MODELING OF FATIGUE CRACK GROWTH IN PLATES UNDER ARBITRARY MODE I STRESS

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Possible simplifications in fatigue crack growth modeling of cracks in plates under arbitrary Mode I stress distribution were investigated using WF2D method based on 2-D weight function. It is important if analytical methods are used, where some restrictions regarding stress distribution and crack shape exist. The performed analyses revealed a small impact of the idealized initial crack shape (aspect ratio) used to replace the actual defect shape. They also confirmed that averaging of stress distribution over the entire crack plane may be a reasonable simplification in analytical crack growth analysis.

Keywords: *fatigue crack, plates, stress distribution, mode I, defects.*

Mechanical and structural components very often contain crack-like defects or cracks initiated due to cyclic loading in operation. In most cases cracks initiate at the surface of components (surface cracks), usually at stress concentration areas, whereas internal defects are modeled as embedded (sub-surface) cracks. Both types of crack-like defects are very common for welded joints of structural components (Fig. 1*a*). Therefore fatigue life of welded joints is very often limited by the crack growth to the critical crack size, whereas crack initiation period is neglected.

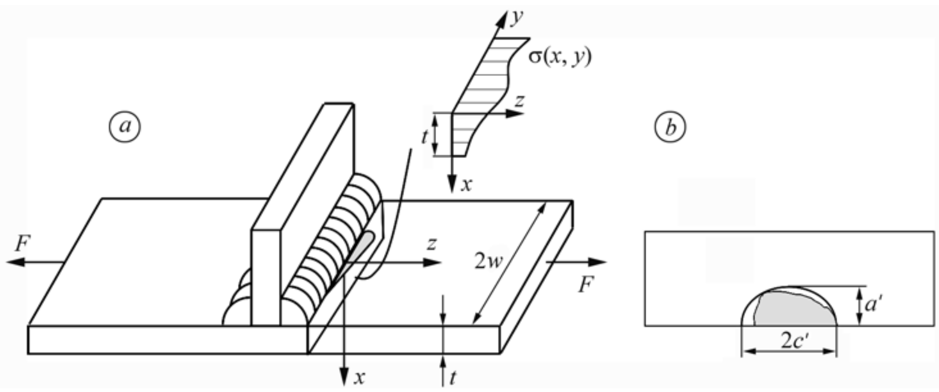

Fig. 1. Crack in structural component under arbitrary stress distribution (*a*) and the simplified model (*b*).

Assessment of fatigue behavior of mechanical components containing crack-like defects is needed in damage tolerant fatigue design of components and structures. Calculations based on analytical methods are usually used for that purpose, mainly due to quick and, in most cases, reasonable predictions.

Crack-like defects in structural components have usually irregular contour shape and/or undergo a non-uniform stress distribution (e.g. varying along x and z axes in Fig. 1*a*). Analysis of crack growth in mechanical components with such cracks needs either numerical methods (BEM, FEM), which are time consuming and require some experience, or analytical methods.

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Analysis of crack growth with the use of analytical methods requires usually simplification regarding the crack shape, component geometry, and stress distribution. The reason is the limited possibility of analytical models used for calculation of stress intensity factors (SIF). An engineering practice in crack growth predictions performed using such models is simplification regarding the stress distribution over the prospective crack surface, which may lead to very conservative life assessments [1].

The purpose of this paper is to propose reasonable simplifications in crack growth modeling and analysis for cracks propagating in plates under mode I loading [2]. The previous work [1] shows that averaging the stress over component cross-section containing the crack provides reasonable results.

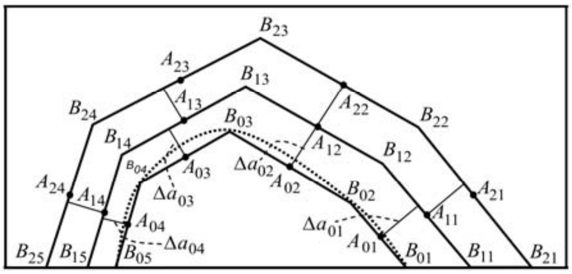

Fig. 2. The idea of crack growth analysis using WF2D.

The analyses were performed using the developed method (WF2D) for crack growth analysis of planar cracks under arbitrary Mode I loading. The method is based on the point-load (2-D) weight function used for the calculation of stress intensity factors. The point-load weight function was developed by Glinka and Reinhard [3]. It makes possible to calculate stress intensity factors at arbitrarily selected points on the actual crack contour of planar convex cracks under any Mode I stress distribution. The actual crack contour is replaced by a number of rectilinear segments, and SIFs are calculated at a mid point of each of them (Fig. 2).

