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MODELING CRACK INITIATION IN A COMPOSITE UNDER BENDING

A mathematical description of a crack initiation model in a binder composite under bending is carried out. The determination of the unknown parameters characterizing an initial crack reduces to solving a singular integral equation. A closed system of nonlinear algebraic equations is constructed, whose solution helps to predict cracks in a composite under bending, depending on the geometric and mechanical characteristics of both the binder and the inclusions. The criterion of crack initiation in a composite under the influence of bending loads is formulated.

Introduction

The creation of new materials of high strength, rigidity and reliability opens up great opportunities for their wide application in various fields of engineering and construction. These include, in particular, fibrous composite materials. Solving various engineering problems requires objective information about the stress-strain state in the structural elements made from composite materials. Such information can only be obtained by taking into account the main features of these materials. At the design stage of new structures made from composite materials, it is necessary to take into account the cases when cracks may appear in the material. In this regard, it is necessary to carry out a limit analysis in order to establish that the expected initial damage will not grow to critical dimensions and will not cause damage during the design life.

At present, one of the main places in the mechanics of composite materials is occupied by the problems related to their structural peculiarities. One of the main features of composite materials, which is important to take into account while studying various kinds of problems in the mechanics of composite bodies, are damages to their structures. These damages can be caused by the construction of composite materials themselves, and can result from various factors in technological processes. In practice, successful application of artificially created composite materials is largely connected with the solution of the problems of determining their stress-strain state, taking into account the their structural features, in particular, damages in both the binder and reinforcing elements. Therefore, studies of the stress-strain state in damaged composite materials should be recognized as very relevant. A large number of papers [1–22 and others] have been devoted to the problems of the stress-strained state and destruction of a fiber composite. It is important to develop a mathematical model that allows predicting the stress-strain state of a composite at the pre-fracture stage (the formation of cracks).

The purpose of this paper is to construct a computational model for a binder-inclusion composite body, which makes it possible to calculate the limiting external bending loads at which cracking occurs in a composite.

Problem formulation

Let the unbounded composite plate (composite) be subjected to bending by the mean moments (bending at infinity) $M_x = M_x^\infty$, $M_y = M_y^\infty$, $H_{xy} = 0$. When a composite is loaded in a binder material, a pre-fracture zone will appear, oriented in the direction of the maximum tensile stresses. A pre-fracture zone is modeled as a region of weakened inter-particle bonding of the material. It is believed that when a composite is loaded in it (a layer of over-stressed material), a zone of plastic flow is formed. The studies [23–25] of the formation of regions with a disturbed structure of the material indicate that at the initial loading stage pre-fracture zones represent a narrow elongated layer, and then, with an increase in the external load, a secondary system of zones of weakened inter-particle bonding of the material suddenly appears.

Let, for definiteness, the external bending load change so that plastic deformation occurs in the zone of weakened inter-particle bonds of the binder material. After a certain number of loading cycles, the possibility of plastic deformation in the zone of weakened inter-particle bonds of the material is exhausted and the opening of plastic flow zone faces sharply increases. If the opening of the pre-fracture zone faces at the point of maximum concentration reaches the limit value δ_c for a given binder material, then a crack (a break in the material inter-particle bonding) arises at this point.

In the process of a composite being subjected to bending moments, a pre-fracture zone will occur in the binder material. For a mathematical description of the interaction of pre-fracture zone faces, it is accepted that in this zone there are bonds between the faces, that restrict the opening of the weakened inter-particle bond zone faces of the material. The interaction of the pre-fracture zone faces is modeled by inserting plastic slip lines (degenerate plastic deformation bands) between its faces. The position and dimensions of the plastic flow zones depend on the type of material and loading. It is believed that in a pre-fracture zone there is plastic flow at constant voltage. In this case, the location and size of the pre-fracture zone are not known in advance and should be determined in the process of solving the problem. The interaction of the binder with the fibers is analyzed on the basis of a single-fiber model. The remaining fibers get 'smeared', and the material outside the isolated fiber appears to be homogeneous and isotropic with the corresponding effective elastic constants (by the 'mixture' rule). The interaction between other smeared fibers and pre-fracture zones is carried out through the corresponding effective elastic constants. At the same time, there are no restrictions on the relative position and relative sizes of the fibers and pre-fracture zones. It is believed that pre-fracture zones do not intersect each other and the fiber.

