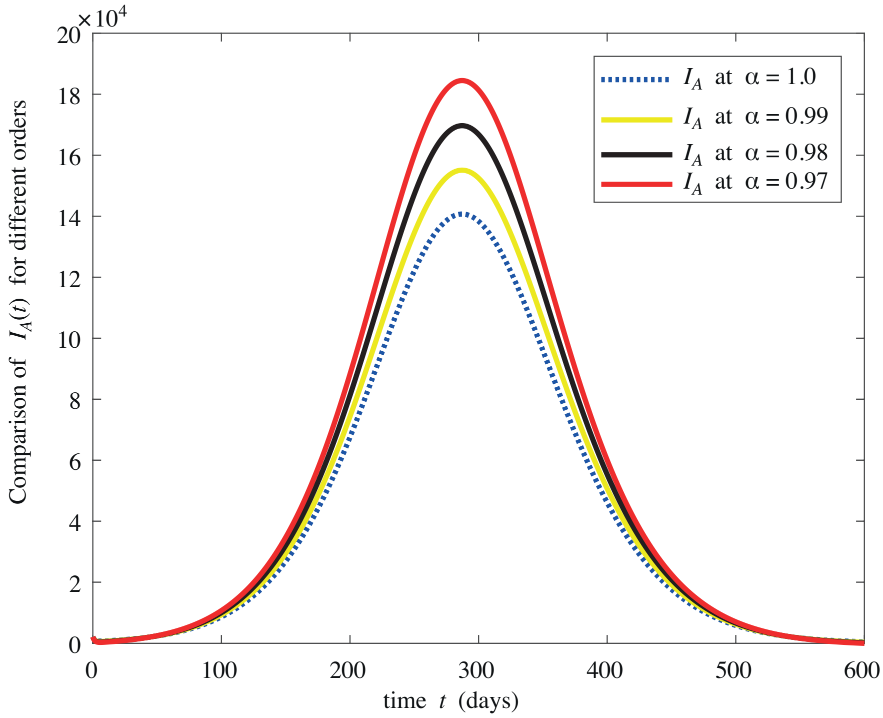

Fig. 4. Comparative analysis for the $I(t)$ for the orders 1.0, 0.99, 0.98, 0.97.

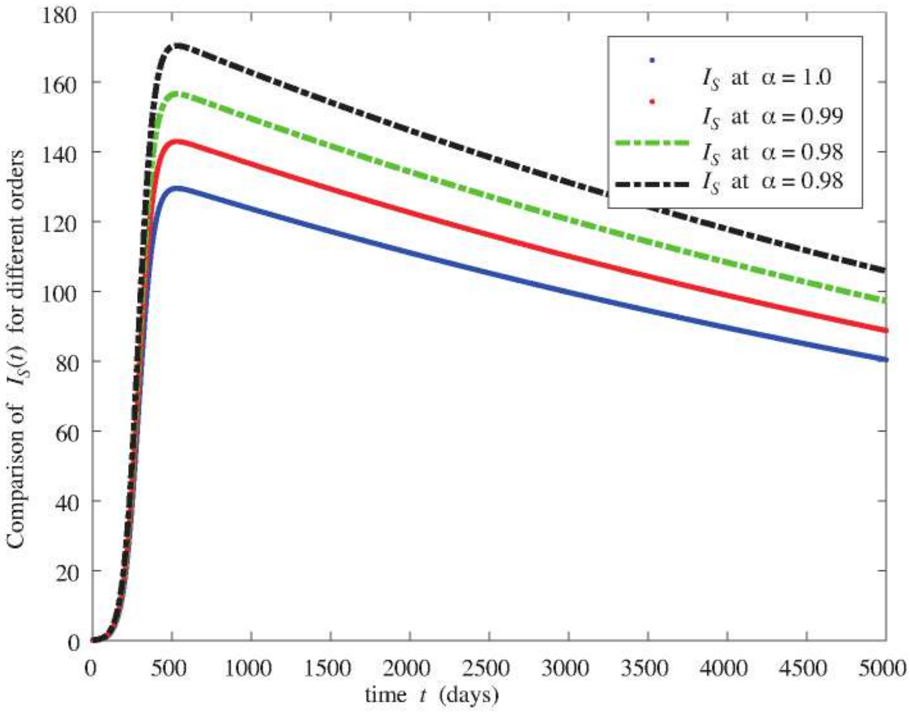

Fig. 5. Comparative analysis for the $I_S(t)$ for the orders 1.0, 0.99, 0.98, 0.97.

