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DETERMINATION OF DISSIPATION FORSE IN OSCILLATORY SYSTEMS

The problem of estimating the dissipation force arising during the motion of a linear dissipative oscillator is considered. A method for determining the oscillator velocity and the viscous friction coefficient using only displacement measurements is proposed. It is based on the synthesis of invariant relations that allow us to relate the variables of a special extended dynamic system and determine the unknown variables as functions of the known ones. The efficiency of the proposed approach for the Duffing oscillator is demonstrated using numerical simulation.

MSC: 34C15, 34D20.

Keywords: *oscillator, identification of dissipation, invariant relations, asymptotic estimation.*

1. Introduction.

Mathematical models of oscillators are widely used for modeling, analysis, and control in various fields such as mechanics, electronics, control systems, biomedical research, geology, etc. This makes the problem of determining both their parameters and state variables particularly important. Empirical methods for determining the dynamic characteristics of oscillatory systems are widely used (see for instance [1]).

With the development of mathematical control theory, publications appeared focusing on the construction of observers for differential equations of oscillators. Many authors have explored methods for online estimation of unknown parameters of a periodic signal. In particular, paper [2] considers an algorithm for the nonlinear representation of a sinusoidal signal with the subsequent use of a nonlinear Luenberger observer to determine its parameters. The authors built a rather complicated construction of the corresponding observer based on the general theory of nonlinear observation.

In [3], after presenting the sinusoidal signal in a linearly parameterized form, several adaptive laws were developed and applied to estimate the sinusoid parameters using the methods of the sliding mode control theory. The use of methods for identifying external sinusoidal signals led to the determination of parameters of oscillating objects, such as phase, amplitude, offset and frequency [4]. In [5], an estimator for a multi-sinusoidal signal was constructed using the standard gradient algorithm. For a biased multi-sinusoidal signal an estimator is designed to identify the frequency, amplitude and phase of each sinusoids as well as the overall offset. External force estimation has been addressed by numerous authors (see, for instance, [6–9]).

The main contribution of this paper is the proposal of an online methodology that allows reconstructing an unknown force of complex nature, namely the dissipative force arising from the motion of an oscillator. The force itself is represented as the product of the velocity and the dissipation coefficient. Here, both factors are unknown, only the oscillator displacement is measured, which is assumed to be known as a function of time. A similar problem for determining nonlinear dissipation of van der Pol oscillator was solved in [7]. In the mentioned work it was shown for the first time that the change of coordinates (the Lienard transformation [10]) reduces the oscillator equation to an affine form with respect to the unknowns. Such a representation of the original system enabled to use the methods of constructing a classical observer in the original problem.

Unlike the article [7], we used the method of invariant relations for a slightly different observer design. This method [11] was developed in analytical mechanics and was intended, in particular, to find particular solutions (dependencies between variables) in problems of rigid body dynamics with a fixed point. Its modification for problems of observation and identification theory enabled the synthesis of additional relations between known and unknown quantities of the original system which arise on the trajectories of its extended dynamic model [12].

2. The problem of observing the dissipating force.

As a model of a self-oscillating system, consider the equation of forced motion of a nonlinear oscillator in general form:

$$\ddot{x}(t) + f(x(t))\dot{x}(t) + g(x(t)) = u(t). \tag{1}$$

Here, $x(t)$ is the displacement of the point from the coordinate origin; $\dot{x}(t)$ is the velocity of this displacement. The product $f(x(t))\dot{x}(t)$ and the function $g(x(t))$ describe the dissipation law and potential forces, respectively. An external periodic force $u(t)$ acting on the oscillator will be further represented as a harmonic function of time

$$u(t) = A\cos(\omega t + \gamma).$$

The output of the system is the displacement $x(t)$ measured during the oscillations, i.e. known as a function of time.

Remark. *At present, in the qualitative theory of differential equations, many questions have been studied related to the equation (1) regarding the boundedness of oscillations, the existence of periodic solutions, the presence of limit cycles, etc. Further, we will assume that the problem of observing and identifying the process of permanent oscillations of a real object is being solved and its solutions are correctly defined in a certain bouunded domain* $\{x(t), \dot{x}(t)\} \subset R^2$.

