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IDENTIFICATION OF VAN DER POL OSCILLATOR NETWORK PARAMETERS

The problem of state observation and parameters identification of an oscillatory system consisting of coupled van der Pol oscillators is considered. The unknowns are: velocity of oscillations and parameters that characterize the threshold values for displacements of network's oscillators at which the damping forces change sign. An invariant relations method for simultaneous of the state and parameters estimation is used. Such approach is based on dynamical extension of original system and synthesis of appropriate invariant relations, from which the unknowns can be expressed as a function of the known quantities on the trajectories of extended system during the observed motion. The stability property is formally checked considering the oscillatory behavior of the system. On the first step the corresponding observation and identification problems are solved for one of autonomous van der Pol oscillator, further, the results obtained are extended to a system of interconnected oscillators. The simulation results confirm efficiency of the proposed scheme of nonlinear observer and identifier design for network of oscillators.

MSC: 34N05.

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1. Introduction. In many practical applications of physics, biology and other sciences an approach based on the concept of model equations is used as an approximate mathematical model of complex nonlinear processes. The basis of this concept is the provision that a small number of characteristic type's movements of simple mathematical models inherent in systems give the key to understanding and exploring a huge number of different phenomena. In particular, it is known that nonlinear oscillatory motion of various systems can be modeled by a system consisting of one or more coupled van der Pol oscillators or some of their modifications [1]. For this reason, oscillators have been widely studied, as a way to model, analyze or even control in various fields such as electronics [2], control [3], biology [4–6], geology [7].

Naturally, when using of such models, the problems may arise in determining their state and parameters from the known data on observed motion. In such cases, parameter identification becomes the basis for a number of engineering tasks, such as: i) gaining knowledge of the behavior of the process; ii) to test theoretical models and adjust related parameters; iii) to develop control algorithms, etc. In particular, the method for reconstruction the numerical values of a nonlinear potential, dissipation and coupling functions for ensembles of coupled van der Pol oscillators from multivariate time series is proposed [8]. A classical approach with exact Kalman-like observer design for simultaneous state and parameter estimation is proposed in [9] for the autonomous van der Pol oscillator. The influence of colored noise in measurements on the identification process was investigated in the [10] on phenomen of thermoacoustic instabilities in gas turbine as an example.

This article discusses one of such identification problems for ensembles of coupled van der Pol oscillators, namely, the problem of determining the threshold values of deviations at which the damping forces in the system change sign. Previously, this problem was solved by authors under the assumption that the system output is a complete phase vector [11]. In this paper, we propose a scheme for the simultaneous estimation of the state and parameters only from the data on the displacements of the oscillators. On the first step the identification problem is solved for single van der Pol oscillator; further, the results obtained are extended to a system of interconnected oscillators. It was used developed in analytical mechanics the method of invariant relations [12], modification of which in the problems of observation, identification allows us to synthesize additional relations arising between known and unknown quantities during the observed motion of the system considered [13–15]. It should be noted that a more general approach, which forms an appropriate method for solving observation problems for nonlinear dynamical systems due to the synthesis of invariant manifold, was proposed as a modification of the method I&I (Input and Invariance) of stabilization of nonlinear systems in [16, 17]. Numerical simulation confirms the effectiveness of this method of a nonlinear observer and identifier design for a network of oscillators.

2. An identification problem of dissipation characteristics for a single van der Pol oscillator.

The van der Pol oscillator is defined by a dynamical equation which we write in the following form [1]

$$\ddot{x} = (\lambda - x^2)\dot{x} - \omega^2 x. \tag{1}$$

Here $x(t)$ is the oscillator displacement from the equilibrium position; λ is the parameter that characterizes alternating dissipation in the system, $\lambda \geq 0$. The absence of the nonlinear term in (1) corresponds to harmonic oscillations without friction with frequency ω. It is assumed that: i) system (1) performs an bounded oscillatory motion in a given area, ii) the trivial motion $x(t) \equiv 0$ is not considered and iii) oscillator movements are measured, i.e. the values of output $y(t) = x(t)$ are known $\forall t \geq 0$. The unknowns are the vibration velocity $\dot{x}(t)$ and the parameter λ.

