DURABILITY ASSESSMENT OF BONE CEMENT PLATES WITH ELLIPTICAL SHAPE DEFECTS: EFFECT OF CONFIGURATION AND POSITION OF DEFECT

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The length critical values of ellipse-like defects in plates made of bone cement manufactured with various additives are assessed. The analysis was made also using previously studied samples subjected to different soaking periods (0; 3; 6 months) in simulated body fluid. This research is focussed on the behaviour of samples made of bone cement containing different additives: diethylaminoethylacrylate, dimethylaminoethylmethacrylate and diethylaminoethylmethacrylate. The models generated at present allow the determination of the critical length of defects from stress intensity factor values. This in turn made it possible to calculate the critical number of loading cycles in specific mechanical elements, containing ellipse-like defects. This class of calculations could be the basis for engineering recommendations for design and risk assessment of real elements, manufactured from this material.

Keywords: *bone cement, crack defect, different geometry, stress intensity factor, engineering monitoring.*

Bone cements have been used clinically for a number of years in orthopaedic surgery for the fixation of artificial joints with encouraging results [1]. Currently several research groups around the world work to improve the mechanical strength and biocompatibility of bone cements for real prosthesis applications in human beings. Several research approaches to assess critical values for bone cement components fracture have been reported [2]. The effect of the incorporation of comonomers containing amine groups on the mechanical and fracture properties of acrylic bone cements were also considered in [2]. Bone cements were prepared with diethylaminoethylacrylate (DEAEA), either diethylaminoethylmethacrylate (DEAEM), either dimethylaminoethylmethacrylate (DMAEM) as comonomers in the liquid phase. The mechanical and fracture properties of cements were also evaluated after soaking the specimens in simulated body fluid (SBF) for 0; 3 and 6 months [2, 3].

Usually structural elements made of bone cements can take diverse forms depending upon the requirements of specific applications for prosthesis. Proper application of fracture mechanics requires a reliable criterion for determining the growth of defects such as ellipse-like defects. This issue has received much attention around the world with studies on the influence of critical parameters determining such criteria. However, this question has been not fully developed, since there are not international standard procedures for determining the criteria for linear and nonlinear mechanics of the bone cements fracture [4].

There are theoretical solutions, which can estimate the stress distribution ahead of the ellipse-like defect but they are very complex and cumbersome. In engineering practice, theoretical models represent considerable difficulties and are not always applicable.

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Therefore, approximate solutions using the relatively simple models could represent excellent simulation approaches to establish criterion for engineering estimations of structural elements and components that contain defects [5].

This paper presents a model for calculations of lifetime of plates with different configurations that contain ellipse-like defects [6]. Based on the results of these studies, this paper proposes recommendations for engineering estimations for bodies of ellipse-like defects of various forms and geometry. Establishing the criteria and evaluation of strength of materials near different defects from the computational models requires experimentally established laws and basic data on the interaction of local strain and working environment. It is important to establish a relationship of physicomechanical and physicochemical parameters of the system “material–environment” and its role in the materials fracture process.

Experimental. Bone cement plates 3×10×75 mm of size containing ellipse-like defects of different form were analysed. Using the stress intensity factors values obtained in [2] (see Table 1) and analytic models of structural elements [6], some models for the calculation of critical length values of the defect are proposed.

Table 1. Values of K_{fc} (MPa Ч$\sqrt{\text{m}}$) in a simulated body fluid of acrylic bone cements with different soaking time in SBF [2, 3]

Material	wt.%	Soaking time in SBF, months		
		0	3	6
DEAEA	4	1.54	1.62	1.76
	6	1.66	1.68	2.03
DMAEM	4	1.76	1.75	1.00
	6	1.75	1.15	1.41
DEAEM	4	1.36	1.54	1.96
	6	1.44	1.66	1.89

Three cases were considered: a plate under tension with a semielliptical surface crack (case I); a plate under tension with a quarterelliptical corner crack (case II) and a plate under tension with an embedded elliptical crack (centred) (case III). Based on these models the critical values a_{fc} of the defect length are shown for three materials. The experimental bone cements were formulated by adding the liquid component to the solid (powder) component at room temperature (25°C). The powder solid components consisted of nictone polymethylmethacrylate beads, benzoyl peroxide and barium sulfate ($BaSO_4$) while the liquid component consisted of methylmethacrylate (MMA) (as the base monomer), dimethyl propiothetin and either DEAEA, DMAEM or DEAEM at 4 and 6 wt.%; these quantities were incorporated by a partial replacement of MMA in the liquid phase.

