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EVOLUTION OF ROTATIONAL MOTIONS IN A RESISTIVE MEDIUM OF A NEARLY DYNAMICALLY SPHERICAL RIGID BODY WITH A MOVABLE MASS

The problem of a rigid body motion about a fixed point is one of the classical problems of mechanics. The interest in the problem of the rigid body dynamics has increased in the second half of the XX century in connection with the development or rocket and space technologies. The study of the motion of a satellite about center of mass is important for creating systems of orientation control, stabilization of motion and for solving the practical problems of astronautics. The paper develops an approximate solution by means of an averaging method for the motion in a resistive medium of a nearly dynamically spherical rigid body containing a viscoelastic element. The numerical integration of the averaged system of equations is conducted for the body motion. The graphical presentations of the solutions are represented and discussed. We received the system of motion equations in standard form, which refined in square approximation by small parameter. The asymptotic approach permits to obtain some qualitative results and to describe evolution of rigid body motion using simplified averaged equations and numerical solution. The paper can be considered as mainstreaming of previous works for the problem of rigid body motion under the action of small torques (cavity filled with a fluid of high viscosity, moving mass, constant body-fixed torques). The advantage of this work is in receiving the original asymptotic and numerical calculations, as well as solutions that describe the evolution of rigid body motion with a moving mass in a resistive medium over an infinite time interval with an asymptotically small error. The paper presents a contribution in the problems of spacecraft motion, and the activities of crew members about the vehicles. The importance of the results is in the moving mass control, and the motions of spinning projectiles.

MSC: 70E15,70E20, 70E40.

Key words: *nearly dynamically spherical rigid body, movable mass, elastic and dissipative elements, resistive medium.*

1. Introduction.

A spacecraft or satellite, while orbiting about its center of mass, experiences torques from forces of diverse physical origins. This includes torques generated by the motion of internal masses, which can arise from factors such as presence of rotating components (like rotors or gyroscopes) within the spacecraft or satellite, and the activities of crew members aboard the crew vehicle.

The dynamics of a rigid body incorporating moving masses is a significant focal point in classical mechanics. Extensive research is dedicated to investigating the rotation of a rigid body featuring motion of internal masses. A number of problems in the indicated field and the works in this direction are described in [1–8].

In [1, 9], scenarios involving the motion of a rigid body containing movement of internal masses are explored. Several problems concerning the motion of a rigid body incorporating elastic and dissipative components are investigated in [10–13]. [14] tackled the issue of minimum-time deceleration in a resistant medium for the rotation of a dynamically symmetric rigid body containing a viscous-elastic element. [15] focused on the challenge of achieving quasi-optimal time-based deceleration for a gyrostat featuring a moving mass in a medium with resistance.

In the paper [16], the influence is estimated of the moving point masses (linear oscillators) on the stability of uniform rotation of the Lagrange top. The possibility of stabilizing unstable uniform spinning of a "sleeping" Lagrange gyroscope, in a resisting medium, is considered in [17].

Paper [18] delved into the motion of a rigid body that is close to dynamically spherical, and houses a cavity filled with a highly viscous fluid. In [19], researchers explored the motion of a nearly dynamically spherical rigid body, also with a cavity containing viscous fluid but at a low Reynolds member. They provided insights both the qualitative and quantitative aspects of its motion in a resistive medium. [20] focused on the motion about the center of mass of a nearly dynamically spherical rigid body with a cavity filled with highly viscous fluid, which was subjected to constant body-fixed torque. In [21] qualitative and quantitative results of motion of a nearly dynamically spherical rigid body with a moving mass attached to the body by means of elastic coupling were presented. Paper [22] extended the investigation of rigid body motion presented in [18] by adding another (third) component of the gyrostatic moment. In paper [23] the results of [18] was generalized to charged rigid body.

2. Formulation of the problem.

Let us examine the motion of a dynamically asymmetric rigid body about its center of inertia, featuring a movable point mass m connected via an elastic linkage to a point O_1, located on one of its principal axes of inertia.

