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CONTACT INTERACTION OF TWO ELASTIC HALF-SPACES WITH A CIRCULAR RECESS

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The method of functions of intercontact gaps extended to axially symmetric contact problems of two half-spaces one of which contains a circular surface recess is presented. This approach consists in construction of the integral representations of the displacements and stresses within the every of mated solids through functions defined on the unknown but bounded region of contact interface. For the case considered in the paper, namely, a frictionless contact of half-spaces, the integral representations contain only one unknown function – height of the gap. As examples, analytical solutions are obtained for some shapes of the initial recess. With some recess shape, the contact normal stresses exhibit peaks at the point which corresponds to the initial defect edge. This distinguished feature of the stress distribution is discussed.

Key words: *contact problem, conforming boundaries, local fault of contact, the method of functions of intercontact gaps, discontinuity of curvature.*

The contact interaction of solids allowing for intersurface gaps has been studied by many researchers. The plane and axially symmetric problems were studied in [1–7]. The thermomechanical opening and closing of interface gaps due to thermal contact resistance were examined in [6–10]. The interaction of solids in the absence of local contact caused by the presence of thin rigid inclusions was investigated in [11–14].

In papers written by Martynyak and co-workers [15–19], the plane frictionless contact problems for elastic isotropic and anisotropic half-planes with locally disturbed boundaries have been formulated and solved. Effect of friction [20, 21], stick-slip phenomena [22–23] and filler of interfacial gaps [24–29] on contact interaction of solids with such boundaries has been investigated. The above-mentioned papers are grounded on a method which is called the method of functions of intercontact gaps. Problems, close to those but dealing with the three-dimensional contact in the axially symmetric case, were solved in our papers [30–32]. The aim of the present paper is to summarize and generalize the theoretical approach suggested in these works. Moreover, some new effects not having been properly analyzed before are discussed.

Formulation of the problem. Consider two dissimilar isotropic elastic semi-infinite solids in frictionless contact due to a uniform normal pressure p applied at infinity. The surface of one of the bodies (body *1* in Fig. 1) contains a local deviation from the plane in the form of a small smooth circular recess with radius b, while the surface of the second body (body *2*) is flat.

Refer the contacting couple to a cylindrical coordinate system (r, θ, z) (see Fig. 1) in such a way that the z-axis coincides with the axis of symmetry and a plane $z = 0$ is the plane of nominal contact.

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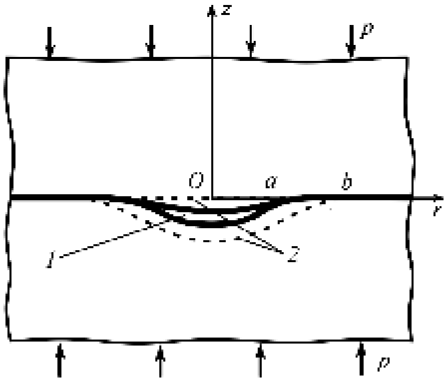

Fig. 1. Contact of two half-spaces with allowance for intersurface gap.

In this system, the shape of the initial surface recess, occupying a region $\{(r, z=0): 0 \le r \le b\}$, is described by a given function $f(r)$.

Due to the presence of the imperfection, the intimate contact of solids occurs not over the whole plane $z = 0$ but in a certain circular region of radius a. Since the initial recess is smooth, the gap location is not fixed and its size depends on the external load (it is found in the process of the problem solution).

Thus, the nominal contact interface $z = 0$ is subdivided into two regions: the region of the gap $\{(r, z=0): 0 \le r \le a\}$ and the region of solid *1* – solid *2* contact $\{(r, z=0): a \le r < \infty\}$.