The analytical expressions for calculating the stress intensity factor (SIF) for each model on which the critical defect length was identified are as follows.

Defect I: Plate under tension with a semi-elliptical surface crack [6, 7] (Fig. 1*a*):

$$K_0 = \sigma F \sqrt{\pi a}\,, \qquad (1)$$

$$F = \frac{M_1 + M_2 (a/t)^2 + M_3 (a/t)^4}{\sqrt{Q}} F_w; \quad F_w = \sqrt{\frac{1}{\cos\left(\frac{\pi c}{2W}\sqrt{a/t}\right)}};$$

for $a/c \le 1$: $Q = 1 + 1.464(a/c)^{1.65}$; $M_1 = 1.13 - 0.09(a/c)$; $M_2 = -0.54 + \dfrac{0.89}{0.2 + a/c}$;

$$M_3 = 0.5 - \frac{1}{0.65 + a/c} + 14(1 - a/c)^{24};\ K_{\mathrm{I}a} = K_0,\ K_{\mathrm{I}c} = K_0 (1.1 + 0.35(a/t)^2)\sqrt{a/c};$$

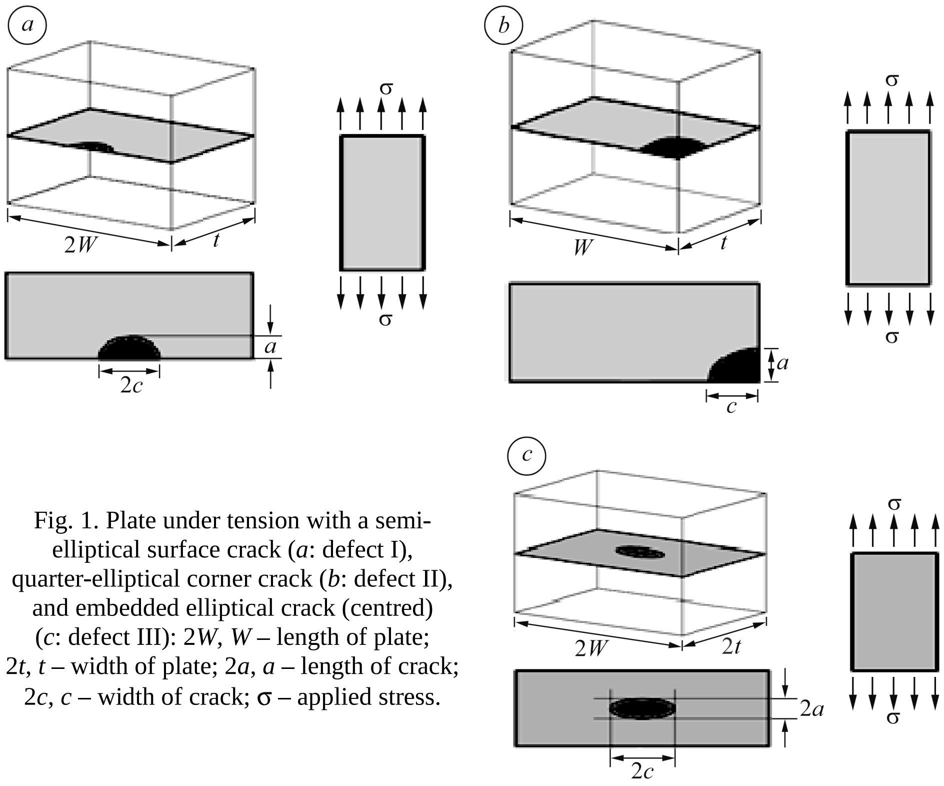


Fig. 1. Plate under tension with a semi-elliptical surface crack (*a*: defect I), quarter-elliptical corner crack (*b*: defect II), and embedded elliptical crack (centred) (*c*: defect III): $2W$, W – length of plate; $2t$, t – width of plate; $2a$, a – length of crack; $2c$, c – width of crack; σ – applied stress.