The purpose of this paper is to find or estimate the numerical values of these unknowns by the known information. The known information is the output - the time function $y(t) = x(t)$, as well as those values that can be obtained by using only the output. In particular, we can assume that any solution of the Cauchy problem is known for any system of differential equations of the form

$$\dot{\xi} = U(\xi, x(t)),\ \xi(0) = \xi_0 \in R^p,\ p \geq 2 \tag{2}$$

where vector function $U(\xi, x(t))$ does not contain unknown variables.

Consider the problem of $\dot{x}(t)$ and parameter λ determination as a classical problem of observation and identification for nonlinear dynamical system. For this purpose, rewrite (1) in the form of a second-order system in a state space representation. To avoid bilinearity with respect to unknowns $\dot{x}(t)$ and λ we express (1) in the Lienard form [14] by performing the following change of variables:

$$x_1(t) = x(t),\ x_2(t) = \dot{x}(t) + \int\limits_0^x (\lambda - \sigma^2) d\sigma. \tag{3}$$

As a result, we obtain

$$\begin{aligned} \dot{x}_1 &= x_2 - \lambda x_1 + x_1^3/3, \\ \dot{x}_2 &= -\omega^2 x_1,\ y(t) = x_1(t). \end{aligned} \tag{4}$$

Note that the transform (3) depends on an unknown function $\dot{x}(t)$, which will be determined if we find variable $x_2(t)$ of the system (4). Further, we will assume that system (4) performs an bounded oscillatory motion: namely, suppose that there exist two positive constants $0 < \rho_1, \rho_2 < \infty$ such that $\forall t \geq 0, (x_1(t), x_2(t)) \in P = \{(x_1, x_2) : x_1 \leq \rho_1, x_2 \leq \rho_2\}$. Next, we will consider

Identification problem 1. It is required to found asymptotically accurate estimates of the parameter λ and component $x_2(t)$ of the phase vector of system (4) by the known output data $y(t) = x_1(t)$.

In order to solve this problem, we synthesis of special kind of invariant relations, which will allow us to express the unknowns as functions on the known quantities on the observed trajectories [9–13].

3. Synthesis of additional relations.

The main idea of this approach is to obtain additional equations for unknowns of the original system. For this purpose, the system of differential equations (4) is supplemented by differential equations (2), where $p = 2$ – the number of unknowns: namely, the constant λ and component of the phase vector – $x_2(t)$. Moreover, the right-hand sides of the auxiliary system – function $U(\xi, x_1(t))$ must be chosen so that the resulting extended system (2), (4) admits invariant relations

$$F_i(x_1, x_2, \xi, \lambda) = 0,\ i = 1, 2 \tag{5}$$

on its solutions with the following properties:

i) Equalities (5) form the additional equations for the unknowns: λ and $x_2(t)$, i.e. $\text{rank}\frac{\partial(F_1,F_2)}{\partial(\lambda,x_2)} = 2$;

ii) The invariant manifold $\{(x_1, x_2, \xi, \lambda) \subseteq R^5 : F_i(x_1, x_2, \xi, \lambda) = 0,\ i = 1, 2\}$ appropriate to equalities (5) has the property of global attraction for any solutions of the extended system (2), (4). In other words, on any trajectories

$$\lim_{t\to\infty} F_i(x_1(t), x_2(t), \xi(t), \lambda) = 0,\ i = 1, 2 \tag{6}$$