To solve this contact problem, we utilize the principle of superposition. Extracting in the solution the basic stress-and-strain state formed as a result of frictionless contact of half spaces with plane boundaries under compression (which solution is trivial), the problem under study can be reduced to the determination of perturbations caused by the given local imperfection and the created intersurface gap. The boundary conditions of this perturbed problem are as follows:

$$z = \pm\infty: \qquad \sigma_{zz}^{(i)} = 0, \ \sigma_{rz}^{(i)} = 0, \tag{1}$$

$$z = 0: \qquad \sigma_{rz}^{(i)} = 0, \qquad 0 < r < \infty, \tag{2}$$

$$\sigma_{zz}^{(1)} = \sigma_{zz}^{(2)}, \qquad 0 < r < \infty, \tag{3}$$

$$\sigma_{zz}^{(2)} = p, \qquad 0 < r < a, \tag{4}$$

$$u_z^{(1)} - u_z^{(2)} = f(r), \quad a < r < \infty. \tag{5}$$

Here and in what follows, σ_{zz}, σ_{rz}, σ_{rr}, $\sigma_{\theta\theta}$ are the components of the stress tensor and u_z, u_r are the components of the displacement vector in the case of axial symmetry. Moreover, superscripts (i), $i = 1, 2$ refer the corresponding quantities to bodies 1 and 2, respectively.

Note, that the radius of the gap a is an unknown parameter. The following condition of smooth closure of the gap faces serves as an additional equation for the determination of a:

$$h'(a) = 0, \tag{6}$$

where $h(r) = f(r) + u_z^{(2)}(r,0) - u_z^{(1)}(r,0)$ is the height of the gap. On account of the physical meaning of this function it vanishes outside the gap, i.e.

$$h(r) = 0, \quad a < r < \infty. \tag{7}$$

Procedure of solution. The solution is based on the method of functions of intercontact gaps that was developed in a number of papers by Martynyak and coworkers on plane elasticity. It consists in determining some integral equation for the function of the gap height by means of introducing the interface jump functions for some displacements and stresses.

Adopting the idea of this approach in the axially symmetric case, let us consider an auxiliary problem with the boundary conditions (1), (2), (3) and supplemented with the following condition:

$$u_z^{(1)}(r,0)-u_z^{(2)}(r,0)=f(r)-h(r), \quad 0\le r<\infty \ . \tag{8}$$

The problem of linear elasticity described by boundary conditions (1)–(3) and (8) can be solved by applying the standard technique of Hankel's integral transformations to the equilibrium equations in displacements. Omitting the details of the quite straightforward procedure, we present the final representations of displacements and stresses within the every body via the Hankel transforms of the height gap $h(r)$ and initial recess shape $f(r)$ given as

$$u_r^{(i)}(r,z)=\frac{M}{2m_i\,(1-\nu_i)}\int_0^\infty \xi(1-2\nu_i-\xi\,|\,z\,|)(F(\xi)-H(\xi))e^{-\xi|z|}J_1(\xi r)\,d\xi\,, \tag{9}$$

$$u_z^{(i)}(r,z)=(-1)^{i+1}\frac{M}{2m_i\,(1-\nu_i)}\int_0^\infty \xi(2(1-\nu_i)+\xi\,|\,z\,|)(F(\xi)-H(\xi))e^{-\xi|z|}J_0(\xi r)\,d\xi, \tag{10}$$

$$\frac{\sigma_{rz}^{(i)}(r,z)}{M}=z\int_0^\infty \xi^3(F(\xi)-H(\xi))e^{-\xi|z|}J_1(\xi r)\,d\xi\,, \tag{11}$$

$$\frac{\sigma_{zz}^{(i)}(r,z)}{M}=\int_0^\infty \xi^2[(1+\xi\,|\,z\,|)(F(\xi)-H(\xi))]e^{-\xi|z|}J_0(\xi r)\,d\xi, \tag{12}$$

$$\begin{aligned}\frac{\sigma_{rr}^{(i)}(r,z)}{M}=&\int_0^\infty \xi^2[(1-\xi\,|\,z\,|)(F(\xi)-H(\xi))]e^{-\xi|z|}J_0(\xi r)\,d\xi-\\ &-\int_0^\infty \xi[(1-2\nu_i-\xi\,|\,z\,|)(F(\xi)-H(\xi))]e^{-\xi|z|}\frac{J_1(\xi r)}{r}d\xi,\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\sigma_{\theta\theta}^{(i)}(r,z)}{M}=&\,2\nu_i\int_0^\infty \xi^2(F(\xi)-H(\xi))e^{-\xi|z|}J_0(\xi r)d\xi+\\ &+\int_0^\infty \xi[(1-2\nu_i-\xi\,|\,z\,|)(F(\xi)-H(\xi))]e^{-\xi|z|}\frac{J_1(\xi r)}{r}d\xi\,,\end{aligned} \tag{14}$$

where $H(\xi)=\int_0^\infty \rho\,h(\rho)\,J_0(\xi\rho)\,d\rho$, $F(\xi)=\int_0^\infty \rho\,f(\rho)\,J_0(\xi\rho)\,d\rho$, J_k are the Bessel functions of the first kind of order *k*, $m_i=\mu_i/(1-\nu_i)$, $M=m_1m_2/(m_1+m_2)$, and μ_i,ν_i stand for shear modulus and Poisson's ratio of the body denoted by $i=1,2$.