Defect II: Plate under tension with a quarter-elliptical corner crack [6, 7] (Fig. 1*b*):

$$K_{\mathrm{I}a}=K_{\varphi}(\varphi=\pi/2)\,; \qquad (2)$$

$$K_{\varphi}=\sigma F\sqrt{\frac{\pi a}{Q}};\quad F=\left[M_1+M_2\left(\frac{a}{t}\right)^2+M_3\left(\frac{a}{t}\right)^4\right]g_1 g_2 f_{\varphi}\,.$$

For $\frac{a}{c}\le 1$: $f_{\varphi}=\left[(a/c)^2\cos^2\varphi+\sin^2\varphi\right]^{1/4}$; $M_1=1.08-0.03a/c$; $M_2=-0.44+\frac{1.06}{0.3+a/c}$; $M_3=-0.5+0.25\frac{a}{c}+14.8(1-a/c)^{15}$; $Q=1+1.464(a/c)^{1.65}$; $g_1=1+\left[0.08+0.4(a/t)^2\right](1-\sin\varphi)^3$; $g_2=1+\left[0.08+0.15(a/t)^2\right](1-\cos\varphi)^3$.

Defect III: Plate under tension with an embedded elliptical crack (centred) [6, 7] (Fig. 1*c*)

$$K_{\mathrm{I}a}=FF_w\sigma\sqrt{\pi a}\;; \qquad (3)$$

$$F=\frac{M_1+M_2(a/t)^2+M_3(a/t)^4}{\sqrt{Q}};\quad F_w=\sqrt{\frac{1}{\cos\left(\frac{\pi c}{2W}\sqrt{\frac{a}{t}}\right)}};$$

$$Q=1+1.464(a/c)^{1.65};\ M_1=1;\ M_2=\frac{0.05}{0.11+(a/c)^{3/2}};\ M_3=\frac{0.29}{0.23+(a/c)^{3/2}}.$$

Result and discussion. Fig. 2 presents the critical length of the defect of type I as a function of immersion time in SBF in bone cement samples manufactured with 4 and 6 wt.% since they are the most realistic cases for the exploitation of the investigated materials.

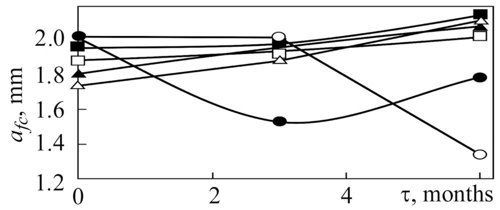


Fig. 2. Critical length of the defect I with dependence on soaking time for materials DEAEA (□ – 4, ■ – 6 wt.%), DMAEM (△ – 4, ▲ – 6 wt.%) and DEAEM (○ – 4, ● – 6 wt.%) in cases 20 MPa for $a/c = 0.5$.

A general trend in the behaviour of the critical length of the ellipse-like defect, depending on the soaking time can be observed in Fig. 4. Results obtained for material DMAEM presented high dispersion to be considered for the calculation of the critical defect length of bone cement samples subjected to fatigue conditions.

Samples of material DEAEA (Fig. 3) presented the safest condition which is 6 wt.% with 6 months of soaking, although the difference is not large enough, and the remaining cases are in approximately the same plane with minimum dispersion. It is worth mentioning that the most dangerous form of the defect for all cases under consideration is that with the ratio $a/c = 0.1$. The best situation for the critical length of an ellipse-like defect in samples with 6 wt.% and 6 months of soaking is for the ratio $a/c = 0.75$.