Let show that for the problem considered the relations of the form (5) exist. In order to satisfy the property i) we will search these invariant relations in the form

$$\begin{aligned} F_1 &= \lambda - \xi_1(t) - \Phi(x_1(t)) = 0, \\ F_2 &= x_2(t) - \xi_2(t) - \Psi(x_1(t)) = 0, \end{aligned} \tag{7}$$

where the variables $\xi_i,\ i=1,2$ are solutions of the system of differential equations (2). On this step we don't set any restrictions on the functions $\Phi(x_1), \Psi(x_1), U(\xi, x_1)$, except for the requirement of continuous differentiability with respect to their arguments. If these functions are chosen so that relations (7) have become invariant on observed solution, then unknowns' values can be found directly from these equalities. We first choose the right-hand side $U(\xi, x_1)$ of differential equations (2) so that for any functions $\Phi(x_1), \Psi(x_1)$ equalities (7) hold identically on some trajectories of the extended system of differential equations (2), (4). Introduce for this purpose the variables $\varepsilon_1, \varepsilon_2$, which characterize the residual in formulas (7) on the solutions of system (2), (4).

$$\begin{aligned} \lambda - \xi_1(t) - \Phi(x_1(t)) &= \varepsilon_1(t), \\ x_2(t) - \xi_2(t) - \Psi(x_1(t)) &= \varepsilon_2(t). \end{aligned} \tag{8}$$

Differentiating (8) with respect to system (2), (4), we obtain a differential equation for the deviations $\varepsilon_1, \varepsilon_2$.

$$\begin{aligned} \dot\varepsilon_1 &= \dot\xi_1 - \acute\Phi[\Psi + \xi_2 + \varepsilon_2 - (\Phi + \xi_1 + \varepsilon_1)x_1 + x_1^3/3], \\ \dot\varepsilon_2 &= \dot\xi_2 - \acute\Psi[\Psi + \xi_2 + \varepsilon_2 - (\Phi + \xi_1 + \varepsilon_1)x_1 + x_1^3/3] - \omega^2 x_1. \end{aligned} \tag{9}$$

To prove that equalities (7) are identities on some solutions of the system (2), (4), it suffices to show that the system of differential equations (9) admits the trivial solution $\varepsilon_1(t) = \varepsilon_2(t) \equiv 0$. For this purpose, we fix the form of the right-hand sides (2), namely: for any $\Phi(x_1), \Psi(x_1)$ we put

$$\begin{aligned} \dot\xi_1 &= -\acute\Phi[\Psi + \xi_2 - (\Phi + \xi_1)x_1 + x_1^3/3], \\ \dot\xi_2 &= -\acute\Psi[\Psi + \xi_2 - (\Phi + \xi_1 +)x_1 + x_1^3/3] - \omega^2 x_1. \end{aligned} \tag{10}$$

As a result, equations (8) become linear and homogeneous with respect to deviations $\varepsilon_1, \varepsilon_2$

$$\begin{aligned} \dot\varepsilon_1 &= \acute\Phi(x_1\varepsilon_1 - \varepsilon_2), \\ \dot\varepsilon_2 &= \acute\Psi(x_1\varepsilon_1 - \varepsilon_2), \end{aligned} \tag{11}$$

and therefore, admits the trivial solution $\varepsilon_1(t) = \varepsilon_2(t) \equiv 0$.

4. Stabilization of deviations.

In order to satisfy the property ii) of invariant relations i.e. stabilizes the trivial solution of system (11) consider the problem of the free functions $\Phi(x_1)$ and $\Psi(x_1)$ synthesis. Take the positive definite function

$$V = \frac{\varepsilon_1^2 + \varepsilon_2^2}{2}. \tag{12}$$

Let the functions $\Phi(x_1)$ and $\Psi(x_1)$ are equal

$$\Phi(x_1) = -kx_1^2/2,\ \Psi(x_1) = kx_1, \tag{13}$$

where k is some positive constant. Then the deviations equations (11) take the form

$$\begin{aligned} \dot{\varepsilon}_1 &= -kx_1(x_1\varepsilon_1 - \varepsilon_2), \\ \dot{\varepsilon}_2 &= k(x_1\varepsilon_1 - \varepsilon_2). \end{aligned} \tag{14}$$

and the derivative of the function (12) becomes negatively semidefinite

$$\dot{V} = -k(x_1\varepsilon_1 - \varepsilon_2)^2 \le 0. \tag{15}$$

We can now show with the use of LaSalle's invariance principle [19] that inequality (15) is sufficient for the asymptotic stability of the trivial solution of the system (14). As an autonomous system under study, consider the system (4), (14) with a phase vector $(x_1(t), x_2(t), \varepsilon_1(t), \varepsilon_2(t)) \in \Omega$.

