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FIRST BAIRE CLASS FUNCTIONS IN THE PLURI-FINE TOPOLOGY

Let $B_1(\Omega, \mathbb{R})$ be the first Baire class of real functions in the pluri-fine topology on an open set $\Omega \subseteq \mathbb{C}^n$ and let $H_1^*(\Omega, \mathbb{R})$ be the first functional Lebesgue class of real functions in the same topology. We prove the equality $B_1(\Omega, \mathbb{R}) = H_1^*(\Omega, \mathbb{R})$ and show that for every $f \in B_1(\Omega, \mathbb{R})$ there is a separately continuous function $g : \Omega^2 \to \mathbb{R}$ in the pluri-fine topology on Ω^2 such that f is the diagonal of g.

Keywords: *plurisubharmonic function, first Baire class, separately continuous function, pluri-fine topology, first functional Lebesgue class.*

This paper is dedicated to Professor Vladimir Gutlyanskii
on the occasion of his 75-th anniversary.

1. Introduction.

The first Baire class functions is a classical object for the studies in Real Analysis, General Topology and Descriptive Set Theory. There exist many interesting characterizations of these functions. Let us denote by I the closed interval $[0, 1]$.

Theorem 1.1. *The following conditions are equivalent for every $f : I \to I$.*

1. *The function f is a Baire one function.*
2. *There is a separately continuous function $g : I \times I \to I$ such that f is the diagonal of g.*
3. *Each nonvoid closed set $F \subseteq I$ contains a point x such that the restriction $f|_F$ is continuous at x.*
4. *The sets $f^{-1}(a, 1]$ and $f^{-1}[0, a)$ are F_σ for every $a \in I$.*
5. *For all a, $b \in I$ with $a < b$ and for every non-void subset $F \subseteq I$, the sets $f^{-1}[0, a]$ and $f^{-1}[b, 1]$ cannot be simultaneously dense in F.*

It is a classical result in the real function theory that the diagonals of separately continuous functions of n variables are exactly the $(n-1)$ Baire class functions. See R. Baire [1] for the original proof in the case where $n = 2$, and H. Lebesgue [9, 10] and H. Hahn [6] for arbitrary $n \geqslant 2$. A proof of the equivalence of (1), (3), (4) and (5) in the situation of a metrizable strong Baire space can be found, for example, in [11, Theorem 2.12, p. 55]. The goal of our paper is to find similar characterizations of the first Baire class functions on the topological space (Ω, τ), where Ω is an open subset of $\mathbb{C}^n$ and τ is the pluri-fine topology on Ω. The pluri-fine topology τ is the coarsets topology on Ω such that all plurisubharmonic functions on Ω are continuous. The topology τ was introduced by B. Fuglede in [5] as a basis for a fine analytic structure in $\mathbb{C}^n$. E. Bedford and B. A. Taylor note in [2] that the pluri-fine topology is Baire and has the quasi-Lindelöf property. S. El. Marzguioui and J. Wiegerinck proved in [14] that τ is locally connected and, consequently, the connected components of open sets are open in τ (see also [15]). It should be noted that τ is not metrizable (see Corollary 1.8 below). Thus, it is not clear whether the above formulated characterizations of the first Baire class functions are valid for (Ω, τ).

Let us recall some definitions.

Let X be an arbitrary nonvoid set. For integer $m \geqslant 2$ the set Δ_m of all m-tuples $(x, ..., x)$, $x \in X$, is by definition, the *diagonal* of X^m. The mapping $d_m : X \to X^m$, $d_m(x) = (x, ..., x)$, is called the *diagonal mapping* and, for every function $f : X^m \to Y$, the composition $f \circ d_m$,

$$X \ni x \longmapsto f(x, ..., x) \in Y$$

is, by definition, the *diagonal* of f.

Let X and Y be topological spaces. A function $f : X \to Y$ is a *first Baire class function* if there exists a sequence $(f_n)_{n\in\mathbb{N}}$ of continuous functions $f_n : X \to Y$ such that the limit relation

$$f(x) = \lim_{n\to\infty} f_n(x) \tag{1}$$

holds for every $x \in X$. Similarly, for an integer number $m \geqslant 2$, a function $f : X \to Y$ belongs to the *m-Baire class functions*, if (1) holds with a sequence $(f_n)_{n\in\mathbb{N}}$ such that each of f_n is in a Baire class less than m. A function $f : X \to Y$ is a *first functional Lebesgue class function*, if for every open subset G of the space Y, the inverse image $f^{-1}(G)$ is a countable union of functionally closed subsets of X. We will denote by $B_1(X, Y)$ (by $H_1^*(X, Y)$) the set of first Baire (first functional Lebesgue) class functions from X to Y and by F_σ^* (G_δ^*) the set of all countable unions (countable intersections) of functionally closed (functionally open) subsets of X. Thus

$$(f \in H_1^*(X, Y)) \Leftrightarrow (f^{-1}(G) \in F_\sigma^* \text{ for all open } G \subseteq Y) \tag{2}$$

$$\Leftrightarrow (f^{-1}(F) \in G_\delta^* \text{ for all closed } F \subseteq Y).$$