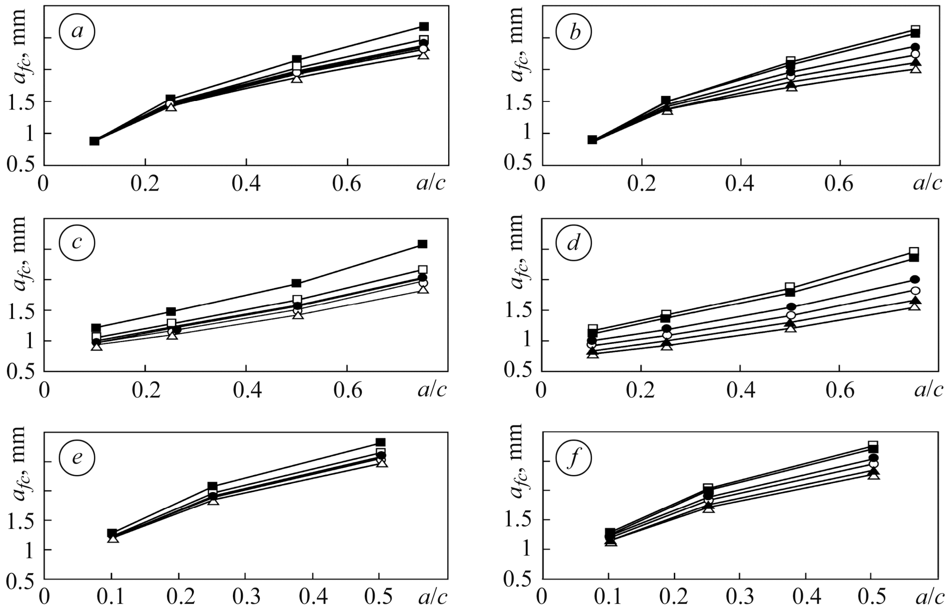


Fig. 3. Critical length of the defect (*a*, *b*: defect I), (*c*, *d*: defect II) and (*e*, *f*: defect III) depending on a/c for materials DEAEA (*a*, *c*, *e*) and DEAEM (*b*, *d*, *f*) in case 20 MPa:
△ – 4 wt.%, 0 months; ▲ – 6 wt.%, 0 months; ○ – 4 wt.%, 3 months;
● – 6 wt.%, 3 months; □ – 4 wt.%, 6 months; ■ – 6 wt.%, 6 months.

Material DEAEM (Fig. 3*a*, *b*) presented a similar trend, but in this case the form of the defect affects the critical length of the ellipse-like defect as in the case of DEAEA. The most secure case is for samples with 4 wt.% and 6 months of immersion in the SBF, although the difference of a_{fc} values for samples with 4 and 6 wt.% is small.

For the defect of type II a general trend of the behaviour of the ellipse-like defect critical length as a function of the a/c ratio is observed as shown in Fig. 3*c*, *d*. Critical a_{fc} values depending on the form of the defect presented similar trend as for the defect of type I with that difference that the gap between the safest cases is higher than in the previous model, as in the case of DEAEA and DEAEM, when the difference between 4 and 6 wt.%, is small, but is larger than in the previous model.

Defect of type III presents a trend of increasing the critical length of the ellipse-like defect as a function of the a/c ratio and of the time of immersion in the SBF as shown in Fig. 3*e*, *f*. Regarding the consideration of the critical values depending on the form of the defect both trends are similar to the case of defect II with that difference that the gap between the safest cases almost disappears and the critical length of the ellipse-like defect in materials DEAEA and DEAEM after 6 months of immersion in SBF and 4 and 6 wt.% respectively are nearly identical.

Using the fracture mechanics approach the values of the critical length of the ellipse-like defect for engineering evaluation can be proposed. It takes into account the above cases I–II described in [6].

For structural element with a crack-like defect of assigned form and location the dimensionless dependences can be presented as

$$(\sqrt{t}/\sigma)\cdot(dK_\mathrm{I}/da)=F(a/t)\,, \tag{4}$$

where t is the size of structural element in the defect growth direction; σ is applied external load. The above relationship indicates that there is some defect size $(a/t)_*$ from which the variation rate of SIF K_I significantly increases. This size was considered a characteristic for the assessment of strength and reliability of the structural elements with crack-like defects. A defect for which the value of $(a/t)_*$ is the lowest is accepted as the most hazardous from the fracture risk point of view.

Tables 2–4 show the length of the ellipse-like defects for different immersion time in SBF and different *a*/*c* ratio. When the defect reaches the critical value it will spontaneously and unstably grow, which is likely to lead to the object fracture. Values obtained with the three different approaches for the calculation of the ellipse-like defect critical length are also presented here.

Table 2. Values of a_{fc} in the SBF with defect I of acrylic bone cements with different soaking time in SBF for 4 and 6 wt.%

Defect I	DEAEA			DEAEM		
a/*c* = 0.1	0 months	3 months	6 months	0 months	3 months	6 months
4	0.87775	0.88042	0.88487	0.86974	0.87775	0.88932
6	0.8822	0.88309	0.8911	0.8733	0.8822	0.88843
a/*c* = 0.25	0 months	3 months	6 months	0 months	3 months	6 months
4	1.27 (a_*)	1.27 (a_*)	1.27 (a_*)	1.27 (a_*)	1.27 (a_*)	1.27 (a_*)
6	1.27 (a_*)	1.27 (a_*)	1.27 (a_*)	1.27 (a_*)	1.27 (a_*)	1.27 (a_*)
a/*c* = 0.5	0 months	3 months	6 months	0 months	3 months	6 months
4	1.88615	1.94112	2.02477	1.74275	1.88615	2.12037
6	1.96502	1.97936	2.14905	1.81206	1.96502	2.0893
a/*c* = 0.75	0 months	3 months	6 months	0 months	3 months	6 months
4	2.24054	2.32725	2.4 ($a_{0.8}$)	2.01629	2.24054	2.4 ($a_{0.8}$)
6	2.36612	2.38705	2.4 ($a_{0.8}$)	2.12094	2.36612	2.4 ($a_{0.8}$)