The actual crack contour (dotted line) is replaced by a number of rectilinear segments (the appropriate number of segments for semi-elliptical cracks is 20...30), and SIFs are calculated at a mid point, $A_i(x, y)$ of each of them. Details of the crack growth method based on the 2-D weight function are presented in [2] and [4].

Having calculated SIFs for points $A_i(x, y)$, the crack extension (increment) Δa_i for each segment can be calculated based on the crack growth rate data and effective range of stress intensity factor, ΔK_{Ai}, corresponding to a considered loading cycle. It is assumed that the increment Δa_i describes the amount of a parallel shift of the i-th segment of the crack contour due to ΔN load cycles (Fig. 2). For the cycle-by-cycle crack growth analysis the increment $\Delta N = 1$, however, it can be increased for faster analysis without losing too much of the accuracy [2] of fatigue crack growth life prediction.

Crack extensions are then calculated for each of these points and the new crack shape is determined. Finally the crack growth limiting condition is met and the number of load cycles is determined. Application of the method to crack growth analysis may significantly reduce the calculation time.

Validation of the developed method [2], [4] indicates that fatigue life obtained for semi-elliptical cracks using the proposed method is conservative by ca. 10...20% depending on the shape of the initial crack.

Modeling of actual shape cracks. Crack-like defects in structural components usually have irregular contour shapes (Fig. 1), which for analytical methods have to be replaced by idealized shapes. The replacement can be made in many different ways, some rules can be found in reference [5]. They are usually conservative; however, it is difficult to assess magnitude of the conservatism a priori.

The method based on 2-D weight function was used to perform analyses showing the effect of an idealized crack shape on the predicted crack growth. Two types of cracks were analyzed: an embedded crack (Fig. 3*a*) and a surface crack (Fig. 3*b*).

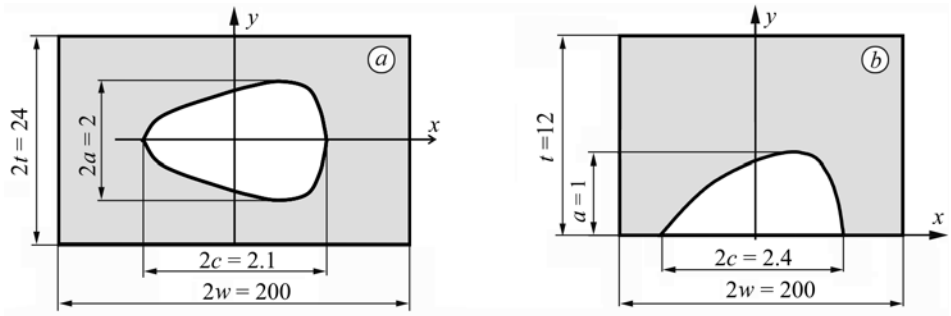

Fig. 3. Actual flaw shapes: *a* – embedded crack; *b* – surface crack.

The idealized crack shapes for the actual contour (KR) of dimensions *a* and *c* were selected as follows (Fig. 4):