It is clear that the representations given by (9)–(14) guarantee the validity of the boundary conditions (1)–(3), whereas the last two contact conditions of the original problem (4) and (5) are reduced, after substituting the corresponding expressions (10) and (12) into (4) and (7), to the following system of dual integral equations for finding the Hankel transform $H(\xi)$:

$$\int_0^\infty \xi^2 H(\xi)\,J_0(\xi r)\,d\xi=-\frac{p}{M}+\int_0^\infty \xi^2 F(\xi)\,J_0(\xi r)\,d\xi, \quad 0<r<a, \tag{15}$$

$$\int_0^\infty \xi H(\xi)\,J_0(\xi r)\,d\xi=0, \quad a<r<\infty\,. \tag{16}$$

The technique of its solution is known [33]. Representing the sought function $H(\xi)$ in the form

$$H(\xi) = \xi^{-1} \int_0^a \gamma(\rho) \sin \xi\rho d\rho \,, \tag{17}$$

equations (15)–(16) can be reduced to the Abel integral equation for an unknown function $\gamma(r)$

$$\frac{1}{r} \frac{\partial}{\partial r} \int_0^r \rho \; \gamma(\rho) \Big/ \sqrt{r^2 - \rho^2} \; d\rho = g(r), \tag{18}$$

whose solution is [34]

$$\gamma(r) = \frac{2}{\pi} \int_0^r \rho \; g(\rho) \Big/ \sqrt{r^2 - \rho^2} \; d\rho, \tag{19}$$

where $g(r)$ stands for right hand side of the equation (15).

To complete the present problem, it is necessary to determine the radius of the gap a. For this purpose, we determine the height of the gap $h(r)$ in terms of the function $\gamma(r)$

$$h(r) = \int_r^a \gamma(\rho) \Big/ \sqrt{\rho^2 - r^2} \; d\rho \tag{20}$$

and we see that the condition of smooth closing of the gap (6) is equivalent to the following equation for a:

$$\gamma(a) = 0 \,. \tag{21}$$

Having known the function $\gamma(r)$ and the radius of the gap a, the stresses and displacements within every solid can be found from relations (9)–(14), (17), (19).

The example. Assume that the shape of the initial recess is given by the formula

$$f(r) = h_0 (1 - r^2 / b^2)^{\,n+1/2}, \; 0 \le r \le b \;\; (h_0 = f(0) = b), \tag{22}$$

where n is a natural number. In the paper our consideration is restricted by the cases n = 1, 2, 3. Though the case n = 1 has been considered in paper [30], here some results of this paper are obtained using more general approach.

The Hankel transform $F(\xi)$ of $f(r)$ is [35]

$$F(\xi) = h_0 b^2 \, 2^{n+1/2} \, \Gamma(n + 3/2) (\xi b)^{-(n+3/2)} \, J_{n+3/2}(\xi b) \,. \tag{23}$$

The function $\gamma(r)$ for $n = 1, 2, 3$ (denoted by $\gamma_n(r)$), calculated from (19), are

$$\gamma_1(r) = -\frac{2}{\pi} \left[\frac{p}{M} - \frac{3\pi}{4} \frac{h_0}{b} \left(1 - \frac{r^2}{b^2} \right) \right] r, \;\; \gamma_2(r) = -\frac{2}{\pi} \left[\frac{p}{M} - \frac{15\pi}{16} \frac{h_0}{b} \left(1 - \frac{r^2}{b^2} \right)^2 \right] r,$$
$$\gamma_3(r) = -\frac{2}{\pi} \left[\frac{p}{M} - \frac{35\pi}{32} \frac{h_0}{b} \left(1 - \frac{r^2}{b^2} \right)^3 \right] r. \tag{24}$$