To determine the law of dissipation, we will assume $f(x(t)) = \delta$ and then viscous friction will be represented by the standard formula $F_{diss} = \delta\dot{x}(t)$, where the part of the phase vector $\dot{x}(t)$ and the coefficient δ are unknown and must be determined from data on the oscillator movement. Other components of the model, namely the function $g(x,t)$, the parameters of the external force A, ω, γ are assumed to be known.

In order for the unknowns $\dot{x}(t)$ and the coefficient δ to enter the equations of motion linearly, we write the original equation (1) as a Liénard system [10], making the next change of variables:

$$x_1(t) = x(t), \quad x_2(t) = \dot{x}(t) + F(x(t)),$$

where $F(x(t)) = \int_0^x f(\sigma)d\sigma$. As a result, taking into account the chosen dissipation law, we get:

$$\dot{x}_1(t) = x_2(t) - \delta x_1(t), \quad \dot{x}_2(t) = -g(x_1(t)) + u(t). \tag{2}$$

Thus, the unknown speed $\dot{x}(t)$ expressed in variables of system (2) will be found by the formula

$$\dot{x}(t) = x_2(t) - \delta x_1(t)$$

after finding $\delta, x_2(t)$. Taking into account the boundness of variables $x(t), \dot{x}(t)$, we will further assume that the phase vector of the system (2) belongs to a certain bounded domain $S_x = \{(x_1(t), x_2(t))\} \subset R^2$).

For this system, the initial problem of determining the magnitude of dissipation looks like the problem of evaluating the expression

$$F_{diss} = (x_2(t) - \delta x_1(t))\delta. \tag{3}$$

Next we will solve the following problem.

Problem. *Find for the system (2) asymptotically exact estimates of the phase vector component $x_2(t)$ and the coefficient δ based on the output data $x_1(t)$.*

3. Invariant relations approach.

Now let us briefly outline the method of invariant relations that we propose. The main idea is, being based on the data about the system motion, to obtain additional relationships for the unknowns. Such information includes the output $x_1(t)$ and only those quantities that can be obtained from the output. In particular, based on the number of unknowns, we introduce into consideration the vector $y = (\xi_1, \xi_2)^T$, which we define as the solution of the Cauchy problem for a certain system of differential equations,

$$\dot{y}(t) = U(t, y(t), x_1(t)), \quad y(0) \in R^2. \tag{4}$$

The next step of the method of invariant relations is to determine additional coupling equations $H(t, x, y, \delta) = 0$ that arise for the known quantities $x_1(t), y(t)$ and the unknown ones $x_2(t), c$ on some trajectories of the extended system of differential equations (2) and (4).

In this case, the selection of the appropriate function $U(t, y, x_1(t))$ must ensure that the following conditions are met:

(i) rank$\frac{\partial H(t,x,y,\delta)}{\partial(x_2,\delta)} = 2$;

(ii) the invariant manifold $\{(x, y, \delta) \in R^2 : H(t, x, y, \delta) = 0\}$ corresponding to these additional equalities has the property of global attraction for any solutions of the extended system. In other words, on any trajectories of the differential equations (2), (4),

$$\lim_{t\to\infty} \|H(t, x(t), y(t), \delta)\| = 0.$$

To satisfy property (i), we will look for additional relations on the trajectories of the system of differential equations (2) and (4) in the form:

$$\delta = \Psi_1(x_1(t)) + \xi_1(t), \quad x_2(t) = \Psi_2(x_1(t)) + \xi_2(t). \tag{5}$$

Similarly to the right-hand side $U(t, y, x_1)$ of the differential equations (4), the functions $\Psi_1(x_1), \Psi_2(x_1)$ are still undefined. It is assumed that these functions are continuously differentiable with respect to their arguments. In the space of variables (x_1, x_2, ξ_1, ξ_2), equalities (5) form a two-dimensional manifold. Let us also introduce into consideration the variables $\varepsilon_1, \varepsilon_2$, that characterize the deviations from relations (5):

$$\varepsilon_1(t) = \delta - \Psi_1(x_1(t)) - \xi_1(t), \quad \varepsilon_2(t) = x_2(t) - \Psi_2(x_1(t)) - \xi_2(t). \tag{6}$$

Thus, the magnitude of the dissipation force acting on the oscillator will be estimated by the formula

$$F_{diss} = [\Psi_2(x_1) + \xi_2 + \varepsilon_2 - (\Psi_1(x_1) + \xi_1 + \varepsilon_1)](\Psi_1(x_1) + \xi_1 + \varepsilon_1). \tag{7}$$

If the equalities $\varepsilon_1(t) = \varepsilon_2(t) \equiv 0$ hold on some trajectories of the extended system for certain functions $U(t, y(t), x_1(t)), \Psi_1(x_1(t)), \Psi_2(x_1(t))$, then the relations (5) become invariant and determine $x_2(t)$ and coefficient δ on these trajectories. Non-zero deviations $\varepsilon_1(t), \varepsilon_2(t)$ appear on other trajectories.