Recall that a subset A of a topological space X is *functionally closed*, if there is a continuous function $f : X \to I$ such that $A = f^{-1}(0)$.

Definition 1.2. Let μ be a topology on the Cartesian product $X = \prod_{i=1}^{m} X_i$ of nonvoid sets $X_1, ..., X_m$, $m \geqslant 2$, and let Y be a topological space. A function $f : X \to Y$ is called separately continuous if, for each m-tuple $(x_1, ..., x_m) \in X$, the restriction of f to any of the sets

$$\{(x, x_2, ..., x_m) : x \in X_1\}, \{(x_1, x, ..., x_m) : x \in X_2\}, ..., \{(x_1, ..., x_{m-1}, x) : x \in X_m\}$$

is continuous in the subspace topology generated by μ.

If $a = (a_1, a_2, ..., a_n) \in \mathbb{C}^n$, $b = (b_1, b_2, ..., b_n) \in \mathbb{C}^n$ and $z \in \mathbb{C}$, then we shall write $a + bz$ the for n-tuple $(a_1 + b_1 z, a_2 + b_2 z, ..., a_n + b_n z)$.

Definition 1.3. Let $\mathbb{C}^n$, $\mathbb{C}$ and $[-\infty, \infty)$ have the Euclidean topologies, $n \geqslant 1$ and let $\Omega \subseteq \mathbb{C}^n$ be a non-void open set. A function $f : \Omega \to [-\infty, \infty)$ is plurisubharmonic (psh) if f is upper semicontinuous and, for all a, $b \in \mathbb{C}^n$, the function

$$\mathbb{C} \ni z \longmapsto f(a + bz) \in [-\infty, \infty)$$

is subharmonic or identically $-\infty$ on every component of the set

$$\{z \in \mathbb{C} \colon a + bz \in \Omega\}.$$

In what follows, τ denotes the pluri-fine topology on Ω, i.e., the coarsest topology in which all psh functions are continuous.

As was noted in [14], many results related to the classical fine topology which were introduced by H. Cartan are valid for the pluri-fine topology. For example, τ is Hausdorff and completely regular. It is well known that Cartan's fine topology is not metrizable and all compact sets are finite in this topology. The topology τ also has these properties.

Let $\pi_j : \Omega^m \to \Omega$ be the j-th projection of Ω^m on Ω, $j \in \{1, ..., m\}$. We identify Ω^m with the corresponding subset of $\mathbb{C}^{mn}$ and denote by τ_m the pluri-fine topology on Ω^m.

Lemma 1.4. *All projections $\pi_j : (\Omega^m, \tau_m) \to (\Omega, \tau)$, $j = 1, ..., m$, are continuous.*

Proof. Let Y be a topological space. It follows form a general result on the continuity of the mappings to a topological space X with a topology generated by a family $\mathfrak{F}$ of functions f on X (see [4, p. 31]), that $\psi \colon Y \to X$ is continuous if and only if the composition $f \circ \psi$ is continuous for every $f \in \mathfrak{F}$. Hence, we need to show that the functions

$$\Omega^m \xrightarrow{\pi_j} \Omega \xrightarrow{f} [-\infty, \infty) \tag{3}$$

are continuous in the topology τ_m for every psh function f. Note that all projections π_j are analytic. Consequently, in (3) we have a composition of an analytic function with a psh function. Since such compositions are psh (see, for example, [7, p. 228]), they are continuous by the definition of pluri-fine topology. □

Substituting $\mathbb{C}$ instead of Ω and n instead of m we obtain the following

Corollary 1.5. *All projections $\pi_j : \mathbb{C}^n \to \mathbb{C}$, $j = 1, ..., n$, are continuous mappings with respect to the pluri-fine topologies on $\mathbb{C}^n$ and $\mathbb{C}$.*

Proposition 1.6. *Let Ω be a non-void open subset of $\mathbb{C}^n$ and let A be a compact set in (Ω, τ). Then A is finite, $|A| < \infty$.*