The solution of equation (21), bearing (24) in mind, yields the following values of the gap radius a for n = 1, 2, 3 (denoted by a_n):

$$a_1 = b\sqrt{1-\frac{4pb}{3\pi M h_0}};\ \ a_2 = b\sqrt{1-\left(\frac{16pb}{15\pi M h_0}\right)^{1/2}};\ \ a_3 = b\sqrt{1-\left(\frac{32pb}{35\pi M h_0}\right)^{1/3}}. \quad (25)$$

Now from these expressions it follows that there is a certain level of external load, namely

$$p_1^* = M3\pi h_0/(4b),\ \ p_2^* = M15\pi h_0/(16b),\ \ p_3^* = M35\pi h_0/(32b), \quad (26)$$

at which the radius of the gap becomes zero. It means that for this magnitude of the pressure the gap is closed and the contact of the solids is realized over the whole contact interface $z = 0$.

Fig. 2 illustrates the influence of the ambient compressive pressure on the value of the where the contact is absent. The dependences $\bar{a} = \bar{a}(\bar{p})$ ($\bar{a} = a/b$, $\bar{p} = p/M$) shown in this figure are for the considered cases of shapes of the initial recess (see (22) for $n = 1, 2, 3$). The numerical calculations have been performed for $h_0/b = 10^{-3}$. As can be seen, the radius of the gap monotonically decreases with increasing pressure. The points of intersection of the curves with the abscissa axis determine the values of the pressure when the gap disappears (see (26)). Note that for a small magnitude of applied pressure $\bar{p}$ ("low pressure") the radius of the gap is the largest for the recess, which corresponds to $n = 1$. This tendency does not hold for all load range. When the pressure is sufficiently large, i.e. $\bar{p}$ is larger than $2\cdot 10^{-3}$ ("high pressure") the situation changes and the biggest gap will be for $n = 3$, while the smallest gap will be for $n = 1$.

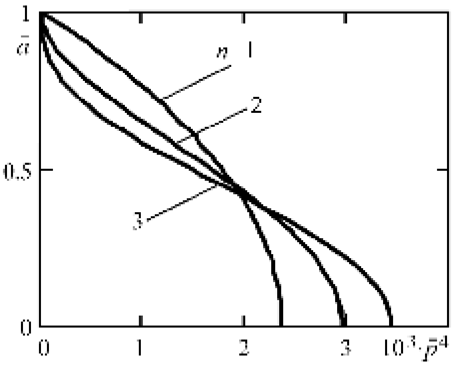

Fig. 2. Dependence of the gap radius on the applied external load for three shapes of the recess.

Now pay attention to the contact parameters of the problem: the height of the gap and the distribution of contact stresses.

The height of the gap can be extracted from formula (20), provided $\gamma(r)$ is given by (24). After calculations of appearing integrals, we obtain for the cases $n = 1, 2, 3$

$$h_1(r) = -\frac{2}{\pi}\left[\left(\frac{p}{M} - \frac{p_1^*}{M}\left(1-\frac{r^2}{b^2}\right)\right)\sqrt{a^2-r^2} + \frac{p_1^*}{3M}\frac{\left(\sqrt{a^2-r^2}\right)^3}{b^2}\right],$$

$$h_2(r) = -\frac{2}{\pi}\left[\left(\frac{p}{M} - \frac{p_2^*}{M}\left(1-\frac{r^2}{b^2}\right)^2\right)\sqrt{a^2-r^2} + \right.$$

$$\left. + \frac{2p_2^*}{3M}\left(1-\frac{r^2}{b^2}\right)\frac{1}{b^2}\sqrt{a^2-r^2} - \frac{p_2^*}{5Mb^4}\left(\sqrt{a^2-r^2}\right)^5\right],$$

$$h_3(r) = -\frac{2}{\pi}\left[\left(\frac{p}{M} - \frac{p_3^*}{M}\left(1-\frac{r^2}{b^2}\right)^3\right)\sqrt{a^2-r^2} + \frac{p_3^*}{Mb^2}\left(1-\frac{r^2}{b^2}\right)^2\left(\sqrt{a^2-r^2}\right)^3 - \right.$$
$$\left. -\frac{3p_3^*}{5Mb^3}\left(1-\frac{r^2}{b^2}\right)\left(\sqrt{a^2-r^2}\right)^5 + \frac{p_3^*}{7Mb^6}\left(\sqrt{a^2-r^2}\right)^7\right]. \quad (27)$$