Note: a_* – by $(a/t)_*$; $a_{0.8}$ – by standard approaches of fracture mechanic.

The reliability of the mathematical models for calculation of SIF in cases I, II and III depends critically upon the definition of the characteristic value a_* for the same model using the original SIF and the classic approaches of fracture mechanics with a length of defect $a_{0.8}$ at the specific zone of stress distribution. These three values chosen minimum, represent a critical length of the ellipse-like defect for the particular cases of the present study. The situation is that for the same objects but with defects of different form and geometry it is necessary to apply different criteria.

Table 3. Values of a_{fc} in the SBF with defect II of acrylic bone cements with different soaking time in SBF for 4 and 6 wt.%

Defect II	DEAEA			DEAEM		
$a/c = 0.1$	0 months	3 months	6 months	0 months	3 months	6 months
4	0.93971	0.9899	1.07355	0.81543	0.93971	1.18349
6	1.0138	1.02575	1.21695	0.8704	1.0138	1.14525
$a/c = 0.25$	0 months	3 months	6 months	0 months	3 months	6 months
4	1.1094	1.17632	1.241 (a_*)	0.94927	1.1094	1.241 (a_*)
6	1.20978	1.22651	1.241 (a_*)	1.02336	1.20978	1.241 (a_*)
$a/c = 0.5$	0 months	3 months	6 months	0 months	3 months	6 months
4	1.43444	1.45 (a_*)	1.45 (a_*)	1.22412	1.43444	1.45 (a_*)
6	1.45 (a_*)	1.45 (a_*)	1.45 (a_*)	1.31972	1.45 (a_*)	1.45 (a_*)
$a/c = 0.75$	0 months	3 months	6 months	0 months	3 months	6 months
4	1.115 (a_*)	1.115 (a_*)	1.115 (a_*)	1.115 (a_*)	1.115 (a_*)	1.115 (a_*)
6	1.115 (a_*)	1.115 (a_*)	1.115 (a_*)	1.115 (a_*)	1.115 (a_*)	1.115 (a_*)

Note: a_* – by $(a/t)_*$.

Table 4. Values of a_{fc} in the SBF with defect III of acrylic bone cements with different soaking time in SBF for 4 and 6 wt.%

Defect III	DEAEA			DEAEM		
$a/c = 0.1$	0 months	3 months	6 months	0 months	3 months	6 months
4	1.20548	1.22472	1.25284	1.15072	1.20548	1.2854
6	1.2336	1.23804	1.29576	1.17588	1.2336	1.27504
$a/c = 0.25$	0 months	3 months	6 months	0 months	3 months	6 months
4	1.437 (a_*)	1.437 (a_*)	1.437 (a_*)	1.437 (a_*)	1.437 (a_*)	1.437 (a_*)
6	1.437 (a_*)	1.437 (a_*)	1.437 (a_*)	1.437 (a_*)	1.437 (a_*)	1.437 (a_*)
$a/c = 0.5$	0 months	3 months	6 months	0 months	3 months	6 months
4	1.699 (a_*)	1.699 (a_*)	1.699 (a_*)	1.699 (a_*)	1.699 (a_*)	1.699 (a_*)
6	1.699 (a_*)	1.699 (a_*)	1.699 (a_*)	1.699 (a_*)	1.699 (a_*)	1.699 (a_*)

Note: a_* – by $(a/t)_*$.

It was also supposed that the crack growth rate diagrams [8] fully define the materials resistance to defects propagation. They are presented analytically using the well-known Paris equation:

$$da/dN = C(\Delta K_{\mathrm{I}})^n, \tag{5}$$

where C and n are the constants of the "material–environment" system. Here parameters are calculated as $n = 7.07$ and $C = 3{\cdot}10^{-4}$ [9].