Consider formulas (6) as a substitution of the variables x_2 and δ in the original system. Then the differential equations for $\dot{x}_2$ and $\dot{\delta}$ are transformed into differential relations (here $(.)'_q$ means the partial derivative $\frac{\partial(.)}{\partial q}$)

$$\dot{\xi}_1 + \dot{\varepsilon}_1 = -\Psi_1{}'_{x_1} v, \quad \dot{\xi}_2 + \dot{\varepsilon}_2 = -g(x_1) + u - \Psi_2{}'_{x_1} v, \tag{8}$$

where

$$v = \Psi_2 + \xi_2 + \varepsilon_2 - (\Psi_1 + \xi_1 + \varepsilon_1)x_1.$$

Let us determine the structure of the additional system of differential equations (4). At the first step of the proposed approach aimed at determining the function $U(t, y, x_1)$, we select from the previous equalities all terms that do not depend on deviations. As a result we get

$$\begin{aligned} \dot{\xi}_1 &= -\Psi_1{}'_{x_1}[\Psi_2 + \xi_2 - (\Psi_1 + \xi_1)x_1], \\ \dot{\xi}_2 &= -g(x_1) + u - \Psi_2{}'_{x_1}[\Psi_2 + \xi_2 - (\Psi_1 + \xi_1)x_1]. \end{aligned} \tag{9}$$

In accordance with this choice, we have that if $\xi_1(t), \xi_2(t)$ are any functions that satisfy (9), then the terms remaining in relationships (8) form the following $\varepsilon_1(t), \varepsilon_2(t)$:

$$\dot{\varepsilon}_1 = -\Psi_1{}'_{x_1}(\varepsilon_2 - \varepsilon_1 x_1), \quad \dot{\varepsilon}_2 = -\Psi_2{}'_{x_1}(\varepsilon_2 - \varepsilon_1 x_1). \tag{10}$$

System (10) has the trivial solution $\varepsilon_1(t) = \varepsilon_2(t) \equiv 0$. So, we have obtained a family of additional systems (9), depending on arbitrary functions $\Psi_1(x_1), \Psi_2(x_1)$ and their derivatives. Thus, the following lemma is proved.

Lemma. *For any functions $\Psi_1(x_1), \Psi_2(x_1)$ differentiable with respect to their arguments the corresponding system (2),(9) on trajectories with initial conditions $\xi_1(0) = \delta - \Psi_1(x_1(0)), \xi_2(0) = x_2(0) - \Psi_2(x_1(0))$ turns the equalities (5) into invariant relations, from which the coordinate $x_2(t)$ and parameter δ can be directly found.*

4. Convergence properties.

We do not know on which trajectories of the system of differential equations (2), (9) relations (5) become invariant. Therefore, it is natural to consider the problem of choosing the functions $\Psi_1(x_1), \Psi_2(x_1)$ in such a way that the deviations in formulas (6) tend to zero. In other words, the trivial solution of equations (10) should become globally asymptotically stable. The second stage of the proposed method is related to solving this problem. There are several approaches to stabilizing non-autonomous systems (see [13], [14]). At this stage, we assume that the appropriate method is selected depending on the analytical form of the original dynamic system.

