UDC

CORRELATION OF UNIAXIAL CYCLIC TORSION AND TENSION-COMPRESSION FOR LOW-CYCLE FATIGUE

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Comparison of fatigue characteristics, in particular the coefficients of tension-compression and cyclic torsion based on available tests under low-cycle fatigue in various materials, were analyzed. The correlation of torsion and tension-compression fatigue strength coefficient does not depend on the relative slope of fatigue diagrams describing plastic strain. On the basis of performed analyses it was concluded that the ratio of the fatigue strength coefficients under tension-compression in most materials varies within the range from 0.5 to $\frac{1}{1+\nu}$. Whereas, the correlation of the strain-based fatigue coefficients in torsion and in tension-compression is strongly dependent on the relative slope of the plastic strain-based fatigue life curves.

Keywords: *LCF, strain, amplitude, fatigue strength.*

A majority of modern stress, strain or the so-called multiaxial fatigue life energy-based criteria are based on the process of determining the equivalent values of stress, strain or energy parameter, respectively. In order to evaluate fatigue strength, the knowledge on the basic characteristics of fatigue is required. In literature simplified methods for determining fatigue characteristics for uniaxial tension-compression are used. More and more often it is also required to be acquainted with the characteristic of pure shear condition, which is usually obtained in cyclic torsion testing of thin-walled specimens. The searched magnitudes are: σ'_f, τ'_f – tension-compression or shear fatigue ductility coefficient, *b*, b_0 – fatigue strength exponent and ε'_f, γ'_f – fatigue ductility coefficient in tension-compression or torsion, *c*, c_0 – fatigue ductility exponent, respectively. These coefficients are used in many fatigue life assessment models. The characteristics of torsion are required in each case, when the multiaxial fatigue criteria are applied by which the shear strain amplitude [7, 11, 17] or the energy criterion [7] is derived as the equivalent value. At the same time, the correlations of fatigue strength coefficients in tension-compression or torsion, and in tension-compression or shear fatigue ductility coefficients are applied in [42, 51]. In literature the proposals can be seen for simple conversion from tension-compression to torsion loading. However, a cursory review of reference works shows that these models often fail. The objective of this research study is a comparison of fatigue characteristics, in particular of the coefficients of these characteristics under tension-compression and cyclic torsion, based on available tests on low-cycle fatigue in various materials.

Strain-based model. Fundamental low-cycle fatigue properties are based on the Manson–Coffine–Basquin strain characteristic, which links the total strain amplitude with the number of cycles to failure. This characteristic is the most popular and commonly applied. This original Manson–Coffine–Basquin characteristic for fatigue life was designed in tension-compression, registering the strain amplitude ε_a, the stress

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amplitude σ_a and the number of cycles to failure N_f and has the following form

$$\varepsilon_a = \frac{\sigma'_f}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c, \tag{1}$$

where E is the longitudinal modulus of elasticity (Young's modulus); σ'_f is fatigue strength coefficient in tension-compression; b is fatigue strength exponent; ε'_f is fatigue ductility coefficient; c is fatigue ductility exponent.

The Manson–Coffine relationship for fatigue in torsion (shear) is similar to Eq. (1)

$$\gamma_a = \frac{\tau'_f}{G}(2N_f)^{b_0} + \gamma'_f(2N_f)^{c_0}, \tag{2}$$

where G is the shear modulus (Kirchhoff's modulus); τ'_f is shear fatigue strength coefficient; b_0 is shear fatigue strength exponent; γ'_f is shear fatigue ductility coefficient; c_0 is shear fatigue ductility exponent.

All fatigue characteristics in Eqs. (1) and (2) are derived on the basis of ASTM standard [2].

A deep review of fatigue characteristics can be found among others in [36]. The formula proposed in [23] requires determination of four material constants just as in the commonly used MCB characteristic.

In the fatigue characteristics of Eqs. (1) and (2), the correlation between parameters appearing in these formulas is worth to notice. Constants are employed in numerous computational models applied for determining fatigue strength of construction materials and components. The correlation of strain amplitudes under biaxial torsion and tension-compression can be designated. This ratio varies according to the type of material. This correlation is presented variously by different authors. There is no single, universally accepted model. Commonly, authors assume parallelism of fatigue characteristic of Eqs. (1) and (2), which means that exponents are equivalent, both in the elastic portion $b = b_0$ and in the plastic portion $c = c_0$. This assumption is crucial for determining other dependences. In previous works [24] it was demonstrated that for a high number of cycles, where the reference is made to the elasticity of the material, most materials have characteristics of the parallel type. However, it was demonstrated that if this assumption is not true, there are obstacles in proper designation of fatigue strength. Whereas, any analysis concerning the correctness of similar assumption that characteristics related to the plastic strain are also parallel, was previously not distinguished.