Here are the examples of calculations of residual durability of structural elements with defects under cyclic loading in working environments. Calculations are performed based on the well-known Eq. (6), which provides conditions for spontaneous fracture of structural elements [6]:

$$N_{fc} = \int_{a_{th}}^{a_{fc}} \frac{da}{F(\Delta K_{\mathrm{I}})}, \tag{6}$$

where N_{fc} is the number of loading cycles to fracture of structural elements; a_{fc} is given in Tables 2–4 and a_{th} is calculated from conditional $da/dN = 10^{-10}$.

Analysis of defects I and II under cyclic loading 20 MPa made of material DMAEM, gives some interesting results. For example if we consider only the critical length of the defect, we can see on defect I a gradual increase in the critical length with increasing ratio of the defect axes (Fig. 4). This trend appears after considering for the calculation all cases of the defects critical length. As for defect II, there is a maximum length of the defect, which becomes critical for the *a*/*c* ratio 0.5 and some decrease in this parameter is observed (Fig. 4).

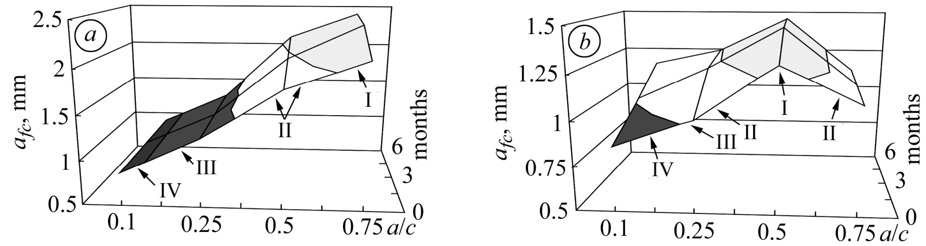
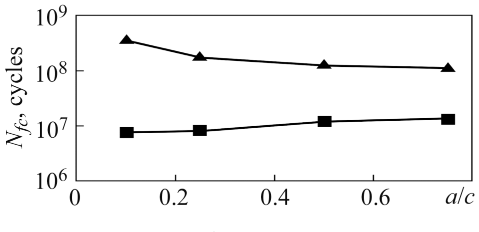


Fig. 4. Diagrams of the defect critical length in case of defect I (*a*) and defect II (*b*), depending on *a*/*c* and time of soaking for DEAEM with 20 MPa and 6 wt.%. *a*: I – 2…2.5, II – 1.5…2, III– 1…1.5, IV – 0.5…1; *b*: I – 1.25…1.5, II – 1…1.25, III– 0.75…1; IV – 0.5…0.75.

Calculation of the number of cycles to achieve the critical defect length indicates that the cases analysed produce somewhat different results (Fig. 5). In defect of type I, the maximum number of cycles is obtained for the *a*/*c* ratio 0.1 and becomes the minimum for the *a*/*c* ratio 0.75. This fact suggests that there is some slowdown defect. This result indicates that additional defects monitoring with lower *a*/*c* ratios are required, because the development of such defects can be much faster than the defects with a higher ratio.

Fig. 5. Number of critical loading cycles for defects I (▲) and II (■) with dependence on *a*/*c* for DEAEM with 20 MPa and 6 wt.%.



In defect II there is a gradual increase in the number of loading cycles with increasing the defect ratio *a*/*c* (Fig. 5). These facts may indicate the adequate control of bodies with defects when the values of the critical length of the ellipse-like defect and the number of loading cycles are known. The knowledge of the ellipse-like defect critical length allows us to conclude that there is a potential threat of real defects appearance in the components and structures. By setting the number of loading cycles it is possible to make predictive assessments of the studied elements lifetime.

CONCLUSIONS

Various cases of structural elements containing defects are simulated. On the basis of these models of the SIF calculation and experimental results the critical length of defects were identified and analysed. This procedure is performed for five structural elements. Here the cases of defects of various form and geometry are considered. Analysis of the data shows the specific defects in the materials which is the safest. It has been also designed for some specific cases, such as values of a_* (the characteristic length of the ellipse-like defect after which a faster defect growth is probable) and also another value of the critical length $a_{0.8}$. Here the critical number of loading cycles is calculated in the case of material DEAEM (with 6 wt.%). These data can be used for engineering recommendations of the test material and concrete structural elements, manufacture form this material. As the most important conclusion we can mention that the best material to consider is DEAEM. It should also be noted that the second case is more unpredictable, because here the defect develops much faster.