To stabilize trivial solution of the system (10), we use the partial derivatives of uncertain functions ${\Psi_1}'_{x_1},\ {\Psi_2}'_{x_1}$ as a control. The purpose of control is to fulfill the condition

$$\lim_{t\to\infty}[\varepsilon_1^2(t)+\varepsilon_2^2(t)]=0. \tag{11}$$

Let us consider the positive definite function $V(t)=\frac{1}{2}[\varepsilon_1^2(t)+\varepsilon_2^2(t)]$ and write its derivative along the solutions of system (10),

$$\dot V(t)=({\Psi_1}'_{x_1}\varepsilon_1+{\Psi_2}'_{x_1}\varepsilon_2)(\varepsilon_1x_1-\varepsilon_2).$$

Let us choose the remaining undefined functions by setting:

$$\Psi_1(x_1)=-\lambda x_1^2/2,\quad \Psi_2(x_1)=\lambda x_1, \tag{12}$$

where λ is a positive constant. Under such functions, the derivative

$$\dot V(t)=-\lambda(\varepsilon_1x_1-\varepsilon_2)^2\le 0.$$

and linear homogeneous differential equations for the deviations have the form

$$\dot\varepsilon_1=-\lambda x_1(\varepsilon_2-\varepsilon_1x_1),\quad \dot\varepsilon_2=-\lambda(\varepsilon_2-\varepsilon_1x_1) \tag{13}$$

That is, the function $V(t)$ becomes a Lyapunov function with a negative semidefinite derivative for system of differential equations (2),(13). In this case, the zero solution of the equations in deviations is uniformly stable.

Let us show that it is asymptotically stable. To do this, we will use the LaSalle's invariance principle [15] for autonomous systems. In our case, equations (2) depend on time, $z_1(t)=u(t)$. However, when this dependence the dynamic expansion of the equations for deviations by the differential equations

$$\dot z_1(t)=z_2(t),\ \dot z_2(t)=-\omega^2z_1(t), \tag{14}$$

Next we will consider the systems of differential equations (13) and (14) as an dynamical extensions of the initial system (2).

Proposition 1. *If the functions $\Psi_1(x_1), \Psi_2(x_1)$ are chosen according to formulas (12), then the trivial solution of the system of differential equations for the deviations (10) is globally exponentially stable.*

Proof. The phase vector of the extended system is

$$(x_1, x_2, \varepsilon_1, \varepsilon_2, z_1, z_2) \in S_x \times S_\varepsilon \times S_z \subset R^6,$$

where

$$S_\varepsilon = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1^2(t) + \varepsilon_2^2(t) \leq \varepsilon_1^2(0) + \varepsilon_2^2(0)\}$$

is a ball in R^2 and

$$S_z = \{(z_1, z_2) : z_1^2(t) + z_2^2(t) = A^2\}$$

is a sphere in R^2. Thus, the set $\overline{S_x} \times S_\varepsilon \times S_z = \Omega$ is compact and positive invariant.

Let E be a set of all points $(\varepsilon_1, \varepsilon_2, z_1, z_2)$ where the derivative of the Lyapunov function is identically equal to zero: $\dot{V} = -\lambda(\varepsilon_1 x_1 - \varepsilon_2)^2 \equiv 0$. It is clear that the set $M = \{(\varepsilon_1 = 0, \varepsilon_2 = 0, z_1(t), z_2(t))\}$ is invariant in E. If M is the largest invariant set, then according to LaSalle's invariant principle, all solutions with their initial values in E tend to M, i.e. condition (11) is satisfied.

Let us show that M is the largest invariant set in E. To this end, let us assume the opposite, i.e., the set E contains trajectories with non-zero deviations $\varepsilon_i(t)$. From the differential equations (13) it follows that these non-zero deviations are constant $\varepsilon_i(t) = \varepsilon_i^*, i = 1, 2$. Hence, our assumption leads to the existence of the relationship

$$\varepsilon_1^* x_1(t) - \varepsilon_2^* = 0.$$

The last equality means that the $x_1(t)$ is a constant, which contradicts our assumption about permanent oscillations of the oscillator. The resulting contradiction proves the statement that the set M is the largest invariant set in E, and therefore, according to LaSalle's invariant principle, conditions (11) are satisfied. Hence, the Proposition is proved.