The origin of the *Oxy* coordinate system is compatible with the geometric center of a fiber (Fig. 1) in the middle plane of the composite plate. It is assumed that an elastic fiber from other material is inserted into the circular hole of the binder.

It is believed that all over the joining region boundary L ($\tau = R\exp(i\theta)$) there is a rigid adherence of various materials. In the center of a rectilinear pre-fracture zone, we place the origin of the local coordinate system $O_1x_1y_1$, whose x_1 axis coincides with the pre-fracture zone line and forms the angle α_1 by the axis x.

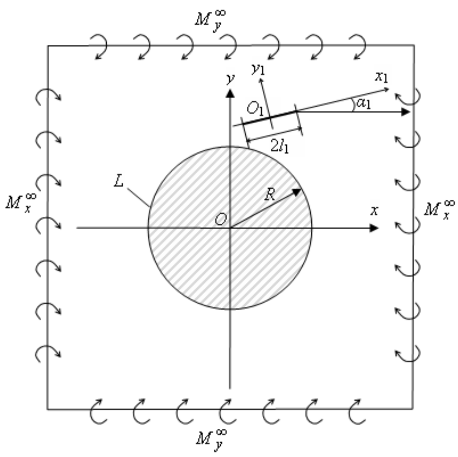

Fig. 1. Design diagram of cracking in a composite binder under bending

On the media separation boundary, the following conditions must be fulfilled

$$w = w_0, \quad \frac{\partial w}{\partial n} = \frac{\partial w_0}{\partial n}, \quad \frac{\partial w}{\partial t} = \frac{\partial w_0}{\partial t}, \quad \frac{\partial^2 w}{\partial n^2} = \frac{\partial^2 w_0}{\partial n^2}, \quad \frac{\partial^2 w}{\partial t^2} = \frac{\partial^2 w_0}{\partial t^2}, \quad \frac{\partial^2 w}{\partial n \partial t} = \frac{\partial^2 w_0}{\partial n \partial t}, \tag{1}$$

where w and w_0 are the deflections of a binder and fibers, respectively, n and t are natural coordinates (normal and tangent to the boundary L).

These relations (1) are a consequence of the continuity of composite deflections, angles of tangent inclination and values of bending moments.

When a composite is subjected to external bending moments in the bonds connecting the pre-fracture zone faces, both normal $\sigma_{y_1} = \sigma_s$ and tangential $\tau_{x_1y_1} = \tau_s$ stresses will arise. Here σ_s is the tensile yield stress of a material, τ_s is the shear yield stress of a material.

The boundary conditions on pre-fracture zone faces will have the following form:

$$M_n = M_S, \quad N_n + \frac{\partial H_{nt}}{\partial t} = H_s,$$

where $M_s = \sigma_s h^2/4$; $H_s = \tau_s h^2/4$; h is the thickness of a composite plate (composite); M_n, H_n are the specific bending and twisting moments; N_n is the specific transverse force.

To determine the values of the external bending load, at which cracking takes place in the binder, it is necessary to supplement the formulation of the problem with a crack appearance criterion (rupture of the material inter-particle bonds). As a condition, we take the criterion of the maximum opening of the weakened inter-particle bond zone faces of the material

$$\left|\left(v_1^+ - v_1^-\right) - i\left(u_1^+ - u_1^-\right)\right| = \delta_c, \tag{2}$$

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where δ_c is the resistance characteristic of the binder material to crack formation; $\left(v_1^+ - v_1^-\right)$ is the normal component of the opening of a pre-fracture zone faces; $\left(u_1^+ - u_1^-\right)$ is the tangential component of the opening of a pre-fracture zone faces.

This additional condition makes it possible to specify the parameters of the composite plate (composite) at which a crack originates in the binder.