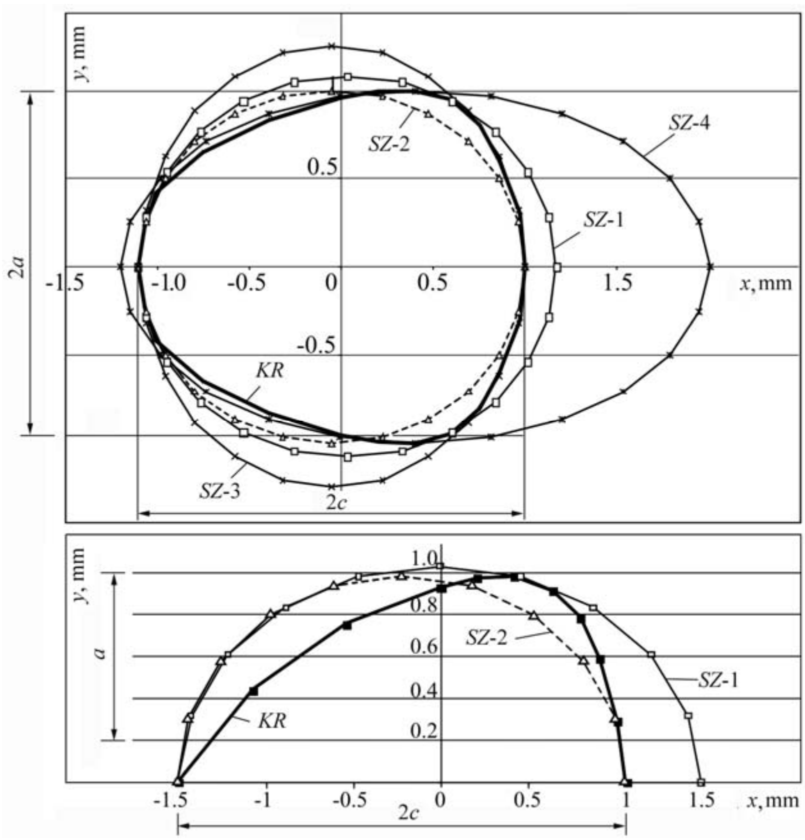

Fig. 4. Actual flaws (KR) and idealized crack shapes (SZ): (—— – KR; □ – SZ-1; △ – SZ-2; –×– – SZ-3; –✳– – SZ-4) (*a*) and (■ – KR; □ – SZ-1; ▲ – SZ-2) (*b*).

1. The smallest crack (SZ-1) covering the actual shape. Dimensions of the ideal crack are $a' \geq a$, and $c' \geq c$.

2. The smallest crack denoted SZ-2 of dimensions $a' = a$, and $c' = c$.

3. Crack covering the actual shape, but containing one of its dimensions: $c' = c$ (SZ-3), or $a' = a$ (SZ-4). Those replacements are made for the embedded crack only.

Crack growth to reach depth of $a = 0.8t$ was performed for the actual crack contour and the idealized shapes using the WF2D method. For all analyses a uniform stress distribution was assumed. The results are shown in Fig. 5 where the bars corresponding to idealized crack shapes show relative life change in respect to that calculated for the actual crack shape (KR).

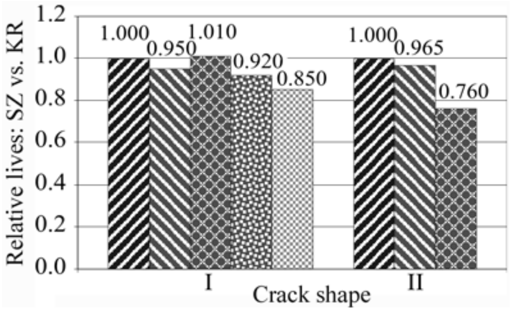

Fig. 5. Comparison of relative fatigue lives for actual flaws (KR) and idealized crack shapes (SZ): – KR; – SZ-1; – SZ-2; – SZ-3; – SZ-4. I – embedded; II – semi-elliptic.

As it was expected, the bigger the idealized crack, the shorter was the calculated life in comparison to life of the actual crack. This is especially visible for the embedded crack. However, for the semi-elliptical surface defect both idealized cracks yielded shorter lives. The interesting observation on those cracks is the shorter life obtained for crack denoted SZ-2, which is bigger than the actual defect. The possible explanation for such behavior is the difference in curvature of both cracks. The curvature of the actual defect is bigger at the localization close to the deepest point. Therefore the actual crack grows slowly in the vertical direction, as the SIF magnitude decreases with the curvature on the crack contour.

It is worth noting that the final crack shapes of both idealized cracks were very similar, i.e. for the final crack depth their aspect ratio a/c was almost the same.

Modeling of complex fatigue growth of cracks. The capability of the crack growth method based on the WF2D weight function can be useful for the analysis of complex growth of cracks in mechanical components, where significant change of crack shape is observed. In such cases a transition from one crack model to another has to be considered, if analytical methods are used for crack growth analysis. In addition, if such transition is necessary, stress distribution has to be modified as well.

Such behavior of growing cracks may especially occur when a shape of structural component is complicated or one of dimensions is much larger than the other (thin plate). An example of such complex crack growth can be observed in a plate, where an initial semi-elliptical crack grows through the plate thickness in the first stage, and then transforms to a through crack.

If analytical methods are used for such crack analysis, the stress distribution at the first stage may change along the y-axis, whereas it must be (is considered as) uniform along the x-axis. When the crack breaks through the plate thickness, the further crack growth analysis using those methods allows for the change of stress along the x-axis (symmetrically with respect to the y-axis), however, they assume a uniform stress distribution along the y-axis. Furthermore, a replacement of the actual crack shape with an equivalent one and estimation of the size of the initial through crack, $2a$, after transition, are needed.