Let us show that Ω – compact set that is positively invariant with respect to this system. Indeed, under assumption $(x_1(t), x_2(t)) \in P \subset R^2$ where P describe the bounded region of oscillations. The values of deviations according (15) lie inside the sphere $S = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1^2 + \varepsilon_2^2 \le \varepsilon_1^2(0) + \varepsilon_2^2(0)\}$. Then $\Omega = P \times S$ is a compact set that is positively invariant with respect to system (4), (14).

Denote $d(t) = x_1(t)\varepsilon_1(t) - \varepsilon_2(t)$. Let E be the set of all points $(x_1, x_2, \varepsilon_1, \varepsilon_2)$ from Ω where $\dot{V} = -kd(t)^2 = 0$. It is obvious that the set $M = \{(x_1(t), x_2(t), 0, 0)\}$ is invariant in E. If M will be the largest invariant set, then by LaSalle's theorem all solutions starting in Ω will tend to M, i.e.

$$\lim_{t\to\infty} \varepsilon_i(t) = 0, \;\; i = 1, 2. \tag{16}$$

We prove this by contradiction. Namely, suppose that $d(t) = 0$ and E contains trajectories with nonzero deviations. Then it follows from (14) that $\varepsilon_i(t) = \varepsilon_i^* -$ *const*, $i = 1, 2$ and there is a stationary linear constraint $\varepsilon_1^* x_1(t) - \varepsilon_2^* = 0$. The last one contradicts the oscillation regime of $x_1(t)$.

5. Non-linear observer and identifier.

All free functions $\Phi(x_1), \Psi(x_1), U(\xi, x_1)$ was chosen in such a way that the deviations $\varepsilon_1, \varepsilon_2$ in formulas (8) tend to zero with increasing t. So finite relations (7) and auxiliary differential equations (10) define a nonlinear identifier:

$$\begin{aligned} \xi_1 &= -kx_1[kx_1 + \xi_2 - (\xi_1 - \frac{kx_1^2}{2})x_1 + \frac{x_1^3}{3}], \\ \xi_2 &= -k[kx_1 + \xi_2 - (\xi_1 - \frac{kx_1^2}{2})x_1 + \frac{x_1^3}{3}] - \omega^2 x_1. \end{aligned} \tag{17}$$

As a result, we have **Proposition 1.** *For any nontrivial solution $x_1(t), x_2(t)$ of system (4), a positive constant k, and any initial value in the Cauchy problem for the auxiliary system of differential equations (17) the formulas*

$$\begin{aligned} \hat{\lambda} &= \xi_1(t) - k\frac{x_1^2(t)}{2}, \\ \hat{x}_2(t) &= \xi_2(t) + kx_1(t) \end{aligned} \tag{18}$$

provide the asymptotic estimates for the parameters λ and variable $x_2(t)$.

6. Non-linear observer and identifier for coupled oscillators.
Consider a system consisting of n interconnected non-identical van der Pol oscillators

$$\ddot{x}_i = (\lambda_i - x_i^2)\dot{x}_i - \omega_i^2 x_i + G_i(t, x_1, ..., x_n),\ i = \overline{1, n}. \tag{19}$$

Here the variables x_i mean the displacement from the equilibrium position of the i-th oscillator $\dot{x}_i$ – the speed of the corresponding displacement; the functions $G_i(t, x_1, ..., x_n)$ formalize the external influence and the influence of connections in the network on the dynamics of the i-th oscillator. In the problems of approximate modeling of oscillations of complex objects using models of the form (19), these functions are selected for various reasons and may not have a physical or mechanical meaning, describing, for example, one-way relationships.