РЕЗЮМЕ. Обчислено критичну довжину тріщини у пластинах з лікувального цементу з додаванням різних домішок. Використано зразки, які заздалегідь просочували (0; 3; 6 місяців) у модельованому розчині для людського тіла. Досліджено конструкційні елементи, виготовлені з лікувального цементу з додаванням дієтиламінометилакрилату, диметиламіно-метилметакрилату та дієтиламінометилметакрилату. За створеними моделями встановлено критичну довжину дефектів за значеннями коефіцієнта інтенсивності напружень, а також критичну кількість циклів навантаження конструкційних елементів з еліпсоподібною тріщиною. Такі дослідження можуть стати основою для інженерних рекомендацій під час проектування та оцінки ризику реальних елементів, виготовлених з цього матеріалу.

РЕЗЮМЕ. Рассчитана критическая длина трещины в пластинах из лечебного цемента с добавлением различных смесей. Использованы образцы, предварительно пропитанные (0; 3; 6 месяцев) в моделированном растворе для человеческого тела. Исследованы конструкционные элементы, изготовленные из лечебного цемента с добавлением диэтиламинометилакрилата, диметиламинометилметакрилата и диэтиламинометилметакрилата. С помощью разработанных моделей установлена критическая длина дефектов по значениям коэффициента интенсивности напряжений, а также критическое количество циклов нагружения конструкционных элементов с эллипсовидной трещиной. Такие исследования могут стать основой для инженерных рекомендаций при проектировании и оценке риска реальных элементов, изготовленных из этого материала.

1. *Modified* acrylic bone cement with high amounts of ethoxytriethyleneglycol methacrylate / B. Pascual, M. Gurruchaga, M. P. Ginebra, F. J. Goñi, J. A. Planell, B. Vázquez, J. San Román, and I. Goñi // Biomaterials. – 1999. – **20**. – P. 453–463.
2. *Comparative* study on the mechanical and fracture properties of acrylic bone cements prepared with monomers containing amine groups / A. May-Pata, W. Herrera-Kaoa, J. V. Cauich-Rodrígueza, J. M. Cervantes-Uca, and S. G. Flores-Gallardob // J. Mechan. Behaior of Biomedical Mat. – 2012. – **6**. – P. 95–105.
3. *Acuña Gonzales N. A., Bilyi O. L., and González Sánchez J. A.* Monitoring of the critical length of cracklike defects in the bone-cement plate with regard for their different geometry // Materials Science. – 2017. – **53**, № 1. – P. 116–122.
4. *Sheafin E. M. and Tanner K. E.* Effects of test sample shape and surface production method on the fatigue behaviour of PMMA bone cement // J. Mechan. Behaviour of Biomedical materials. – 2014. – **29**. – 2014. – P. 91–102.
5. *Panasyuk V. V., Luchko I. I., and Pan’ko I. N.* Deformation Model of Fracture of Concrete // Problems of Strength. – 2003. – № 2. – P. 18–28.
6. *Fracture* mechanics and strength of materials: reference manual / Editor-in-Chief Academician of NAS Ukraine V. V. Panasyuk. Vol. 13: Workability of materials and structural elements with sharp-tipped stress concentrators / I. M. Dmytrakh, L. Tóth, O. L. Bilyy, A. M. Syrotyuk. – Lviv: Spolom, 2012. – 316 p.
7. *Newman J. C. and Raju I. S.* Stress intensity factor equations for cracks in three-Dimensional Finite Bodies / Eds.: J. C. Lewis, G. Sines // Fract. Mech.: Fourteenth Symp.: Vol. I: Theory and Analysis (STP-791). – ASTM, 1983. – P. 238–265.
8. *Fracture* mechanics and strength of materials: reference book / Editor-in-Chief Academician of NAS Ukraine V. V. Panasyuk. Vol. 7: Reliability and durability of structural elements for hear-and-power engineering equipment / I. M. Dmytrakh, A. B. Vainman, M. H. Stashchuk, L. Tóth. – Kyiv: PH “Academperiodyka”. – 2012. – 378 p.
9. *Physicochemical,* mechanical and biological properties of bone cements prepared with functionalized methacrylates / M. A. Sabino, D. Ajami, V. Salin, N. Nazhat, R. Vargas-Coronado, J. V. Cauich-Rodriguez, and M. P. Ginebra // J. Biomaterials Applications. – 2004. – **19**. – P. 147–161.

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