By eliminating p and p_n^*, $n=1,2,3$ with the aid of (25) and (26) these expressions can be rewritten in the following simple forms:

$$h_1(r) = h_0\left(\frac{a}{b}\right)^3\left(1-\frac{r^2}{a^2}\right)^{3/2},$$
$$h_2(r) = h_0\left(\frac{5}{2} - \frac{3}{2}\frac{a^2}{b^2} - \frac{r^2}{b^2}\right)\frac{a^3}{b^3}\left(1-\frac{r^2}{a^2}\right)^{3/2}, \quad (28)$$
$$h_3(r) = h_0\left(\frac{35}{8} - \frac{21}{4}\frac{a^2}{b^2} + \frac{15}{8}\frac{a^4}{b^4} - \frac{7}{2}\frac{r^2}{b^2} + \frac{3}{2}\frac{a^2r^2}{b^4} + \frac{r^4}{b^4}\right)\frac{a^3}{b^3}\left(1-\frac{r^2}{a^2}\right)^{3/2}.$$

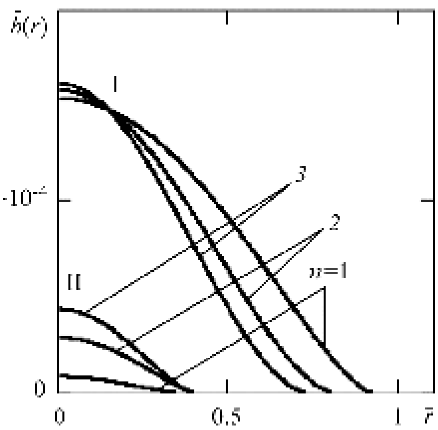

Fig. 3. The shapes of half the gap for two prescibed loads (I – $10^3 \cdot \bar{p} = 0.382$; II – 2.078) and the cases n = 1, 2, 3.

Fig. 3 demonstrates the variations of the gap height $\bar{h}(r) = h(r)/b$ $(\bar{r} = r/b)$ under some external loads. We can see that the gap becomes smaller with increasing pressure.

Now, analyze contact normal stress distribution. To do this, let us substitute (19) into (12). As a result, we find after straightforward manipulation

$$\frac{\sigma_{zz}^{(i)}(r,0)}{M} = -\frac{p}{M} + \int_0^\infty \xi^2 F(\xi) J_0(\xi r)\,d\xi - \frac{1}{r}\frac{d}{dr}\int_0^{\min(r,a)} \rho\,\gamma(\rho)\Big/\sqrt{r^2-\rho^2}\,d\rho\,. \quad (29)$$

Computation of the corresponding integrals in the above formula, by using relations (23) and (24), is performed to yield [35]:

$$\int_0^\infty \xi^2 F(\xi) J_0(\xi r)\,d\xi = \quad (30)$$

$$n=1: \quad \frac{1}{M}p_1^* \begin{cases} 1 - \dfrac{3}{2}\dfrac{r^2}{b^2}, & r<b, \\ \dfrac{2}{\pi}\left[\arcsin\dfrac{b}{r}\left(1-\dfrac{3}{2}\dfrac{r^2}{b^2}\right) + \dfrac{3}{2}\dfrac{1}{b}\sqrt{r^2-b^2}\right], & r>b, \end{cases}$$

$$n=2:\quad \frac{1}{M}p_2^*\begin{cases}1-\dfrac{3r^2}{b^2}+\dfrac{15}{8}\dfrac{r^4}{b^4}, & r<b,\\[2mm] \dfrac{2}{\pi}\left[\arcsin\dfrac{b}{r}\left(1-\dfrac{3r^2}{b^2}+\dfrac{15}{8}\dfrac{r^4}{b^4}\right)+\left(\dfrac{7}{4}-\dfrac{15}{8}\dfrac{r^2}{b^2}\right)\dfrac{\sqrt{r^2-b^2}}{b}\right], & r>b,\end{cases}$$