Proof. If f is a psh function on $\mathbb{C}^n$, then the restriction $f|_\Omega$ is psh on Ω. Hence it is sufficient to show that $|A| < \infty$ for the case $\Omega = \mathbb{C}^n$. By Corollary 1.5 every projection π_j is continuous. Hence the sets $A_j = \pi_j(A)$, $j = 1, ..., n$, are compact. As was mentioned above, every compact set in $(\mathbb{C}, \tau)$ is finite. Consequently, we have $|A_j| < \infty$, $j = 1, \ldots, n$. These inequalities and $|A| \leqslant \prod_{j=1}^{n} |A_j|$ imply that A is finite. □

Proposition 1.7. *Let Ω be a non-void open subset of $\mathbb{C}^n$. The pluri-fine topology τ is not first-countable for any $n \geqslant 1$.*

Proof. Suppose, contrary to our claim, that τ is first-countable. The topology τ is Hausdorff. Since (Ω, τ) is not discrete, Ω contains an accumulation point a which is the limit of a non-constant sequence $(a_k)_{k\in\mathbb{N}}$ of points of Ω. It is clear that the set

$$A = \{a\} \cup \left(\bigcup_{k=1}^{\infty} \{a_k\} \right)$$

is an infinite compact subset of Ω. The last statement contradicts Proposition 1.6. □

Corollary 1.8. *The pluri-fine topology τ on a non-void open set $\Omega \subseteq \mathbb{C}^n$ is not metrizable for any integer $n \geqslant 1$.*

Proof. Since every metrizable topological space is first countable, the corollary follows from Proposition 1.7. □

M. Brelot in [3] considers a fine topology generated by a cone of lower-semicontinuous functions of the form $f : X \to (-\infty, \infty]$. Every plurisuperharmonic function satisfies these conditions and such functions are just the negative of plurisubharmonic functions. Thus, the pluri-fine topology τ is an example of fine topologies studied in [3].

2. Separately continuous functions and the first Baire functions in the pluri-fine topology.

The following is a result from Mykhaylyuk's paper [17] (see also [16]).

Lemma 2.1. *Let X be a topological space and let X^m be a Cartesian product of $m \geqslant 2$ copies of X with the usual product topology. Then for every $(m-1)$-Baire class function $g : X \to \mathbb{R}$ there is a separately continuous function $f : X^m \to \mathbb{R}$ such that $f(x, ..., x) = g(x)$ holds for every $x \in X$.*

Let us denote by t^m the Tychonoff topology (= product topology) on the product of m copies of the topological space (Ω, τ). The topology t^m is the coarsest topology on Ω^m making all projections $\pi_j : \Omega^m \to \Omega$, $j = 1, \ldots, m$, continuous. Lemma 2.1 directly implies the following.

Lemma 2.2. *Let $m \geqslant 2$ be an integer. For every $(m-1)$-Baire class function $g : \Omega \to \mathbb{R}$ in the pluri-fine topology τ there is a separately continuous function $f : \Omega^m \to \mathbb{R}$ in the Tychonoff topology t^m such that g is the diagonal of f.*

The following theorem gives a "pluri-fine" analog of the first implication from Theorem 1.1.

Theorem 2.3. *Let Ω be a non-void open subset of $\mathbb{C}^n$, $n \geqslant 1$ and let $m \geqslant 2$ be an integer. For every $(m-1)$ Baire class function $g : \Omega \to \mathbb{R}$, in the pluri-fine topology τ, there is a separately continuous function $f : \Omega^m \to \mathbb{R}$, in the pluri-fine topology τ_m, such that*

$$g = f \circ d_m, \tag{4}$$

where d_m is the corresponding diagonal mapping.

Proof. By Lemma 2.2, it is sufficient to show that t^m is weaker than τ_m. From the definition of Tychonoff topology it follows at once that t^m is weaker than τ_m if and only if all projections $\pi_j\colon \Omega^m \to \Omega$, $j \in \{1, ..., m\}$, are continuous mappings on (Ω^m, τ_m). The continuity of these projections follows from Lemma 1.4. □

Proposition 2.4. *The equality*

$$H_1^*(X, \mathbb{R}) = B_1(X, \mathbb{R}) \tag{5}$$

holds for every topological space X.

Proof. Let X be a topological space and let Y be an arcwise connected, locally arcwise connected, metrizable space. Then every $f \in H_1^*(X, Y)$, with separable $f(X)$, belongs to $B_1(X, Y)$ (see [8]). Hence $H_1^*(X, \mathbb{R}) \subseteq B_1(X, \mathbb{R})$ holds.