5. Final evaluation of unknowns.

In the proposed algorithm, all degrees of freedom have been fixed. Specifically, the structure of additional differential equations is defined by equation (9). The choice of functions $\Psi_1(t, x_1), \Psi_2(t, x_1)$ according to formulas (12) determines the final form of these equations:

$$\begin{aligned} \dot{\xi}_1 &= \lambda x_1[(\lambda x_1 + \xi_2 + (\lambda x_1^2/2 - \xi_1)x_1], \\ \dot{\xi}_2 &= -g(x_1) - [(\lambda x_1 + \xi_2 + (\lambda x_1^2/2 - \xi_1)x_1] + u. \end{aligned} \tag{15}$$

According to this scheme, the terms in formulas (6) become known and the deviations $\varepsilon_1(t), \varepsilon_2(t)$ asymptotically tend to zero, which allows us to directly obtain asymptotic estimates for $x_2(t)$ and parameter δ. Thus, the statement formulated below is proved.

Proposition 2. *The formulas*

$$\begin{aligned} \hat{\delta} &= -\lambda x_1^2/2(t) + \xi_1(t), \\ \hat{x}_2(t) &= \lambda x_1(t) + \xi_2(t), \end{aligned} \tag{16}$$

where $\xi_1(t), \xi_2(t)$ are the solutions of any Cauchy problem for system of differential equations (14) give asymptotic estimates of the oscillation velocity $x_2(t)$ and coefficient δ for the system (2).

Taking into account (3) the estimate of the dissipation force, we find it using the formula

$$\hat{F}_{diss} = (\hat{x}_2(t) - \hat{\delta}x_1(t))\hat{\delta}. \tag{17}$$

6. Simulation results. Duffing oscillator.

Below are the results of some tests carried out to identify the possibility of implementing the proposed methodology for the Duffing oscillator. The Duffing oscillator model is a physical model of an oscillating system that consists of a material point attached to a nonlinear spring and a linear shock absorber. The equation of forced oscillations can be written as follows:

$$\ddot{x}(t) + \delta\dot{x}(t) + \alpha x(t) + \beta x^3(t) = u(t). \tag{18}$$

Here, $x(t)$ is the displacement of the material point; δ characterizes dissipation in the system; α and β are the linear and nonlinear stiffness coefficients, respectively; $\dot{x}(t)$ and δ are unknowns. It should be noted that if β is negative, the dynamics of the trajectories are trivial, as all solutions of the differential equation (18) exponentially tend to infinity. Hence, we consider the case $\beta \geq 0$. As in the general case, we assume that solutions (18) are correctly defined in the interval $[0, \infty)$.

Let us rewrite equation (18) in the form of a system in the space of states by substituting the Liénard variables. We obtain

$$\begin{aligned} \dot{x}_1 &= x_2 - \delta x_1, \\ \dot{x}_2 &= -\alpha x_1 - \beta x_1^3 + u(t). \end{aligned} \tag{19}$$

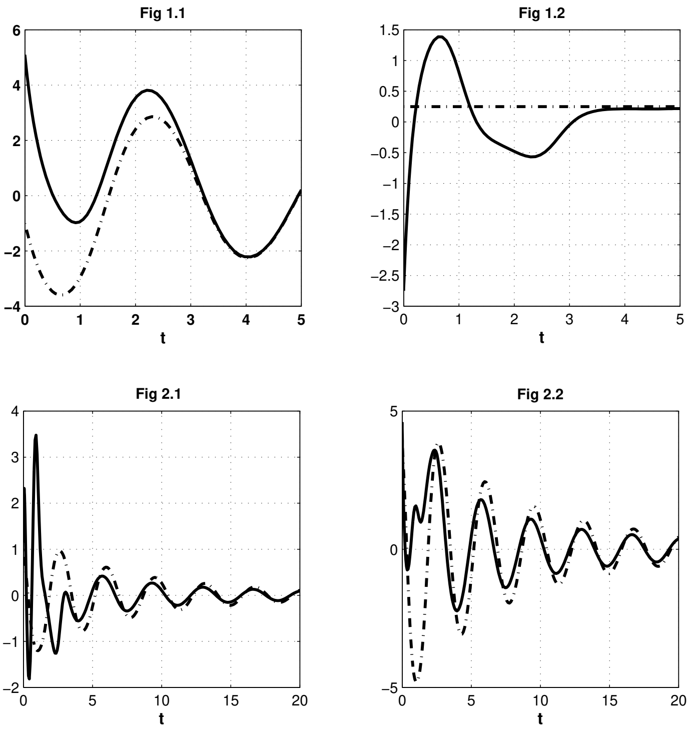
The scheme proposed in this work was numerically tested within a wide range of initial conditions and parameters of the dynamic system (19). This system was used to obtain the output $x_1(t)$ and to find the asymptotic estimates of $x_2(t)$ and δ. The parameters of the simulated system are equal