Solution method

Let in the binder there be one rectilinear zone of weakened inter-particle bonds in the state of plastic flow at a constant voltage (Fig. 1). The moments M_x, M_y, H_{xy}, transverse forces N_x, N_y and deflection w in the technical theory of plate bending can be represented using the Kolosov-Muskhelishvili complex potentials [26]. On the media separation boundary

$$\varphi(\tau) + \tau\overline{\Phi(\tau)} + \overline{\psi(\tau)} = \varphi_0(\tau) + \tau\overline{\Phi_0(\tau)} + \overline{\psi_0(\tau)}, \tag{3}$$

$$n_*\varphi(\tau) + \tau\overline{\Phi(\tau)} + \overline{\psi(\tau)} = \frac{D_0(1-\nu_0)}{D(1-\nu)}\left\{n_0\varphi_0(\tau) + \tau\overline{\Phi_0(\tau)} + \overline{\psi_0(\tau)}\right\}. \tag{4}$$

Here, $\varphi(z)$, $\psi(z)$ and $\varphi_0(z)$, $\psi_0(z)$ are the complex potentials for the binder and fiber, respectively; τ is a variable point on the media separation boundary, $\tau=\exp(i\theta)$ $n_*= -(3+\nu)/(1-\nu)$; D and D_0 are the cylindrical rigidity of the binder and fiber, respectively; ν and ν_0 are the Poisson coefficients of the binder and fiber material; $n_0= -(3+\nu_0)/(1-\nu_0)$.

On the rectilinear pre-fracture zone faces we have the condition

$$n_*\Phi(x_1) + \overline{\Phi(x_1)} + x_1\overline{\Phi'(x_1)} + \overline{\Psi(x_1)} = f_1^0 + iC_1, \tag{5}$$

where; $f_1^0 = M_s - iH_s$; x_1 is the affix of the pre-fracture zone points; C_1 is the real constant determined in the course of solving the problem from the condition that the deflection jump at the top of the pre-fracture zone is zero.

Under the accepted assumptions of the Kirchhoff theory, the problem of determining the stress-strain state of a composite plate reduces to finding two pairs of the functions $\Phi(z)$, $\Psi(z)$, and z), $\Psi_0(z)$ of the complex variable $z = x + iy$, analytical in the corresponding regions and satisfying the boundary conditions (3)–5).

We seek the complex potentials $\varphi_0(z)$ and $\psi_0(z)$ describing the stress-strain state of the fiber in the form

$$\varphi_0(z) = \sum_{k=1}^{\infty} a_k z^k, \quad \psi_0(z) = \sum_{k=1}^{\infty} b_k z^k. \tag{6}$$

We denote the left-hand side of the boundary condition (3) as f_1+if_2 and assume that on the boundary L this complex function can be expanded into a Fourier series

$$f_1 + if_2 = \sum_{k=-\infty}^{\infty} A_k e^{ik\theta}. \tag{7}$$

Based on the boundary condition (3) and relations (6), (7), using the power series method [26], we find the coefficients a_n, b_n of the functions $\varphi_0(z)$ и $\psi_0(z)$

$$a_n = \frac{A_n}{R^n} \quad (n>1), \quad \operatorname{Re} a_1 = \frac{A_1}{2R}, \quad b_n = \frac{\overline{A}_{-n}}{R^n} - (n+2)\frac{A_{n+2}}{R^n} \quad (n \ge 0).$$

We seek the values of the coefficients A_n in the course of solving the problem for a binder. Using the complex potentials $\varphi_0(z)$ and $\psi_0(z)$, after some elementary transformations, we write the boundary conditions on the media separation boundary $\tau=\exp(i\theta)$ as follows:

$$\varphi(\tau) + \tau\overline{\Phi(\tau)} + \overline{\psi(\tau)} = \sum_{k=-\infty}^{\infty} A_k e^{ik\theta}, \tag{8}$$

$$n_*\varphi(\tau) + \tau\overline{\Phi(\tau)} + \overline{\psi(\tau)} = \frac{D_0(1-\nu_0)}{D(1-\nu)}\left\{n_0\sum_{k=1}^{\infty} a_k R^k e^{ik\theta} + \overline{a}_1 \operatorname{Re}^{i\theta} + \sum_{k=0}^{\infty}(k+2)\overline{a}_{k+2}R^{2k+2}e^{-ik\theta} + \sum_{k=0}^{\infty}\overline{b}_k R^k e^{-ik\theta}\right\} \tag{9}$$