Regardless of the type and structure of these functions, we will require the following assumptions to construct the observer and identifier of the system (19): a) the values of the functions $G_i(t, x_1, ..., x_n)$ are known at any instant of time; b) solutions of systems of differential equations, which contain in their right-hand parts these functions are bounded. Changing variables according to the formulas (3), we obtain the Lienard form of oscillator network equations

$$\begin{aligned} &\dot{x}_{i1} = x_{i2} - \lambda_i x_{i1} + \frac{x_{i1}^3}{3}, \\ &\dot{x}_{i2} = -\omega_i^2 x_{i1} + G_i(t, x_{11}, ..., x_{n1}), \\ &y(t) = (x_{11}(t), ..., x_{1n}(t)),\ i = \overline{1, n}. \end{aligned} \tag{20}$$

Identification problem 2. It is required to found asymptotically accurate estimates of the parameters λ_i and components $x_{i2}(t)$ of the phase vector of system (20) by the known output data $y(t) = (x_{11}(t), ..., x_{1n}(t)),\ i = \overline{1, n}$.

The solution of problem 2 is carried out according to the scheme of the autonomous oscillator described above. We consider a dynamic extension of system (20) – an auxiliary system of differential equations (2) of dimension $p = 2n$. Similarly (8) we present the unknowns in the form of the sum of uncertain values

$$\begin{aligned} &\lambda_i = \xi_{i1} + \Phi_i(x_{i1}) + \varepsilon_{i1}, \\ &x_{i2} = \xi_{i2} + \Psi_i(x_{i1}) + \varepsilon_{i2},\ i = \overline{1, n}. \end{aligned} \tag{21}$$

where $\Phi_i(x_{i1}), \Psi_i(x_{i1})$ - free functions, ξ_{i1}, ξ_{i2} - components of the solution ξ of system (2) with an indefinite right-hand side $U(\xi, y, G_1, ..., G_n)$, $\varepsilon_{i1}, \varepsilon_{i2}$ - deviations in the equations of invariant relations. Differentiating (21), we obtain

$$\begin{aligned} &\dot{\varepsilon}_{i1} = \xi_{i1} - \acute{\Phi}_i[\Psi_i + \xi_{i2} + \varepsilon_{i2} - (\Phi_i + \xi_{i1} + \varepsilon_{i1})x_{i1} + \frac{x_{i1}^3}{3}], \\ &\dot{\varepsilon}_{i2} = \xi_{i2} - \acute{\Psi}_i[\Psi_i + \xi_{i2} + \varepsilon_{i2} - (\Phi_i + \xi_{i1} + \varepsilon_{i1})x_{i1} + \frac{x_{i1}^3}{3}]. \end{aligned} \tag{22}$$

Let, by analogy with (13),

$$\Phi_i(x_{i1}) = -k\frac{x_{i1}^2}{2},\ \Psi_i(x_{i1}) = kx_{i1}. \tag{23}$$

For the system of equations in deviations (22) to allow a trivial solution, we write the auxiliary system of differential equations (2) in the form

$$\begin{aligned} \xi_{i1} &= kx_{i1}[kx_{i1} + \xi_{i2} + (k\frac{x_{i1}^2}{2} - \xi_{i1})x_{i1} + \frac{x_{i1}^3}{3}],\\ \xi_{i2} &= -k[x_{i1} + \xi_{i2} + (k\frac{x_{i1}^2}{2} - \xi_{i1})x_{i1} + \frac{x_{i1}^3}{3}] + G_i - \omega_i^2 x_{i1},\ i = \overline{1,n}. \end{aligned} \tag{24}$$

We note that functions describing the interactions among oscillators are contained in auxiliary equations (24). As a result, the system of equations for deviations is decomposed into n identical subsystems of differential equations, which completely coincide with the similar system for the autonomous van der Pol oscillator (14).