Considering the described simplifications necessary for analytical methods of complex crack growth analysis, one may conclude that fatigue life prediction using those methods may be inaccurate in such case.

The developed method of crack growth based on the WF2D does not require any of simplifications mentioned above. Examples of such complex crack growth analyses of a semi-elliptical crack in a finite plate using the developed method are presented below.

The objective was to find out, how big is the difference in predicted fatigue life if one makes simplified assumptions regarding the trough crack size (initial crack for stage II analysis) and/or the stress distribution. The comparisons were performed for uniform and two arbitrary stress distributions (linear), shown in Fig. 6.

Notation of crack and component dimensions is shown in Fig. 6. A semi-elliptical crack of initial dimensions $a = 1$ mm and $c = 1$ mm in a rectangular plate of width $2w = 200$ mm and thickness $t = 12$ mm was considered.

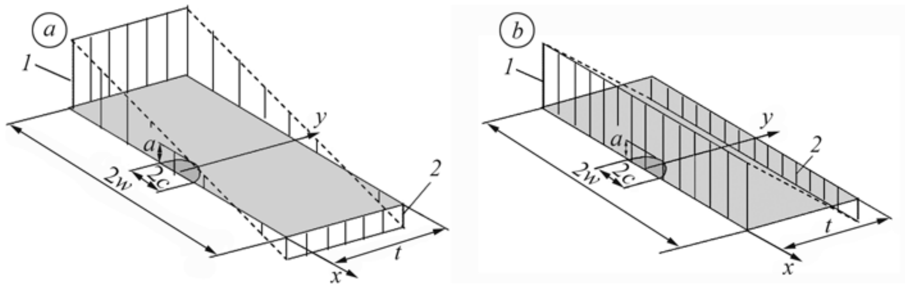

Fig. 6. Crack dimensions and two (*a*, *b*) arbitrary (linear) stress distributions: *1* – $\sigma/\sigma_{ref} = 1$; *2* – $\sigma/\sigma_{ref} = -0.111$.

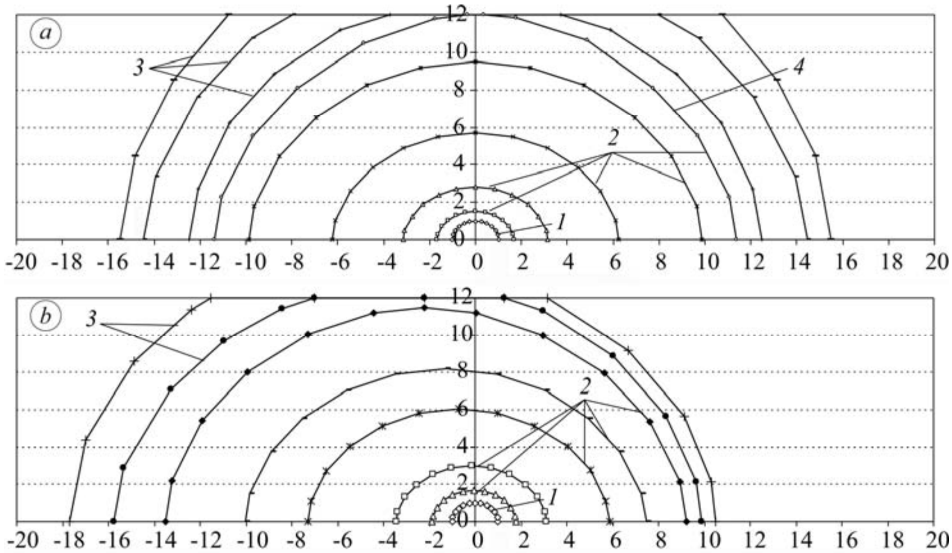

Fig. 7. Modeling of complex crack growth under uniform (*a*) and arbitrary (*b*) stresses distribution: *1* – initial crack; *2* – stage 1; *3* – stage 2; *4* – final semi-elliptical crack.

All calculations presented below were performed using the developed method WF2D in order to eliminate differences between various methods and concentrate on the problem of simplifying the crack contour and/or stress distribution.