The correlations of the strain-based fatigue coefficients ε'_f and γ'_f are most frequently analyzed. Less frequently analyzed are the correlations of fatigue strength coefficients σ'_f and τ'_f. In the second case, Li et al. [28] suggest the Huber–Mises–Hencky hypothesis and Galileo's respectively

$$\frac{\tau'_f}{\sigma'_f} = \frac{1}{\sqrt{3}}, \qquad \frac{\tau'_f}{\sigma'_f} = \frac{1}{1+\nu} \tag{3), (4}$$

or the contents

$$\frac{\tau'_f}{\sigma'_f} = \frac{1}{2}. \tag{5}$$

Therefore, most of the results should be in the range of

$$\frac{1}{2} < \frac{\tau'_f}{\sigma'_f} < \frac{1}{1+\nu}. \tag{6}$$

For strain fatigue coefficients ε'_f and γ'_f more analyses can be found in the research literature. There are proposals formulated by:

– Kim et al. [21] depending on the adopted criterion:

Huber–Mises–Hencky

$$\frac{\gamma'_f}{\varepsilon'_f} = \sqrt{3}\,, \tag{7}$$

Tresca

$$\frac{\gamma'_f}{\varepsilon'_f} = 1.5\,; \tag{8}$$

– Shamsaei and Fatemi [41] propose formulas (7) and (8), and for maximum normal strain

$$\frac{\gamma'_f}{\varepsilon'_f} = 2\,; \tag{9}$$

– Kim and Park [20] suggest general formula in the form

$$\frac{\gamma'_f}{\varepsilon'_f} = 1.5 + 0.5S\,, \tag{10}$$

where S depends on the material type;

– Liu and Mahadevan [31] suggest that the correlation is within the range of

$$\sqrt[2]{\frac{(1+\nu_{\text{eff}})^2}{4-(1-\nu_{\text{eff}})^2}} < \frac{\gamma'_f}{\varepsilon'_f} < 2\,, \tag{11}$$

which means that by adopting perfect plasticity, that is $\nu_{\text{eff}} = 0{,}5$, it finally gives

$$1.5492 < \frac{\gamma'_f}{\varepsilon'_f} < 2\,. \tag{12}$$

Comparison of material constants subjected to tension-compression and torsion. On the basis of the available research literature, material constants present in the Manson–Coffine–Basquin fatigue characteristics under torsion (shear) and tension-compression are listed in the Table. In some cases, there is a lack of material constants for tension-compression tests. For this purpose these constants were derived for the same materials from the collective work [4]. Due to the fact, as it was already noted in other works [32–34], that possible nonparallelism of fatigue characteristics is significant in fatigue strength calculations for high-cycle fatigue loading, it should be assumed, that the same is true for the low number of cycles. Based those assumptions, Figs. 1 and 2 summarize respectively the correlation of torsion fatigue strength and tension-compression fatigue strength coefficient, depending on the relative slope

$$b_w = \frac{b-b_0}{b}\cdot 100\% \tag{13}$$

and the ratio of torsion and tension-compression fatigue ductility coefficient according to the relative slope

$$c = \frac{c-c_0}{c}\cdot 100\%\,. \tag{14}$$

By analyzing Fig. 1 it can be seen that the correlation of torsion fatigue strength and tension-compression fatigue strength is not dependent on the relative slope of fatigue life curves describing elastic deformation. This relationship for most materials varies within the range of 0.5 to $\frac{1}{1+\nu}$. Whereas, by analyzing Fig. 2 it can be noticed that the ratio of torsion and tension-compression fatigue ductility coefficient depends primarily on the relative slope of fatigue curves describing ductility. This ratio ranges from almost 0 to about 15 and can be described by the dependence

$$\frac{\gamma'_f}{\varepsilon'_f} = 1.788 \cdot e^{-2^{(c-c_0)/c}} . \quad (15)$$

This in the case of perfect parallelism leads to $\frac{\gamma'_f}{\varepsilon'_f} = 1.788$ which is consistent with formula (12).

It should be noted that the type of materials does not affect the relative slopes of (13) and (14) as well as the dependences searched.