We assume that the torque of the resistance forces is proportional to the angular momentum of the body [1, 15, 24]

$$\mathbf{M}^r = -\varepsilon^2 \lambda \mathbf{J} \omega \tag{1}$$

where λ is a positive coefficient of proportionality that depends on the properties of the medium and on the shape of the body $0 < \varepsilon \ll 1$ is a small parameter.

The origin of Cartesian coordinate system, connected with the rigid body, is placed at the center of inertia of the body with point mass, whereas the basic vectors $\mathbf{e_1}$, $\mathbf{e_2}$, $\mathbf{e_3}$ of the system are directed so that basic vector $\mathbf{e_3}$ coincides with axes on which point O_1 is located. Then radius-vector of point O_1, $\boldsymbol{\rho} = \rho \boldsymbol{e_3}$ where without loss of generality, we assume $\rho > 0$.

In references [1, 9], a vector equation was derived to describe the alteration of the vector m within the coordinate system linked to the body.

$$\mathbf{J}_0^* \cdot \omega + (\omega \times \mathbf{J}_0^* \cdot \omega) = \mathbf{\Phi}(\omega) + O(\mathbf{\Omega}^{-4}, \lambda^2 \mathbf{\Omega}^{-8}) \tag{2}$$

Here $\mathbf{J}_0^*$ denotes the inertia tensor of a rigid body containing a moving mass when referenced with respect to point O, ω is the absolute angular velocity of the body, vector function $\mathbf{\Phi}$ includes the terms of the orders $\mathbf{\Omega}^{-2}$ of and $\lambda^2 \mathbf{\Omega}^{-4}$. The quantities

$\Omega^2 = c/m$, $\lambda = \delta/m$, having the dimensions t^{-1}, characterize the frequency and decay time of free oscillations, c is a stiffness coefficient, and δ is a viscous friction coefficient.

Perturbation torques in (2) are small, provided

$$\Omega^2 \gg \lambda\omega \gg \omega^2 \tag{3}$$

Free oscillations of the system decaying long time before the body performs one revolution [1, 9].

To obtain equations (2) and assess their level of error, please refer to [1, 9]. Function $\mathbf{\Phi}(\omega)$ is a polynomial containing the fourth and fifth order of ω [1, 9].

Having evaluated the vector function $\mathbf{\Phi}$ as per [1, 9], the equation (2) for our specific problem, when expressed in terms of projections on axes $\boldsymbol{e_1}$, $\boldsymbol{e_2}$, $\boldsymbol{e_3}$, takes the following shape

$$\begin{aligned}
A\frac{dp}{dt} + (C-B)qr &= -\rho^2 m\{\Omega^{-2}qr(Q_1p^2 + K_1q^2 + L_1r^2) + \\
&\quad + \lambda\Omega^{-2}p[q^2(M_1p^2 + N_1q^2 + R_1r^2) + r^2(S_1p^2 + T_1r^2)]\} - \varepsilon^2\lambda Ap \\
B\frac{dq}{dt} + (A-C)pr &= -\rho^2 m\{\Omega^{-4}pr(Q_2q^2 + K_2p^2 + L_2r^2) + \\
&\quad + \lambda\Omega^{-4}q[r^2(M_2q^2 + N_2r^2 + R_2p^2) + p^2(S_2q^2 + T_2p^2)]\} - \varepsilon^2\lambda Bq \\
C\frac{dr}{dt} + (B-A)pq &= -\rho^2 m\lambda\Omega^{-4}r^3(A+C-B)(B+C-A)\times \\
&\quad \times A^{-1}B^{-1}[(A-C)B^{-1}p^2 + (B-C)A^{-1}q^2] - \varepsilon^2\lambda Cr
\end{aligned} \tag{4}$$

The tensor of inertia $\mathbf{J}_0^*$ of the rigid body, whose point mass m is superposed with O_1, obtains the following form

$$\mathbf{J}_0^* = \begin{vmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{vmatrix}, \tag{5}$$

where A, B and C are the principal moment of inertia of the rigid body, p, q, r are the components of the absolute angular velocity ω.