We seek the solution to the boundary value problem (5), (8), (9) in the form

$$\Phi(z)=\varphi_1'(z)+\Phi_2(z)\,,\quad \Psi(z)=\psi_1'(z)+\Psi_2(z)\,, \tag{10}$$

$$\varphi_1(z)=-\frac{M_x^\infty+M_y^\infty}{4(1+\nu)D}z+\sum_{k=1}^{\infty}c_k z^{-k}\,,\qquad \psi_1(z)=\frac{M_y^\infty-M_x^\infty}{2(1+\nu)D}z+\sum_{k=1}^{\infty}d_k z^{-k}\,, \tag{11}$$

$$\Phi_2(z)=\frac{1}{\pi i(1+\kappa)}\int_{-l_1}^{l_1}\frac{g_1(t)dt}{t-z_1},\quad \Psi_2(z)=\frac{1}{\pi i(1+\kappa)}e^{-2i\alpha_1}\int_{-l_1}^{l_1}\left[\frac{\kappa\overline{g_1(t)}}{t-z_1}-\frac{\overline{T}_1e^{i\alpha_1}}{(t-z_1)^2}g_1(t)\right]dt, \tag{12}$$

where $T_1=te^{i\alpha_1}+z_1^0$; $z_1=e^{-i\alpha_1}\left(z-z_1^0\right)$; $\kappa=(3-\nu)/(1+\nu)$; $g_1(x_1)$ is the sought-for function characterizing the discontinuity of the rotation angles when the pre-fracture zone line is crossed

$$\pm g_1(t)=\frac{d}{dt}\left(\frac{\partial w^\pm}{\partial x}+i\frac{\partial w^\pm}{\partial y}\right).$$

Satisfying the boundary conditions (9), (10) by the functions (10)–(12) and comparing the coefficients of equal powers exp($i\theta$), we obtain a system of algebraic equations for determining the coefficients c_k, d_k,, and A_k.. These equations are such that we can explicitly find formulas for a_k, b_k, c_k, d_k,, A_k via the function $g_1(x_1)$.

Satisfying the boundary conditions (5) on the faces of a rectilinear pre-fracture zone by complex potentials (10)–(12), we obtain a complex singular integral equation with respect to the unknown function $g_1(x_1)$

$$\int_{-l_1}^{l_1}\left[R_{11}(t,x_1)g_n^*(t,x_1)+S_{11}(t,x_1)\overline{g_n^*(t,x_1)}\right]dt=\pi F_1(x)\quad |x|\le l_1\,, \tag{13}$$

where $g_n^*(t)=\dfrac{g_1(t)}{i(1+\kappa)}$, $F_1(x)=-\left[n_*\Phi_1(x)+\overline{\Phi_1(x)}+x\overline{\Phi_1'(x)}+\overline{\Psi_1(x)}\right]+f_1^0+iC_1$; x, t, z_1^0, l_1 are dimensionless values related to R; R_{11}, S_{11} determined by the known relations ([27] of the formula (VI.62) at n=k=1).

The singular integral equation (13) requires adding the equality

$$\int_{-l_1}^{l_1}g_1(t)dt=0\,, \tag{14}$$

providing uniqueness of the rotation angles of the plate medium plane when the pre-fracture zone boundary is bypassed.

To determine the constant C_1 (in the general case of a piecewise constant function) we have the relation [28]

$$\mathrm{Re}\left[\int_{-l_1}^{l_1}tg_1(t)dt\right]=0\,,$$

ensuring a zero deflection at the pre-fracture zone tips.

The complex singular integral equation (13) under the additional condition (14) reduces [23, 27] to the system of algebraic equations with respect to the approximate values of the sought-for function $g_1(x_1)$ at the nodal points:

$$\frac{1}{M}\sum_{m=1}^{M}l_1\left[g_1^*(t_m)R_{11}(l_1t_m,l_1x_r)+\overline{g_1^*(t_m)}S_{11}(l_1t_m,l_1x_r)\right]=F_1(x_r)\,,\quad \sum_{m=1}^{M}g_1^*(t_m)=0\,, \tag{15}$$

where r=1, 2, …, $M-1$, $t_m=\cos\dfrac{2m-1}{2M}\pi$, $x_r=\cos\dfrac{\pi r}{M}$.