A list of low-cycle fatigue behaviour of material constants under tension-compression and biaxial torsion

Material	σ'_f	τ'_f	b	b_0	ε'_f	γ'_f	c	c_0
	MPa							
1	2	3	4	5	6	7	8	9
1045 [47]	948	505	–0.092	–0.097	0.26	0.413	–0.445	–0.445
1045 [11]	1027	424	–0.107	–0.074	0.335	0.325	–0.494	–0.42
SAE 1045 [50]	948	505	–0.09	–0.1	0.26	0.413	–0.44	–0.44
SNCM630 [20]	1270	858	–0.073	–0.061	1.54	1.51	–0.823	–0.706
SNCM439 [20]	1380	969	–0.072	–0.085	1.89	3.68	–0.801	–0.765
SCM440 [20]	1400	754	–0.088	–0.081	0.675	0.315	–0.65	–0.54
SCM435 [20]	1100	512	–0.067	–0.045	0.996	0.36	–0.708	–0.519
SFNCM85S [20]	1040	533	–0.092	–0.071	0.316	0.251	–0.522	–0.406
SF60 [20]	978	504	–0.082	–0.067	0.187	0.286	–0.439	–0.417
45 [20]	843	559	–0.105	–0.108	0.327	0.496	–0.546	–0.469
S45C [20]	1400	630	–0.107	–0.08	0.449	1.22	–0.564	–0.564
S25C [20]	821	426	–0.096	–0.074	0.216	0.249	–0.458	–0.376
S45C [21]	932	451	–0.098	–0.058	0.359	0.704	–0.519	–0.514
S45C [22]	923	685	–0.099	–0.12	0.359	0.198	–0.519	–0.12
S460N [3]	1005	476	–0.097	–0.075	0.142	0.316	–0.483	–0.469
S460N [44]	834	529	–0.079	–0.096	0.157	0.213	–0.493	–0.096
SNCM630 [15]	1272	858	–0.073	–0.061	1.54	1.51	–0.823	–0.706
42CrMo [6]	860	817	–0.105	–0.102	0.317	3.212	–0.546	–0.852
316 [51]	722	506	–0.13	–0.13	0.377	0.6381	–0.63	–0.563
304 [48]	1227	333			0.14	0.266		
304 [47]	930	505	–0.106	–0.097	0.298	0.413	–0.49	–0.445
304 [43]	1000	709	–0.114	–0.121	0.171	0.413	–0.402	–0.353
304 [37]	691	1137	–0.169	–0.215	0.101	1.055	–0.377	–0.566
304 (650K) [38]	529	275	–0.112	–0.097	0.092	0.132	–0.428	–0.356
Mild steel [9]	1009	431			0.152	0.322		
Mild steel [16]	1735	407	–0.2	–0.103	0.04	0.176	–0.333	–0.403
Inconel 718 [3]	1640	2164	–0.06	–0.148	2.67	18	–0.82	–0.922
Inconel 718 [50]	1640	1030			2.67	3.62		

Continuation of the Table

1	2	3	4	5	6	7	8	9
Haynes 188 (760K) [50]	823	635			0.489	1.78		
IN–718 [45]		2146		–0.148		18		–0.922
Hayness 188 (538K) [24]	1045	548	–0.071	–0.053	0.358	0.805	–0.541	–0.523
Waspaloy [50]	2610	1640			0.381	0.516		
1Cr–18Ni–9Ti [7]	1124	644	–0.091	–0.088	0.807	0.812	–0.665	–0.088
1Cr–Mo–V [32]	1616	740	–0.12	–0.16	1.568	2.389	–0.9	–0.9
30CrNiMo8HH [3]	951	608	–0.041	–0.057	1.064	0.277	–0.733	–0.47
4340 (34CrNiMo6) [14]	1206	1975	–0.095	–0.08	0.536	0.364	–0.568	–0.724
A533B [35]	847	586	–0.083	–0.115	1.201	1.554	–0.64	–0.615
AZ31B [1]	616	144	–0.149	–0.12	0.419	0.131	–0.791	–0.429
EN8 [39]	728	466	–0.051	–0.058	0.095	0.303	–0.328	–0.322
EN24 [49]	1650	1150			1.14	1.69		
Ni–Cr–Mo–V [50]	680	444			1.14	1.69		
Titanium TC4 [3]	1117	716.9	–0.049	–0.06	0.579	2.44	–0.679	–0.8
Titanium [40]	647	485	–0.033	–0.069	0.548	0.417	–0.646	–0.523
Titanium BT9 [40]	1180	881	–0.025	–0.082	0.278	0.18	–0.665	–0.47
Titanium BT1-0 [12]	693	484	–0.041	–0.064	0.477	9.995	–0.617	–0.906
Mar-M247LC (1173K) [42]	1425	802	–0.137	–0.087	0.004	0.019	–0.311	–0.387
6061-T6 [29]	369	285	–0.311	–0.05	0.09	0.388	–0.452	–0.642
6061-T6 [30]	373	245	–0.033	–0.048	0.104	1.475	–0.473	–0.675
2007 [52]	271	158	–0.069	–0.099	0.645	0.3779	–0.721	–0.555
7075-T6 [52]					0.078	0.269	–0.256	–0.32
7075-T6 [14]	776	210	–0.095	–0.037	2.565	0.919	–0.987	–0.173
1100-0 [14]	159	62	–0.092	–0.126	0.467	7.049	–0.613	–0.599