$$n=3:\quad \frac{1}{M}p_3^*\begin{cases}1-\dfrac{9}{2}\dfrac{r^2}{b^2}+\dfrac{45}{8}\dfrac{r^4}{b^4}-\dfrac{35}{16}\dfrac{r^6}{b^6}, & r<b,\\[2mm] \dfrac{2}{\pi}\left[\arcsin\dfrac{b}{r}\left(1-\dfrac{9}{2}\dfrac{r^2}{b^2}+\dfrac{45}{8}\dfrac{r^4}{b^4}-\dfrac{35}{16}\dfrac{r^6}{b^6}\right)+\right.\\[2mm] \left.+\left(\dfrac{23}{12}-\dfrac{25}{6}\dfrac{r^2}{b^2}+\dfrac{35}{16}\dfrac{r^4}{b^4}\right)\dfrac{\sqrt{r^2-b^2}}{b}\right], & r>b,\end{cases}$$

$$\frac{1}{r}\frac{d}{dr}\int_0^{\min(r,a)}\rho\,\gamma(\rho)\Big/\sqrt{r^2-\rho^2}\,d\rho= \tag{31}$$

$$n=1:\quad \begin{cases}-\dfrac{1}{M}p+\dfrac{1}{M}p_1^*\left(1-\dfrac{3}{2}\dfrac{r^2}{b^2}\right), & r<a,\\[2mm] \dfrac{2}{\pi}\left[-\dfrac{1}{M}p\arcsin\dfrac{a}{r}+\dfrac{1}{M}p_1^*\left(\arcsin\dfrac{a}{r}\left(1-\dfrac{3}{2}\dfrac{r^2}{b^2}\right)+\dfrac{3}{2}\dfrac{a}{b^2}\sqrt{r^2-a^2}\right)\right], & r>a;\end{cases}$$

$$n=2:\quad \begin{cases}-\dfrac{1}{M}p+\dfrac{1}{M}p_2^*\left(1-3\dfrac{r^2}{b^2}+\dfrac{15}{8}\dfrac{r^4}{b^4}\right), & r<a,\\[2mm] \dfrac{2}{\pi}\left[-\dfrac{1}{M}p\arcsin\dfrac{a}{r}+\dfrac{1}{M}p_2^*\left(\arcsin\dfrac{a}{r}\left(1-3\dfrac{r^2}{b^2}+\dfrac{15}{8}\dfrac{r^4}{b^4}\right)+\right.\right.\\[2mm] \left.\left.+\dfrac{\sqrt{r^2-a^2}}{b}\left(\dfrac{3a}{b}-\dfrac{5}{4}\dfrac{a^3}{b^3}-\dfrac{15}{8}\dfrac{ar^2}{b^3}\right)\right)\right], & r>a;\end{cases}$$

$$n=3:\quad \begin{cases}-\dfrac{1}{M}p+\dfrac{1}{M}p_3^*\left(1-\dfrac{9}{2}\dfrac{r^2}{b^2}+\dfrac{45}{8}\dfrac{r^4}{b^4}-\dfrac{35}{16}\dfrac{r^6}{b^6}\right), & r<a,\\[2mm] \dfrac{2}{\pi}\left[-\dfrac{1}{M}p\arcsin\dfrac{a}{r}+\dfrac{1}{M}p_3^*\left(\arcsin\dfrac{a}{r}\left(1-\dfrac{9}{2}\dfrac{r^2}{b^2}+\dfrac{45}{8}\dfrac{r^4}{b^4}-\dfrac{35}{16}\dfrac{r^6}{b^6}\right)+,\right.\right.\\[2mm] \left.\left.+\dfrac{\sqrt{r^2-a^2}}{b}\left(\dfrac{9}{2}\dfrac{a}{b}-\dfrac{15}{4}\dfrac{a^3}{b^3}-\dfrac{45}{8}\dfrac{ar^2}{b^3}+\dfrac{7}{6}\dfrac{a^5}{b^5}+\dfrac{35}{24}\dfrac{a^3r^2}{b^5}+\dfrac{35}{16}\dfrac{ar^4}{b^5}\right)\right)\right] & r>a;\end{cases}$$

Thus, the contact parameters of the problem, namely the gap height and the normal contact stresses, are found analytically for the three considered particular shapes of the initial recess n = 1, 2, 3.