It still remains to make sure that

$$H_1^*(X, \mathbb{R}) \supseteq B_1(X, \mathbb{R}) \tag{6}$$

is valid for every topological space X. The following is a simple modification of well known arguments.

Let $f \in B_1(X, \mathbb{R})$. Consider a sequence $(f_n)_{n\in\mathbb{N}}$ of continuous real valued functions on X such that the limit relation $f(x) = \lim\limits_{n\to\infty} f_n(x)$ holds for every $x \in X$. Let $(\varepsilon_m)_{m\in\mathbb{N}}$ be a strictly decreasing sequence of positive real numbers with

$$\lim_{m\to\infty} \varepsilon_m = 0. \tag{7}$$

Let us prove the equality

$$f^{-1}(-\infty, a) = \bigcup_{m=1}^{\infty} \bigcup_{p=1}^{\infty} \left(\bigcap_{k=p}^{\infty} f_k^{-1}(-\infty, a - \varepsilon_m]) \right) \tag{8}$$

for every $a \in \mathbb{R}$. It is sufficient to show that for every $x \in f^{-1}(-\infty, a)$ there are m, $p \in \mathbb{N}$ such that

$$x \in \bigcap_{k=p}^{\infty} f_k^{-1}(-\infty, a - \varepsilon_m]. \tag{9}$$

Let $x \in f^{-1}(-\infty, a)$. Then we have $\lim\limits_{n\to\infty} f_n(x) < a$. The last inequality and (8) imply $\lim\limits_{n\to\infty} f_n(x) < a - \varepsilon_{m_1}$ for some m_1. Consequently, there is $p \in \mathbb{N}$ such that $f_n(x) < a - \varepsilon_{m_1}$ for all $n \geqslant p$, that is

$$x \in \bigcap_{k=p}^{\infty} f_k^{-1}(-\infty, a - \varepsilon_m).$$

Since the sequence $(\varepsilon_m)_{m\in\mathbb{N}}$ is strictly decreasing, the inclusion

$$(-\infty, a - \varepsilon_m) \subseteq (-\infty, a - \varepsilon_{m+1}]$$

follows for every m. Hence (9) holds with $m = m_1 + 1$.

Note now that $f_k^{-1}(-\infty, a-\varepsilon_m]$ is functionally closed as a zero-set of the continuous function

$$g_{k,m,a}(x) := \min(\max(f(x) - f(a-\varepsilon_m); 0); 1).$$

Since each countable intersection of functionally closed sets is functionally closed [4, p. 42–43], equality (8) implies $f^{-1}(-\infty, a) \in F_\sigma^*$. Moreover, if $g = -f$ and $b = -a$, then $f^{-1}(a, \infty) = g^{-1}(-\infty, b)$ holds. Hence, the set $f^{-1}(a, \infty)$ belongs to F_σ^*.

We can now easily prove (6). Indeed, it is sufficient to show that $\{x : a < f(x) < b\}$ is a countable union of functionally closed sets for every $f \in B_1(X, \mathbb{R})$ and every interval $(a, b) \subseteq \mathbb{R}$. Using (8), we obtain

$$f^{-1}(a, b) = \left(\bigcup_{i=1}^{\infty} H_i\right) \cap \left(\bigcup_{i=1}^{\infty} F_i\right) = \bigcup_{i,j=1}^{\infty} (H_i \cap F_j), \tag{10}$$

where all H_i and F_j are functionally closed. It was mentioned above that the countable intersection of functionally closed sets is functionally closed. Hence, by (10), $f^{-1}(a, b) \in F_\sigma^*$, so that (6) follows. □

Corollary 2.5. *The equality $B_1(\Omega, \mathbb{R}) = H_1^*(\Omega, \mathbb{R})$ holds if the non-void open set $\Omega \subseteq \mathbb{C}^n$ is endowed by the pluri-fine topology τ.*

This corollary and Theorem 2.3 imply the following result.

Theorem 2.6. *Let Ω be a non-void open subset of $\mathbb{C}^n$, $n \geqslant 1$, and let $g : \Omega \to \mathbb{R}$ be a first functional Lebesgue class function on (Ω, τ). Then there is a separately continuous function $f : \Omega^2 \to \mathbb{R}$ in the pluri-fine topology τ_2 on Ω^2 such that g is the diagonal of f.*

The proof of the next proposition is a variant of Lukeš–Zajiček's method from [12, 13].