The right-hand side of the initial two equations (2) comprises coefficients Q_1, K_1, L_1, N_1, R_1, S_1, T_1, Q_2, K_2, L_2, N_2, R_2, S_2, T_2 that are specific expressions containing A, B, C. For instance,

$$Q_1 = -1 - 5\frac{A-C}{B} - 6\frac{C-B}{A} - 3\frac{B-A}{C} + 7\frac{(A-C)(B-A)}{BC} + 4\frac{(B-A)(C-B)(A-C)}{ABC} \tag{6}$$

We have chosen not to reference in our paper other expressions from (4) such as $K_1, \ldots, T_2$ due to their drawbacks.

If $A = B$ and $\varepsilon = 0$ the system (4) reduces to the corresponding system in [1, 9].

Take into consideration a nearly dynamically spherical rigid body, where the principal central moments of inertia of the unperturbed body are closely aligned and can be expressed in the following manner

$$A = J_0 + \varepsilon A', B = J_0 + \varepsilon B', C = J_0, \tag{7}$$

where $0 < \varepsilon \ll 1$ is a small parameter of the same order just in (1).

According to (3) Ω^{-2}, $\lambda\Omega^{-4}$ are small parameters in equations of motion (4). We assume that, i.e. $\Omega^{-2} \sim \varepsilon^2$, $\lambda\Omega^{-4} \sim \varepsilon^2$, $\lambda\Omega^{-2} \sim 1$.

For $\varepsilon = 0$ equations of motion (4) depict the motion of a body exhibiting spherical symmetry.

We also make an assumption that there are estimates

$$|A - B| = O(\varepsilon^2 J_*), |A' - B'| = O(\varepsilon J_*, J_* \sim J_0. \tag{8}$$

Then, after (7), (8) the following equations are provided

$$A - B = \varepsilon(A' - B') = \varepsilon^2 J_*, A - C = \varepsilon A', B - C = \varepsilon B'. \tag{9}$$

After applying transformations to the system (4), we derive the perturbed Euler system. Relations (7)-(9), and transfer to show time $\tau = \varepsilon t$ are considered (terms of order ε^3 and higher are rejected):

$$\begin{aligned}
\frac{dp}{d\tau} &= \frac{B'}{J_0}\left(1 - \varepsilon\frac{A'}{J_0}\right) qr + \varepsilon f_{1p}(p, q, r), && p(0) = p_0,\\
\frac{dq}{d\tau} &= -\frac{A'}{J_0}\left(1 - \varepsilon\frac{B'}{J_0}\right) pr + \varepsilon f_{1q}(p, q, r), && q(0) = q_0,\\
\frac{dq}{d\tau} &= \frac{A' - B'}{J_0} pq + \varepsilon f_{1r}(p, q, r), && r(0) = r_0.
\end{aligned} \tag{10}$$

In the given equation, r is a slow variable. The set of differential equations in (10) constitutes an inherently nonlinear system, wherein the frequency is contingent upon the slow variable. The perturbations were incorporated within (10).

$$\begin{aligned}
\varepsilon f_{1p}(p,q,r) &= -\frac{\rho^2 m}{J_0}\left\{\Omega^{-1}qrh_{11}+\frac{\Omega^{-2}}{J_0}qr(h_{12}-A'h_{11})+\frac{\lambda\Omega^{-4}}{J_0}p(q^2h_{21}+r^2h_{31})\right\}-\\
&-\varepsilon\lambda p,\\
\varepsilon f_{1q}(p,q,r) &= -\frac{\rho^2 m}{J_0}\left\{\Omega^{-1}prg_{11}+\frac{\Omega^{-2}}{J_0}pr(g_{12}+B'g_{11})+\frac{\lambda\Omega^{-4}}{J_0}q(r^2g_{21}+p^2g_{31})\right\}-\\
&-\varepsilon\lambda q,\\
\varepsilon f_{1r}(p,q,r) &= -\frac{\rho^2 m}{J_0^2}\lambda\Omega^{-4}r^3(A'p^2+B'q^2)-\varepsilon\lambda r,\\
g_{11} &= q^2+p^2+r^2, g_{12}=A'(p^2+3r^2)+(3A'-2B')q^2,\\
g_{12} &= 2(A'-B')p^2-B'r^2, g_{31}=(A'-B')(q^2+p^2),\\
h_{11} &= -g_{11}, h_{12}=B'(q^2+3r^2)-(2A'-3B')p^2,\\
h_{21} &= (B'-A')(q^2+p^2+r^2), h_{31}=-A'r^2.
\end{aligned} \tag{11}$$

The perturbation torque of the influence of a movable mass in the rigid body is small [1, 9].