First, the same initial semi-elliptical crack in the rectangular plate was considered under arbitrary stress distribution (linear decrease along the x-axis) (Fig. 6*a*). The crack extension under the non-uniform stress distribution modeled as a continuous crack growth with the actual crack contour is presented in Fig. 7*b*. The analysis was performed cycle-by-cycle, but for a better visibility, only some selected crack contours are shown in the diagram. One may observe that as the through crack progresses, the number of linear segments on the crack contour decreases, and the crack shape becomes more rectangular. The crack extension is not symmetrical; however, the crack shape remains semi-elliptical. As the through crack progresses the number of linear segments on the crack contour decreases unevenly on both sides of the crack.

The subsequent analysis was aimed at examining the effect of simplifying the stress distribution on the predicted fatigue life. Such simplification is necessary for life prediction using analytical methods, which do not allow for calculations using stress distribution shown in Fig. 6*a*. The arbitrary stress distribution must be replaced by a uniform distribution.

The crack extension under the uniform stress distribution modeled as a continuous crack growth with the actual crack contour is presented in Fig. 7*a*. Crack extension is symmetrical in that case; however, an interesting observation is that the size $2a$ of the final semi-elliptical crack for stage 2 is the same as for the non-uniform stress distribution.

Since there is no an accepted method for calculating the equivalent stress magnitude, a conservative assumption is usually made that the uniform stress is equal to the maximum stress of the arbitrary stress distribution. However, the equivalent uniform stress distribution should be such that the calculated life equals to that obtained for arbitrary stress distribution.

Therefore, another assumption, tested here is that magnitude of the equivalent stress equals to the average value of the arbitrary stress calculated over the entire considered cross-section of the component. In the considered case magnitude of the average stress is $\sigma_{av} = 0.45\ \sigma_{ref}$.

Finally, fatigue life predicted for the actual crack contour and actual stress distribution (AS) was compared with lives obtained for two magnitudes of the equivalent uniform stress:

- the maximum value, $\sigma = \sigma_{ref}$ (ES-a1), and
- the average value, $\sigma = \sigma_{av}$, (ES-a2).

In the crack growth analyses performed for the equivalent stress distribution the average equivalent through crack for stage II was used. The initial equivalent through crack length was calculated to give the same area as the final semi-elliptical crack, i.e. $2a = \pi \cdot c/2$.

Comparison of the life predicted for non-uniform stress distribution and actual crack contour with lives for uniform stress distributions is presented in Fig. 8*a*. Fatigue life predicted for the equivalent stress equal to the maximum value of the non-uniform stress is very conservative, as it is 10 times smaller than the life predicted for the actual stress distribution, whereas life predicted for the uniform stress of the average value of the actual stress differs by ca. 5%.

The following analysis was related to studying the effect of simplifying the stress distribution shown in Fig. 6*b* on the predicted fatigue life. In this case crack growth of a semi-elliptical crack (stage I) does not require any simplification of the stress distribution. Such simplification is necessary for life prediction using analytical methods, when the semi-elliptical crack transforms into the through crack (II stage of crack growth). At this stage of crack growth the stress distribution along the thickness must be constant, so the arbitrary stress distribution must be replaced by a uniform distribution.

Similarly to the previous case, a conservative assumption can be made that the uniform stress is equal to the maximum stress of the arbitrary stress distribution. Another assumption, tested here, was that magnitude of the equivalent stress equals to the average value of the arbitrary stress over the tensile portion of the considered component cross-section. In the considered case the magnitude of the average stress is $\sigma_{av} = 0.45\ \sigma_{ref}$.

In this case fatigue life was also compared with lives obtained for two magnitudes of the equivalent uniform stress:

- the maximum value, $\sigma = \sigma_{ref}$ (ES-b1), and
- the average value, $\sigma = \sigma_{av}$, (ES-b2).

Comparison of life predicted for non-uniform stress distribution and actual crack contour with lives for uniform stress distributions is presented in Fig. 8*b*. Total lives were calculated as a sum of stage I (same for all calculations) and stage II of the crack growth. Total fatigue life predicted for the equivalent stress equal to the maximum value of the non-uniform stress is ca 10% smaller than value calculated for the actual case, whereas life predicted for the uniform stress of the average value of the actual stress (ES-b2) differs by ca. 3%. The relatively small difference in predicted lives is due to the fact, that life spent on growth of a semi-elliptical crack was prevailing in the performed analyses. The differences in lives spent at stage II of crack growth (through crack) are much bigger: 80% for the maximum stress (ES-b1-II) and 24% for the average stress (ES-b2-II).