Distributions of the normal compressive stresses $\bar{\sigma}_{zz}^2(r,0) = \sigma_{zz}^2(r,0)/M$ along the contact interface for five cases of the gap radius are presented in Fig. 4 for the cases n = 1, 2, 3. It is seen that the contact pressures $-\bar{\sigma}_{zz}^2(r,0)$ are zero within the gap region, first they increase with the distance from the gap reaching their maxima in the vicinity of the initial recess edge, and then decrease monotonically approaching to the level of the external load p. For the case n = 1 the maximal value of the normal contact pressure is attained exactly at the point $\bar{r} = 1$ on the whole range of loading, while for cases n = 2 and n = 3 the points of maximum are located toward the center of the gap. Moreover, for n = 1 the contact pressure exhibits the peak at the point $\bar{r} = 1$, which corresponds to the edge of the initial defect. Such a behavior is not observed for the remaining cases $n = 2, 3$. It can be explained by the fact that the curvature of smooth curves given by (22) is discontinues at the point $\bar{r} = 1$ (i.e. r = b) only for n = 1. It is worth noting that some similar aspects of the influence of the curvature discontinuity in the surface profiles on the contact pressure distribution in two-dimensional frictionless contact problems have been discussed in [36].

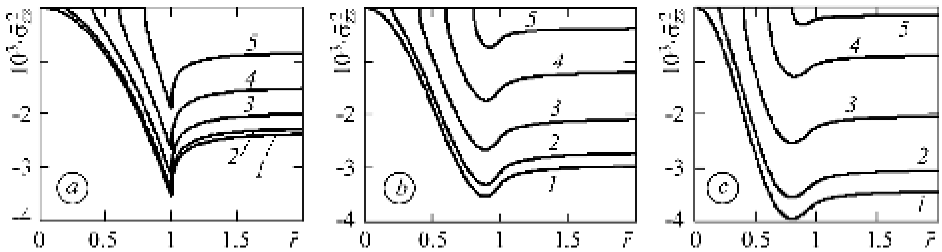

Fig. 4. Contact normal stress distributions (a – n = 1; b – n = 2; c – n = 3).
1 – $\bar{a} = 0$; *2* – 0.2; *3* – 0.4; *4* – 0.6; *5* – 0.8.

CONCLUSIONS

The method of functions of intercontact gaps is extended to axially symmetric elasticity contact problems of two half-spaces one of which contains a circular surface recess is presented. This method implies: *i*) construction of the integral representations of the displacements and stresses within every of mated solids by functions defined on the unknown but bounded region of the contact interface (for the case considered, only one such a function – the height of the intersurface gap); and *ii*) reduction of the problem to dual integral equations for Hankel transform of these functions. Analytical solutions are obtained for some shapes of the initial recess. The results for the contact characteristics are discussed and presented graphically. For some recess shape, the contact normal stresses exhibit peaks at the point which corresponds to the initial defect edge. This distinguished feature is associated with the specific geometry of the surface defect, namely with the discontinuity of the curvature.

РЕЗЮМЕ. Узагальнено метод міжконтактних зазорів для розв'язання осесиметричних контактних задач теорії пружності для півбезмежних тіл з локальним геометричним збуренням поверхні. Підхід передбачає побудову інтегральних подань напружень та переміщень у кожному з тіл контактної пари через функції, що визначені на локальній області поверхні контакту. Для безфрикційної взаємодії тіл за наявності однієї кругової виїмки відповідні подання отримані через одну таку функцію, зокрема, висоту міжконтактного просвіту, і задачу зведено до парних інтегральних рівнянь. Розв'язок останніх отримано в аналітичному вигляді для певних профілів початкового дефекту. Проаналізовано контакт-

ні параметри задачі. Встановлено, що через розрив кривини поверхні у функції розподілу нормальних контактних напружень з'являються піки.

РЕЗЮМЕ. Обобщен метод межконтатных зазоров для решения осесимметричных контактных задач теории упругости для полубесконечных тел с локальным геометрическим возмущением поверхности. Подход предполагает построение интегральных представлений перемещений и напряжений в каждом из взаимодействующих тел контактной пары через функции, определенные на локальной области поверхности сопряжения. Для бесфрикционного взаимодействия при наличии одной круговой выемки соответствующие представления получены через одну такую функцию, в частности, высоту межконтактного зазора, и задача сведена к парным интегральным уравнениям. Решение последних получено в аналитическом виде для некоторых профилей начального дефекта. Проанализированы контактные параметры задачи. Выявлено, что пики в распределении нормальных контактных напряжений обусловлены разрывом кривизны поверхности.

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