3. Methods of investigation.

The solution of the unperturbed system (10) for $\varepsilon = 0$ is as follows

$$p = acos\varphi, q = -\frac{J_0awsin\varphi}{B'r}, r = r_0. \tag{12}$$

In the given equation, $a = \sqrt{p_0^2 + (\dot{p}_0/w)^2}$ is the amplitude (slow variable), $\varphi = w\tau + \varphi_0$ is the phase, $w = r\sqrt{A'B'}/J_0$, $A'B' > 0$, φ_0 is the initial phase, $cos\varphi_0 = p_0/a$, $sin\varphi_0 = -q_0\sqrt{B'/A'}/a$ supposedly.

We transition from the slow variables p, q, r to the updated standard slow variables a, r and the phase φ by implementing a change in variables:

$$p = acos\varphi, q = -\frac{J_0awsin\varphi}{B'r}, r = r. \tag{13}$$

We differentiate equations (13) considering a perturbed system. Through a series of transformations, we arrive at the system in its standard form, where the point represents the time derivative of τ

$$
\begin{aligned}
&\dot{a}cos\varphi - a\dot{\varphi}sin\varphi = -aw(r)sin\varphi + \varepsilon f_{2p},\\
&\dot{a}sin\varphi + a\dot{\varphi}cos\varphi = aw(r)cos\varphi - \sqrt{\frac{B'}{A'}}\varepsilon f_{2q},\\
&\dot{r} = \frac{B'-A'}{J_0}a^2\sqrt{\frac{A'}{B'}}sin\varphi cos\varphi + \varepsilon f_{2r}, w(r) = \frac{r}{J_0}\sqrt{A'B'},\\
&\varepsilon f_{2p} = \varepsilon\frac{aA'}{J_0}w(r)sin\varphi - \varepsilon\lambda a cos\varphi + \varepsilon f_{1p},\\
&\sqrt{\frac{B'}{A'}}\varepsilon f_{2q} = \varepsilon\frac{aB'}{J_0}w(r)cos\varphi + \varepsilon\lambda a sin\varphi + \sqrt{\frac{B'}{A'}}\varepsilon f_{1p},\\
&\varepsilon f_{2r} = \varepsilon f_{1r} = -\frac{\rho^2 m}{J_0}\lambda\Omega^{-4}r^3(A'p^2 + B'q^2) - \varepsilon\lambda r.
\end{aligned}
\tag{14}
$$

We will address the equations (14) with a focus on solving for $\dot{a}$ and $\dot{\varphi}$, resulting in the derivation of a system

$$
\begin{aligned}
&\dot{a} = \varepsilon f_{2p}cos\varphi - \varepsilon f_{2q}\sqrt{\frac{B'}{A'}}sin\varphi,\\
&\dot{\varphi} = w(r) - \frac{1}{a}\varepsilon f_{2p}sin\varphi - \frac{1}{a}\varepsilon f_{2q}\sqrt{\frac{B'}{A'}}cos\varphi,\\
&\dot{r} = \frac{B'-A'}{J_0}a^2\sqrt{\frac{A'}{B'}}sin\varphi cos\varphi + \varepsilon f_{2r}.
\end{aligned}
\tag{15}
$$