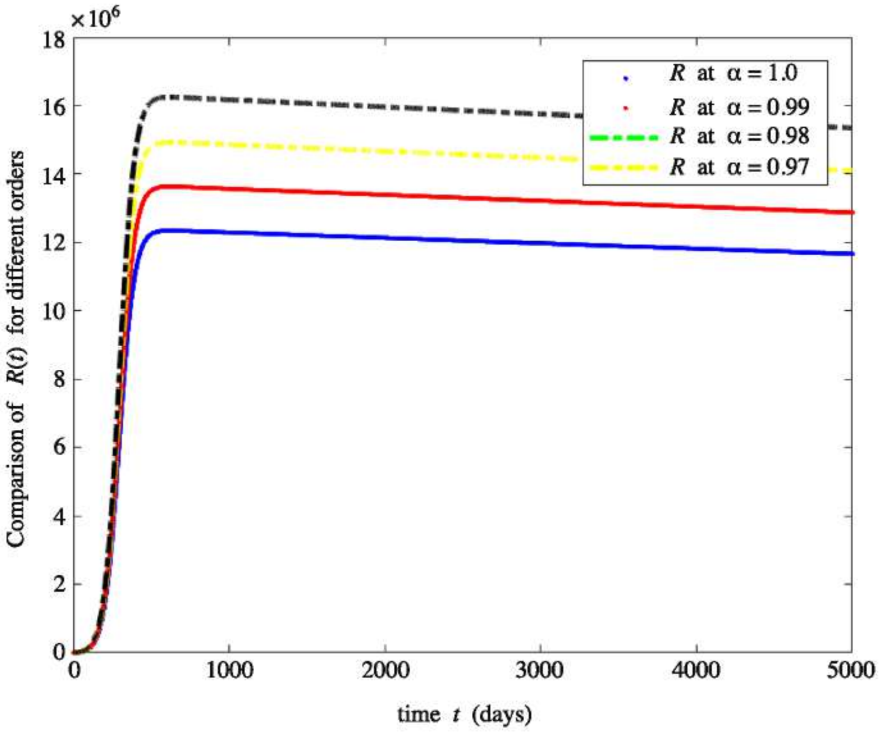

Fig. 6. Comparative analysis for the $R(t)$ for the orders 1.0, 0.99, 0.98, 0.97.

6. Conclusion. In this article, we have focused on the theoretical as well computational studies of the fractional order Covid-19 model in the ABC-sense of derivative. The existence, uniqueness results were carried out with the help of iterative sequential approach with the limiting point as the solution of the suggested model (1). We also estimated the Hyers – Ulam stability and a numerical scheme based on the Lagrange's interpolation was obtained. The numerical scheme was then tested and very similar results like the integer order was obtained. The numerical results were interpreted via six graphs. The details are: in Fig. 1, we have a joint comparative simulation for the two classes $S(t)$ and $S_P(t)$ for the orders $1.0, 0.99, 0.98, 0.97$. The second figure that is Fig. 2, represents a graphical study of the $S(t)$ class for various orders $1.0, 0.99, 0.98, 0.97$ for a long time of 5000 days. There is a decrease in the population of the class. Also, as much the order is decreasing a comparative large decrease is observed in the population while the behavior of the class remains similar. Figure 3, shows a comparative analysis of the $S_P(t)$ for the mentioned orders and a gradual increase can be seen in the graph. The Fig. 4 is for the infection population which shows an increase upto the 300 days while a decrease is observed after 300 to 600. Figure 5 shows a numerical representation of the $I_S(t)$ class for the various orders and the Fig. 6 is for the $R(t)$ class. The reader of the paper can work on the comparative analysis of different fractional operators for more accuracy and better results.

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