$$\begin{aligned} \dot\varepsilon_{i1} &= -kx_{i1}(x_{i1}\varepsilon_{i1} - \varepsilon_{i2})\\ \dot\varepsilon_{i2} &= k(x_{i1}\varepsilon_{i1} - \varepsilon_{i2}),\ i = \overline{1,n}. \end{aligned} \tag{25}$$

Using the results of the previous section, we obtain **Proposition 2.** *For any nontrivial solution $x(t)$ of system (20) and any initial value of $\xi(0)$ in the Cauchy problem for the auxiliary system of differential equations (24) the formulas*

$$\begin{aligned} \hat\lambda_i &= \xi_{i1}(t) - k\frac{x_{i1}^2(t)}{2},\\ \hat x_{i2}(t) &= \xi_{i2}(t) + kx_{i1}(t), \end{aligned} \tag{26}$$

determine asymptotic estimates of parameters λ_i and variables $x_{i2}(t)$, $i = \overline{1,n}$.

7. Numerical simulation.

The scheme proposed in the work was numerically modeled for a wide range of initial conditions and parameters of the dynamic system, which is formed by a chain of three non-identical van der Pol oscillators

$$\ddot x_i = (\lambda_i - x_i^2)\dot x_i - \omega_i^2 x_i + G_i(t, x_1, x_2, x_3),\ i = \overline{1,3}. \tag{27}$$

connected by elastic connections given by functions

$$G_1 = \nu(x_2 - x_1),\ G_2 = \nu(x_1 - x_2) + \nu(x_3 - x_2),\ G_3 = \nu(x_2 - x_3).$$

Here $\nu = 1.2$ is the stiffness coefficient, $\lambda_1, \lambda_2, \lambda_3$ are equal to 1.0, 3.0, 5.0. System output - vector $(x_1(t), x_2(t), x_3(t))$ was obtained as a result of numerical solution of the system of differential equations (27). The system considered is transformed to the form (20) and variant of calculation simulates the case when the chain of oscillators is brought

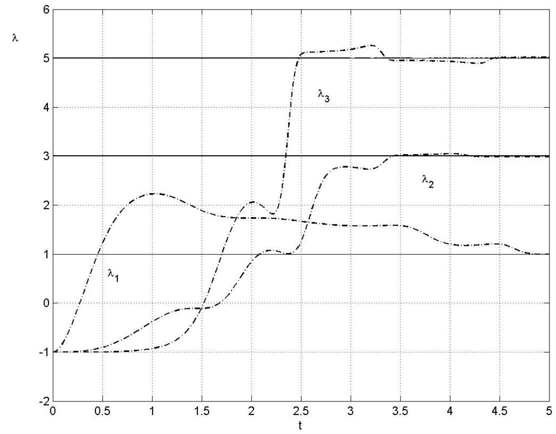

Fig. 1. Asymptotical estimation of parameters $\lambda_i,\ i=\overline{1,3}$ in the chain of three coupled oscillators.

out of equilibrium by shifting the position of the first of them by 1 unit of length. i.e. the initial conditions in the Cauchy problem are taken equal to (1.0; 0.0; 0.0; 0.0 ; 0.0; 0.0). The initial conditions for the variables of the auxiliary system of differential equations (24) are chosen arbitrarily, in this case: $\xi_{i1}(0) = -1.0,\ \xi_{i2}(0) = -15.0,\ i = 1,2,3$. Other parameters: $k = 2.5, \omega_1^2 = \omega_2^2 = \omega_3^2 = 8.0$.

Figure 1 shows the graphs of functions $\xi_{i1}(t) - \frac{kx_{i1}^2(t)}{2},\ i = 1,2,3$, which, according to Proposition 2, asymptotically tend to the required parameters $\lambda_1 = 1.0, \lambda_2 = 3.0$, $\lambda_3 = 5.0$ respectively. The dash-dot lines on Figure 2 show the graphs of functions $\xi_{i2}(t) + kx_{i1}(t), i = 1,2,3$ which, according to Proposition 2, asymptotically tend to unknowns $x_{i2}(t), i = 1,2,3$ (solid lines). The simulation results confirm the efficiency of the proposed method for solving nonlinear problems of observation and identification for coupled van der Pol oscillators.