We insert (13) into the third equation (11) for the variable r. With the standard transformations considered, we arrive at the system of equations:

$$
\begin{aligned}
\dot{a} &= \varepsilon\frac{A'-B'}{J_0}awsin\varphi cos\varphi + \varepsilon f_{1p}cos\varphi - \varepsilon f_{1q}\sqrt{\frac{B'}{A'}}sin\varphi,\\
\dot{r} &= \frac{B'-A'}{J_0}a^2\sqrt{\frac{A'}{B'}}sin\varphi cos\varphi - \frac{\rho^2 m}{J_0^2}\lambda\Omega^{-4}r^3(A'a^2cos^2\varphi + B'(-\frac{J_0aw}{B'r})^2sin^2\varphi),\\
\varepsilon f_{1p}cos\varphi &= -\frac{\rho^2 m}{J_0}cos\varphi\Bigg\{-\frac{J_0awsin\varphi}{B'}[(\Omega^{-1} - \frac{\Omega^{-2}}{J_0}A')(-r^2 - a^2cos^2\varphi - (\frac{J_0aw}{B'r})^2sin^2\varphi)]+\\
&+\frac{\Omega^{-2}}{J_0}[B'(3r^2 + (\frac{J_0aw}{B'r})^2sin^2\varphi) - (2A'-3B')a^2cos^2\varphi]+\\
&+\frac{\lambda\Omega^{-4}J_0}{a}cos\varphi[(\frac{J_0aw}{B'r})^2sin^2\varphi(-2r^2 - a^2cos^2\varphi - (\frac{J_0aw}{B'r})^2sin^2\varphi) + A'r^4]\Bigg\} - \varepsilon\lambda a,
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
\varepsilon f_{1q} sin\varphi = & -\frac{\rho^2 m}{J_0} sin\varphi \left\{ racos\varphi \left[(\Omega^{-1} - \frac{\Omega^{-2}}{J_0} B')(r^2 + a^2 cos^2\varphi + (\frac{J_0 aw}{B'r})^2 sin^2\varphi) - \right.\right. \\
& - \left. \frac{\Omega^{-2}}{J_0}(A'(3r^2 + a^2 cos^2\varphi) + (3A' - 2B')a^2(\frac{J_0 aw}{B'r})^2 sin^2\varphi) \right] + \\
& + \frac{\lambda\Omega^{-4}}{J_0}(-\frac{J_0 aw}{B'r} sin\varphi) \left[r^2(2a^2 cos^2\varphi(A' - B') - B'r^2) + a^2 cos^2\varphi(A' - B') \times \right. \\
& \times \left.\left. (a^2 cos^2\varphi + (\frac{J_0 aw}{B'r})^2 sin^2\varphi) \right] \right\} - \varepsilon\lambda r.
\end{aligned} \tag{17}
$$

In the given system of equations, the value $w(r) = r\sqrt{A'B'}/J_0$ represents the perturbed frequency of the converted system. If we average system (16) over the phase φ [25], we obtain:

$$
\begin{aligned}
\dot{a} &= \beta\gamma(\alpha a^5 + \eta a^3 r^2 - ar^4) + ca, \\
\dot{r} &= \beta A' a^2 r^3 + cr.
\end{aligned} \tag{18}
$$

The notations are introduced at this point.

$$
\begin{aligned}
\beta &= \frac{-\rho^2 m}{J_0^2}\lambda\Omega^{-4}, \quad \eta = \frac{1}{2}(1 - \frac{A'}{B'}), \\
\alpha &= \frac{1}{8}(1 - \frac{A'^2}{B'^2}), \quad \gamma = \frac{1}{2}(A' - B'), c = -\varepsilon\lambda.
\end{aligned}
$$

We transform system (17) to:

$$
\begin{aligned}
\frac{dx}{d\tau} &= 2x[\beta\gamma(\alpha x^2 + \eta xy - y^2 + c)], \\
\frac{dy}{d\tau} &= 2y(\beta A' xy + c).
\end{aligned} \tag{19}
$$

Here we include the slow variables $x = a^2$, $y = r^2 > 0$ in system (18).

4. Numerical integration.

System (18) was numerically resolved using the initial conditions $x(0) = 1$, $y(0) = 1$ and task factors $m = 1$, $p = 1$, $\varepsilon = 0.1$, $\lambda = 1.25 rad/s$.