$$\delta = 1;\ \alpha = 3;\ \beta = 0.2;\ \omega = 0.5;\ \lambda = 1.$$

The auxiliary system looked like

$$\begin{aligned} \dot{\xi}_1 &= \lambda x_1[(\lambda x_1 + \xi_2 + (\lambda x_1^2/2 - \xi_1)x_1], \\ \dot{\xi}_2 &= \alpha x_1 + \beta x_1^3 - [(\lambda x_1 + \xi_2 + (\lambda x_1^2/2 - \xi_1)x_1] + u. \end{aligned} \tag{20}$$

The corresponding modeling results (solid lines for estimated unknowns and dotted lines for their exact values) can be seen in Figures 1 and 2. These estimates, in complete accordance with the formulas (15), asymptotically tend to the true values of the unknowns. In particular, the figures show graphs of the following functions:



For the transformed system (19)

Fig.1.1 $\hat{x}_2(t)$ vs $x_2(t)$; Fig.1.2 $\hat{\delta}(t)$ vs δ.

For the initial system (18)

Fig.2.1 $\hat{x}_2(t) - \hat{\delta}(t)x_1(t)$ vs $\dot{x}(t)$; Fig.2.2 $(\hat{x}_2(t) - \hat{\delta}(t)x_1(t))\hat{\delta}(t)$ vs $F_{diss} = \dot{x}(t)\delta$.

As shown in these graphs, the simulation results confirm the effectiveness of the proposed method of asymptotic estimation of the velocity oscillator and parameters of an external periodic force.

7. Conclusions.

A dynamic system has been considered that describes the oscillations of the Liénard oscillator under the influence of an external load. The movement of the oscillator is measured, and its position is a known function of time. It is required to solve the problem of online reconstruction of unknown quantities, namely, the oscillation speed and the coefficient of viscous friction. A method of asymptotic estimation is proposed, based on the synthesis of invariant relations for the trajectories of an auxiliary dynamic system with invariant relations reflecting the relationship between unknown and known quantities on the family of solutions under study. The convergence of estimates of unknown variables to their exact values is proven based on LaSalle's principle of invariance. An example of numerical simulation of the proposed approach using the Duffing oscillator as an example has been given.

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Визначення дисипативної сили в коливальних системах.

Математичні моделі осциляторів широко використовуються як спосіб моделювання, аналізу або навіть керуванні у різних сферах, таких як механіка, електроніка, керування, медико-біологічні дослідження, геологія та ін. Природньо, що в багатьох випадках на практиці виникають проблеми визначення стану та параметрів таких моделей за результатами вимірювання вихідних сигналів у режимі функціювання реальних об'єктів. Тим паче, що одна із важливих складових динаміки руху осциляторів, а саме, дисипація енергії коливань має складну фізичну природу. Проблему визначення сили демпфірування розглянуто у цій статті. Сама сила предсавлена як добуток швидкості на коефіцієнт дисипації. При цьому обидва множники є невідомими, вимірюється лише переміщення осцилятора, яке вважається відомим як функція часу. Проведено заміну змінних вихідної системи, після якої невідомі компоненти сили дисипації входять до перетворених диференціальних рівнянь лінійним чином. Запропонована схема визначення асимптотичних оцінок швидкості коливань та параметра в'язкості спочатку для перетвореної, а потім і для вихідної систем. Для отримання оцінок невідомих був використаний розроблений в аналітичній механіці метод інваріантних співвідношень, модифікація якого у задачах спостереження, ідентифікації дозволяє синтезувати додаткові співвідношення, що виникають між відомими та невідомими величинами під час спостережуваного руху побудованої спеціальним чином розширеної динамічної системи. В результаті в роботі було запропоновано новий суттєво нелінійний метод отримання асимптотичних оцінок стану та параметрів для перетвореною в результаті заміни змінних системи. Чисельне моделювання проведене для осцилятора Даффінга підтверджує ефективність цього способу конструкції онлайн ідентифікатора для визначення залежності сили дисипації від часу за даними про переміщення.

Ключові слова: *осцилятор, ідентифікація дисипації, інваріантні співвідношення, асимптотичне оцінювання.*

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