8. Resume.

In this paper we have proposed a new observer scheme for state and parameter estimation in the system of coupled van der Pol oscillators, with a convergence analysis. The method of invariant relations is used that allow us to synthesize additional relationships between known and unknown components of initial mathematical model on the trajectories of special dynamical system. The approach developed will be further used in the problems of parameters identification of a Lienard system − a second-order system $\ddot{x} + F(x)\dot{x} + G(x) = 0$ where $F(x)$ and $G(x)$ are functions that represent various nonlinear phenomena. In order to determine these nonlinearities, identifying the parameters characterizing their behaviors is essential. Parameters identification is the basis for a series of engineering tasks such as: i) to gain knowledge about the process behavior; ii) to validate theoretical models and to tune appropriate parameters; iii) for control algorithms design, etc.

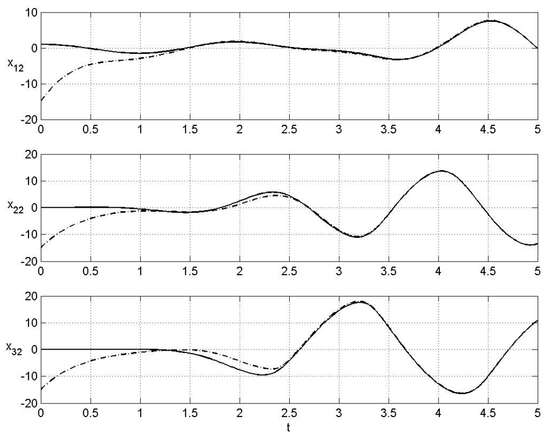

Fig. 2. Asymptotical estimation of variables $x_{i2}(t),\ i=\overline{1,3}$ in the chain of three coupled oscillators.

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Параметри осциляторів ван дер Поля.

Відомо, що нелінійний коливальний рух складних систем може бути приблизно промодельовано рухом системи, яка складається з одного або декількох пов'язаних між собою осциляторів ван дер Поля. З цієї причини осцилятори широко вивчаються як спосіб моделювання, аналізу або навіть контролю у різних сферах, таких як електроніка, керування, медико-біологічні дослідження, геологія та ін. Природньо, що при цьому на практиці виникають проблеми визначення стану та параметрів таких моделей за результатами вимірювання вихідних сигналів у режимі реального часу. Одну з таких проблем, а саме проблему визначення порогових значень відхилень, при яких значення сил демпфірування в компонентах системи осциляторів змінюють знак, забезпечуючи тим самим автоколивальнй режим руху, розглянуто у цій статті. В роботі запропонована схема визначення асимптотичних оцінок цих параметрів, а також швидкості коливань осиляторів системи за даними про їх положення. На першому кроці відповідна задача ідентифікації та спостереження була вирішена для одного автоколивального осцилятора ван дер Поля; надалі отримані результати поширено на систему взаємопов'язаних осциляторів. Для отримання оцінок невідомих був використаний розроблений в аналітичній механіці метод інваріантних співвідношень, модифікація якого у задачах спостереження, ідентифікації дозволяє синтезувати додаткові співвідношення, що виникають між відомими та невідомими величинами під час спостережуваного руху побудованої спеціальним чином розширеної динамічної системи. В результаті в роботі було запропоновано новий суттєво нелінійний метод отримання асимптотичних оцінок стану та параметрів для мережі пов'язаних між собою осциляторів ван дер Поля. Чисельне моделювання підтверджує ефективність цього способу спостереження та конструкції віповідного ідентифікатора для розглянутої системи.

Ключові слова: *ідентифікація, нелінійні коливання, пов'язані осцилятори ван дер Поля, інваріантні співвідношення, асимптотичні оцінки.*

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