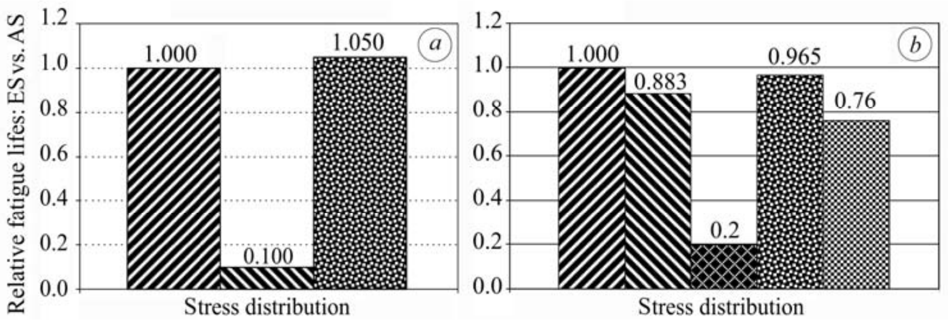

Fig. 8. Comparison of life prediction for arbitrary (AS) and equivalent (ES) stress distributions:
a: – AS; – ES-a1; – ES-a2;
b: – AS; – ES-b1; – ES-b2; – ES-b2-II; – ES-b2-II.

Generally, the difference in lives obtained in continuous and two-stage crack growth modeling depends on crack and/or component shapes and stress distribution. The observed effect of modeling the through crack may be different (bigger) for longer growth of a through crack. Such effect can be expected for thin components, where stage I of crack growth may be short in comparison to stage II crack growth.

The obtained results show how big can be the conservatism of life prediction for inaccurate assessment of the equivalent stress distribution. One may note that this is a usual engineering practice in crack growth predictions performed using analytical methods. A reasonable alternative can be using the average stress distribution, however, in some cases this can be not possible, e.g. for purely bending loading the average stress value equals to zero. Therefore it is suggested to calculate the equivalent stress based on the tensile portion of the stress distribution only.

CONCLUSIONS

Crack growth predictions using the method based on the point-load weight function (WF2D) for calculation of stress intensity factors are presented in the paper. The usefulness of this method in comparison to analytical methods is especially visible while modeling a complex crack growth in arbitrary non-linear stress fields. The advantage of the developed method is predicting the crack growth life in CPU time significantly shorter to that needed by numerical methods.

The presented results show that modeling of the transition between the semi-elliptical and through crack growth in plates, which is needed in classical methods, does not require much concern, since assumption regarding the initial through crack length does not affect the predicted life significantly.

On the other hand, care must be taken when simplifying the non-uniform stress distribution. The presented examples show that the conservative assumption in that respect, which is usual in engineering practice, may result in a very conservative life assessment. However, in a general case this conservatism in life prediction depends on the non-uniform stress distribution, and is difficult to estimate.

Based on the presented results of fatigue life assessments for semi-elliptical crack growth, one may conclude that suggested replacement of arbitrary stress distribution in analytical methods of life prediction can yield more realistic life assessments. Using the uniform stress distribution of the average value of the non-linear stress is promising; however, this conclusion needs confirmation in more extensive research.

РЕЗЮМЕ. Спрогнозовано розвиток тріщини за методом, що ґрунтується на ваговій функції (WF2D) обчислення інтенсивності напруження. Його перевага порівняно з аналітичними у тому, що моделюють складний розвиток тріщини в довільних нелінійних полях напружень. Встановлено, що моделювання переходу від напівеліптичного до прямого розвитку тріщини в пластинах (необхідне в класичних методах) не потребує особливих зусиль, тому що пропоновані довжини початкової прямої тріщини істотно не впливають на прогнозовану міцність. За результатами визначення міцності під час розвитку півеліптичної тріщини можна зробити висновки, що доцільніше замінити довільний розподіл напружень, використовуючи аналітичні методи прогнозування.

РЕЗЮМЕ. Спрогнозировано развитие трещины с помощью метода, основанного на весовой функции (WF2D) вычисления интенсивности напряжений. Его преимущество в сравнении с аналитическими в том, что моделируется сложное развитие трещины в произвольных нелинейных полях напряжений. Выявлено, что моделирование перехода от полуэллиптического к прямому развитию трещины в пластинах (необходимое в классических методах) не требует особых усилий, потому что предлагаемые длины начальной прямой трещины существенно не влияют на предсказываемую прочность. За результатами определения прочности при развитии полуэллиптической трещины можно сделать выводы, что целесообразнее заменить произвольное распределение напряжений с помощью аналитических методов прогнозирования.

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