If we go over to the complex conjugate quantities in the system (15), we obtain another the system of algebraic equations algebraic equations.

For the closedness of the obtained algebraic equations, two complex equations determining the location of a pre-fracture zone (coordinates of the pre-fracture zone tips) are lacking. Since the solution to the singular integral equation (13) must be sought in the class of everywhere bounded functions (stresses), the system (15) requires adding the conditions of bounded stresses at the ends of the pre-fracture zone $x_1=\pm l_1$. These are the solvability conditions for a singular integral equation in the class of everywhere bounded functions.

The above mentioned additional conditions have the form

$$\sum_{m=1}^{M}(-1)^{M+m} g_1^*(t_m)\,\mathrm{tg}\frac{2m-1}{4M}\pi = 0\,, \quad \sum_{m=1}^{M}(-1)^{m} g_1^*(t_m)\,\mathrm{ctg}\frac{2m-1}{4M}\pi = 0\,, \tag{16}$$

Adding these two complex equations (16) to the equations obtained earlier, we obtain a closed combined algebraic system. Because of the unknown length of the pre-fracture zone, the combined system of algebraic equations is nonlinear. The numerical solution of the combined system allows one to find the coordinates of the pre-fracture zone tips (location) and its size, the values $g_1^*(t_m)$ (m=1, 2, …, M). It is obvious that, having determined the coordinates of the pre-fracture zone tips, and, using the known formulas of analytic geometry, we can find the said zone size, coordinates z_1^0 of its center, and angle α_1 with the axis x (Fig. 1).

The obtained systems of equations with respect to $g_1(t_m)$ (m=1, 2, …, M) allow one, at a given external bending load, to determine the stress-strain state of a composite when there is a zone of weakened inter-particle bonds of the material in the binder. A numerical calculation was made for the fiber ν=0.30; μ_0=4.5·10^5 MPa and the binder ν=0.32; μ_0=2.6·10^5 MPa.

Fig. 2 shows a graph of the dependence of a pre-fracture zone length l_1/R on the external bending load M_y^∞ / M_s. For this case, α_1=42°, $z_1^0 = 1{,}17Re^{i\pi/8}$ was found.

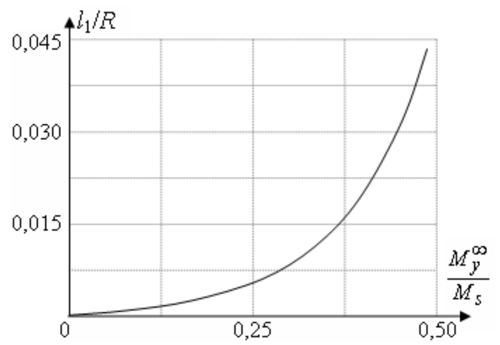


Fig. 2. Dependence of a pre-fracture zone length on the external bending load

With the help of the solution obtained, we calculate the displacements on the pre-fracture zone faces and, using the crack formation criterion (2), we find

$$\left| -\int_{-l_1}^{x_1^0} g_1^*(x_1)\,dx_1 \right| = \delta_c\,, \tag{17}$$

where x_1^0 is the coordinate of the pre-fracture zone point at which the inter-particle bonds of the material break.

The value of the external bending load causing the appearance of a crack is determined from the relation (17). The combined resolving system of equations due to the unknown quantity l_1 turned out to be nonlinear. To solve it, the method of successive approximations is used. A combined system of equations is solved at some definite value l_1^* with respect to the unknowns c_k, d_k, A_k, and $g_1^*(t_m)$. The value l_1^* and the obtained values c_k, d_k, A_k and c_k, d_k, A_k are substituted into (16), i.e. into the unused equations of the combined system. The taken value of the parameter l_1^* and the corresponding values c_k, d_k, A_k, and $g_1^*(t_m)$, generally speaking, will not satisfy the equations (16). Selecting the values of the parameter l_1^*, we will repeat the calculations over and over again until the equations (16) of the combined system are satisfied with the required accuracy. The combined system of equations in each approximation was solved by the Gauss method with the choice of the principal element for different values of M.