Proposition 2.7. *Let X be a topological space. Then, for every $f : X \to \mathbb{R}$, the following conditions are equivalent.*

1. *The function f belongs to $B_1(X, \mathbb{R})$.*
2. *For each couple of real numbers a, b with $a < b$ there are $H_1, H_2 \in F_\sigma^*$ such that*

$$f^{-1}(a, +\infty) \supseteq H_1 \supseteq f^{-1}(b, +\infty), \tag{11}$$

$$f^{-1}(-\infty, b) \supseteq H_2 \supseteq f^{-1}(-\infty, a). \tag{12}$$

Proof. It suffices to show that $f \in B_1(X, \mathbb{R})$ if (11) and (12) hold. (The converse implication follows from (5) and (2).)

Using (6) and (10), it is easy to see that we need only to make sure the statements

$$f^{-1}(a, +\infty) \in F_\sigma^* \quad \text{and} \quad f^{-1}(-\infty, a) \in F_\sigma^*$$

for every $a \in \mathbb{R}$. Suppose (11) holds. Then, for every $m \in \mathbb{N}$, there is $H^m \in F_\sigma^*$ such that

$$f^{-1}\left(a + \frac{1}{m}, +\infty\right) \subseteq H^m \subseteq f^{-1}\left(a + \frac{1}{m+1}, +\infty\right).$$

Consequently,

$$f^{-1}(a,+\infty) = \bigcup_{m=1}^{\infty} f^{-1}\left(a+\frac{1}{m},+\infty\right) \subseteq \bigcup_{m=1}^{\infty} H^m$$

$$\subseteq \bigcup_{m=1}^{\infty} f^{-1}\left(a+\frac{1}{m+1},+\infty\right) = f^{-1}(a,+\infty).$$

Thus,

$$f^{-1}(a,+\infty) = \bigcup_{m=1}^{\infty} H^m.$$

It implies $f^{-1}(a,+\infty) \in F_\sigma^*$, because every countable union of sets from F_σ^* belongs to F_σ^*.

Similarly, using (12), we can prove that $f^{-1}(-\infty,a) \in F_\sigma^*$. □

In the following corollary we consider the classes $B_1(\Omega,\mathbb{R})$ and F_σ^* with respect to the pluri-fine topology τ on Ω.

Corollary 2.8. *Let Ω be a non-void open subset of $\mathbb{C}^n$, $n \geqslant 1$, and let f be a real valued function on Ω. Then f belongs to $B_1(\Omega,\mathbb{R})$ if and only if, for each couple a, $b \in \mathbb{R}$, with $a < b$, double inclusions* (11) *and* (12) *hold for some H_1, $H_2 \in F_\sigma^*$.*

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Функции первого класса Бэра в топологии, порожденной плюрисубгармоническими функциямии.

Пусть $B_1(\Omega, \mathbb{R})$ – множество функций первого класса Бэра в топологии, порожденной плюрисубгармоническими функциями на открытом множестве $\Omega \subseteq \mathbb{C}^n$, и пусть $H_1^*(\Omega, \mathbb{R})$ – первый функциональный класс Лебега вещественнозначных функций в той же топологии. Мы доказываем равенство $B_1(\Omega, \mathbb{R}) = H_1^*(\Omega, \mathbb{R})$ и показываем, что для всякой $f \in B_1(\Omega, \mathbb{R})$ существует раздельно непрерывная функция $g : \Omega^2 \to \mathbb{R}$ в топологии, порожденной плюрисубгармоническими функциями и такая, что f является диагональю g.

Ключевые слова: *плюрисубгармоническая функция, первый класс Бэра, раздельно непрерывная функция, порожденная плюрисубгармоническими функциями топология, первый функциональный класс Лебега.*

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Функції першого класу Бера у топології, що породжена плюрісубгармонійними функціями.

Нехай $B_1(\Omega, \mathbb{R})$ – множина функцій першого класу Бера у топології, породженій плюрісубгармонійними функціями на відкритій множині $\Omega \subseteq \mathbb{C}^n$ та нехай $H_1^*(\Omega, \mathbb{R})$ – перший функціональний клас Лебега дійсних функцій у тій же топології. Ми доводимо рівність $B_1(\Omega, \mathbb{R}) = H_1^*(\Omega, \mathbb{R})$ та показуємо, що для кожної $f \in B_1(\Omega, \mathbb{R})$ існує нарізно неперервна функція $g : \Omega^2 \to \mathbb{R}$ у топології, що породжена плюрісубгармонійними функціями та така, що f є діагоналлю g.

Ключові слова: *плюрісубгармонійна функція, перший клас Бера, нарізно неперервна функція, породжена плюрісубгармонійними функціями топологія, перший функціональний клас Лебега.*

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