The graphical representations of the varying values $x = a^2$ and $y = r^2$ (the squared equatorial and axial components of the rigid body angular velocity vector) are represented when parameters are $\lambda = 98$, $\Omega = 10$ in the first case and $\lambda = 9$, $\Omega = 3$ in the second case.

In the first case $J_0 = 1$, $A' = 5.1$, $B' = 5$, in the second case $J_0 = 3$, $A' = 8.3$, $B' = 8$.

The increase of $x = a^2$ in the first and second cases (parameter J_0, A', B') is faster with values of $\lambda = 9$, $\Omega = 3$.

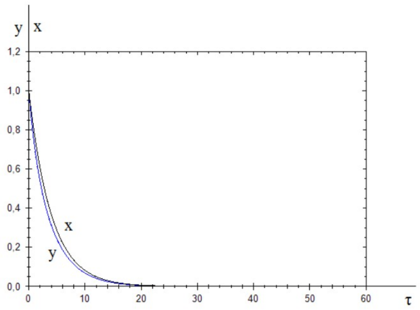

Fig. 1.

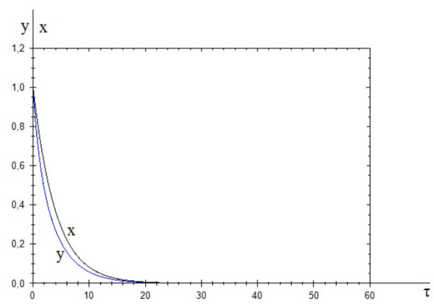

Fig. 2.

In the second case the variable $y = r^2$ asymptotically approaches zero faster with values $\lambda = 9$, $\Omega = 3$.

5. Conclusion.

We investigate the motion in a resistive medium of a nearly dynamically spherical rigid body with a moving mass, connected to it with elastic linkage.

The use of an asymptotic approach allows us to acquire qualitative insights, depict the motion's evolution through streamlined averaging equations, and derive analytical and numerical solutions over the infinite time interval. These outcomes enable us to assess the dynamic consequences of having a moving mass attached to the body.

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Еволюція рухів в середовищі з опором близького до динамічно сферичного тіла з рухомою масою.

Проблема руху твердого тіла відносно нерухомої точки є однією з класичних задач механіки. Інтерес до задач динаміки твердого тіла посилився в другій половині XX сторіччя в зв'язку з розвитком ракетно-космічної техніки. Дослідження руху супутників навколо центра мас важливо для створення систем управління орієнтацією, стабілізації руху і для розв'язування практичних задач космонавтики. В статті з допомогою усереднення одержується наближене розв'язання задачі про рух в середовищі з опором близького до динамічно сферичного твердого тіла з в'язкопружним елементом. Асимптотичний підхід дозволяє одержати деякі якісні результати і описувати еволюцію руху твердого тіла з допомогою спрощених усереднених рівнянь та чисельного розв'язку. В роботі проведена процедура усереднення та одержані усереднені рівняння, які простіше початкових, та описують рух на великому інтервалі часу. Одержані кількісні та якісні результати досліджень руху в середовищі з опором близького до динамічно сферичного твердого тіла з рухомою масою та був представлений опис еволюції руху тіла. Проведено чисельне інтегрування усередненої системи рівнянь руху твердого тіла. Графічні зображення розв'язків представлені та обговорені. Одержано систему рівнянь руху в стандартній формі, яка уточнена в квадратичному наближенні за малим параметром. Досліджено еволюцію руху Ейлера-Пуансо під дією малих внутрішніх та зовнішніх моментів. Стаття може розглядатися як розвиток попередніх задач про рух твердого тіла під дією малих моментів (порожнини, яка заповнена рідиною великої в'язкості, рухомої маси, сталих моментів в зв'язаних з тілом осях). Стаття вносить вклад в вивчення задач руху космічних кораблів і рухів членів екіпажів відносно цих тіл. Одержані результати важливі для управління з допомогою рухомих мас, для рухів обертових снарядів.

Ключові слова: *близьке до динамічно сферичного твердого тіла, рухома маса, пружні та дисипативні елементи, середовище з опором.*

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