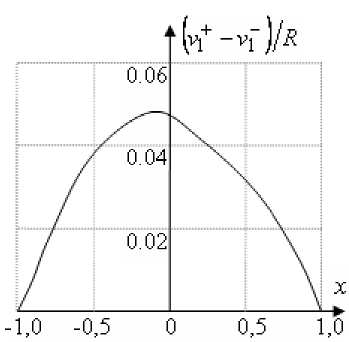
Thus, the joint solution of the combined algebraic system and the condition (17) allows one (at the required characteristics of the binder crack resistance) to determine the limiting value of the external bending load, the location and size of a pre-fracture zone for the state of limiting equilibrium at which cracking takes place.

Fig. 3 shows the graphs of the distribution of the normal $\left(v_1^+ - v_1^-\right)/R$ and tangential $\left(u_1^+ - u_1^-\right)/R$ components of the displacement vector. Dimensional coordinates $x_1' = x_1/l_1$ were used in the calculation.

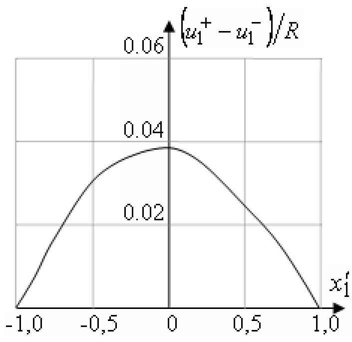
Fig. 4 shows the dependence of the ultimate bending load $M_y^{\infty c}/M_s$ on the relative opening of the faces δ_*/l_1 in the center of the pre-fracture zone. Here α_1=42°, $\delta_* \dfrac{\pi\delta_c\mu}{(1+\kappa)M_s}$.

The resulting combined algebraic system of equations of the problem allows one to obtain a solution with any pre-assigned accuracy.

The analysis of the crack formation model in a composite binder in the process of its being subjected to a bending load reduces to a parametric joint study of the combined solving algebraic system of the problem and the criterion for crack appearance (17) at different values of the composite plate free parameters. These are various geometric and mechanical characteristics of the binder and fiber materials.



a



b

Fig. 3. Distribution of the displacement vector components: a – normal; b –tangent

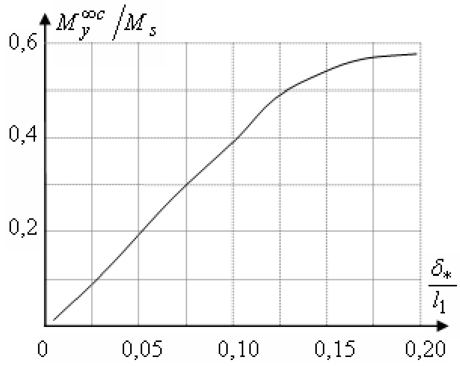


Fig. 4. Dependence of the limiting bending load on the relative opening of the pre-fracture zone center faces

Conclusions

The practice of using fibrous composites shows that at the design stage it is necessary to take into account the cases when cracks may appear in the binder. The existing methods of strength calculation of fibrous composites, as a rule, ignore this circumstance. Such a situation makes it impossible to design a composite with a minimum material consumption, with guaranteed reliability and durability. In this connection, it is necessary to carry out an ultimate analysis of a composite in order to establish the limiting bending loads at which cracking takes place in the binder. The size of the limiting minimum zone of the material weakened inter-particle bonds, at which crack initiation occurs is recommended to be considered as a design characteristic of the binder material. Based on the proposed design model, taking into account the presence of damage in a composite (zones of weakened inter-particle bonds in the material), a method has been developed for calculating the parameters of a composite, under which cracking occurs. Knowing the basic values of the limiting parameters of crack formation and the influence of the material properties on them, it is possible to reasonably control the phenomenon of crack formation through design and technological solutions at the design stage.

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