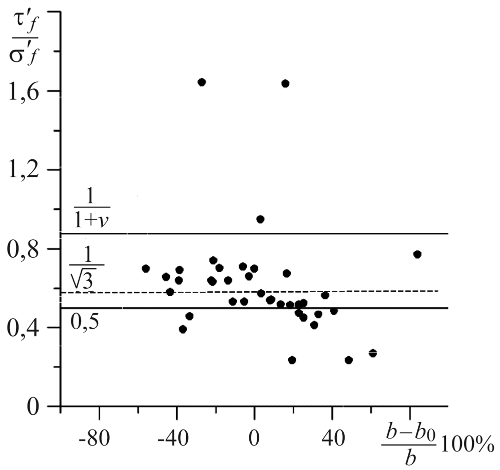

Fig. 1.

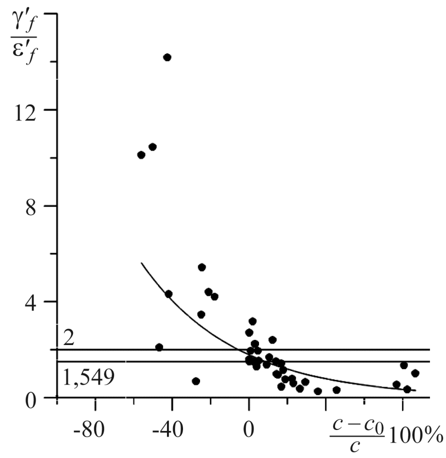

Fig. 2.

Fig. 1. Torsion and tension-compression strength coefficient vs. relative slope.

Fig. 2. Correlation of torsion and tension-compression fatigue ductility coefficient vs. relative slope.

CONCLUSIONS

The correlation of torsion fatigue strength coefficient and tension-compression fatigue strength coefficient is not dependent on the relative slope of fatigue resistance curves describing elastic strain; the correlation of torsion and tension-compression fatigue strength coefficient for most of the materials varies within the range from 0.5 to $\frac{1}{1+\nu}$; the correlation of torsion and tension-compression fatigue ductility coefficient is strongly dependent on the relative slope of fatigue life curves describing plastic strain.

The ratio of tension-compression and shear fatigue ductility coefficients is ranging from 0 to ~15 and can be described by the dependence $\frac{\gamma_f}{\varepsilon'_f} = 1.788 \cdot e^{-2^{(c-c_0)/c}}$; the ratio of tension-compression and torsion (shear) fatigue ductility coefficients explicitly for characteristics of parallel type is within the range given in the research literature and is on average 1.788.

РЕЗЮМЕ. Порівняно характеристики втоми, зокрема коефіцієнти розтягу–стиску та циклічного кручення на основі досліджень за малоциклової втоми для різних матеріалів. Співвідношення коефіцієнтів втомної міцності за умов кручення та розтягу–стиску не залежить від відносного нахилу кривих пластичної деформації. На основі виконаних аналізів зроблено висновок, що відношення коефіцієнтів втомної міцності за умов кручення та розтягу–стиску у більшості матеріалів є в межах від 0,5 до $\frac{1}{1+\nu}$. Водночас співвідношення коефіцієнтів втомної міцності за умов кручення та розтягу–стиску суттєво залежить від умовного нахилу кривих втомної довговічності.

РЕЗЮМЕ. Сравнены характеристики усталости, в частности коэффициенты растяжения–сжатия и циклического кручения на основе исследований при малоцикловой усталости для различных материалов. Соотношение коэффициентов усталостной прочности в условиях кручения и растяжения–сжатия не зависит от относительного наклона кривых пластической деформации. На основе проведенных анализов сделан вывод, что отношение коэффициентов усталостной прочности в условиях кручения и растяжения–сжатия в большинстве материалов находится в пределах от 0,5 до $\frac{1}{1+\nu}$. В то же время соотношение коэффициентов усталостной прочности в условиях кручения и растяжения–сжатия существенно зависит от условного наклонения кривых усталостной долговечности.

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