DOI: https://doi.org/10.15407/pmach2020.02.033

UDC 539.375

WHEELED VEHICLE BRAKE DRUM COMPUTATION BY FRACTURE TOUGHNESS CRITERIA

Sahib A. Askerov
hssh3883@gmail.com
ORCID: 0000-0002-5468-5303

Azerbaijan Technical University,
25, G. Javid Ave., Baku,
Azerbaijan, AZ1073

To ensure vehicle safety at the design stage, of primary importance is the development of a mathematical model, within whose framework it is possible to effectively predict crack initiation in the break drum of a braking wheeled vehicle. Considered is the contact fracture mechanics problem of the initiation of a cohesive crack in the brake drum of a wheeled vehicle. It is believed that during the multiple braking of a wheeled vehicle, the vehicle material is destroyed during friction due to contact interaction. It is considered that the real surface of the brake drum is never absolutely smooth, but it has technological micro- or macroscopic irregularities that make the surface rough. A mathematical model is proposed, within whose framework crack initiation in the brake drum of a braking wheeled vehicle is described. The crack initiation zone is modeled as a region of weakened interparticle bonds of the material (pre-fracture zone). The location and size of the pre-fracture zone are not known in advance and must be determined in the process of solving the problem. Both the perturbation method and the apparatus of the theory of singular integral equations are used. The equilibrium problem of the wheeled vehicle brake drum with an embryonic crack reduces to the solution, in each approximation, of a nonlinear integro-differential equation of Cauchy type. When a collocation solution scheme is used in each approximation, the singular integral equation reduces to a system of nonlinear algebraic equations. To solve them, both the method of successive approximations and an iterative algorithm of elastic solutions are used. From the solution of the obtained system of equations, normal and tangential stresses in the pre-fracture zone are found. The condition for the initiation of a cohesive crack in the brake drum is formulated taking into account the criterion of the ultimate stretching of material bonds.

Keywords: *brake drum, pre-fracture zone, crack initiation, rough surface.*

Introduction

The friction pair "drum-lining" of drum brake shoes works [1–3] in conditions of complex stress. To ensure vehicle safety at the design stage, it is important to develop a mathematical model within which it is possible to predict crack initiation in the drum of the breaking system in a braking wheeled vehicle.

The fracture toughness of materials and machine elements determines their resistance to the initiation and growth of cracks leading to a partial or complete fracture. Strength and durability calculations according to the fracture toughness criteria are performed in the cases where there is a real danger of crack initiation during operation of machines, or when the traditional calculation without taking into account crack initiation is not able to provide answers to the questions regarding brittle fracture prevention measures. Thus, for a wheel drum brake, the prediction of crack initiation plays a decisive role during prolonged multiple braking.

Problem Formulation

As the friction pair "drum-lining" of a wheeled vehicle is used, a pre-fracture zone will appear in the metal brake drum. We will model it as a region of weakened interfacial bonds of the material, and the drum itself, as a real brittle body. In the process of deformation, at some points of the drum, there may appear zones in which Hooke's law is not satisfied, i.e. stresses exceed the elastic limit.

In the intermittent braking mode, the drum of the braking mechanism of a wheeled vehicle experiences multiple cyclic loading. It is accepted that in the brake drum, there is a stress concentrator – a region (zone) of weakened interparticle bonds of the drum material. In the region of weakened interparticle bonds of the material, inelastic deformation occurs. The region of weakened interparticle bonds is small compared to the elastic part of the brake drum. Therefore, the region can be mentally removed by replacing it with a cut whose surfaces interact with each other according to a certain law corresponding to the action of the material removed.

After some of brake applications, in the zone of weakened interparticle bonds, the material loses the ability to deform, and the opening of crack faces occurs. Crack initiation will occur when the opening of the faces of the zone of weakened interparticle bonds of the material reaches the limit value for this material [4]. In the case under study, the occurrence of an embryonic crack is a process of transition of the pre-fracture zone to the zone of broken bonds between the surfaces of the brake drum material, with the location and size of the pre-fracture zone not being known in advance and having to be determined in the process of solving the problem.

Thus, the cohesion crack initiation zone is modeled as a pre-fracture zone (zone of weakened interparticle bonds of the material). At the center of the pre-fracture zone, there is the origin of the local coordinate system $x_1O_1y_1$ (Fig. 1). The x_1 axis coincides with the line of the pre-fracture zone, and forms the angle α_1 with the Ox axis. The brake drum of a wheeled vehicle is modeled as an isotropic homogeneous elastic body. It is accepted that the conditions of plane strain are satisfied.

It is known that the real surface of the brake drum is never absolutely smooth, but it has technological micro- or macroscopic irregularities that make the surface rough. Despite their extremely small sizes, such irregularities significantly affect the operational properties of the drum [5, 6].

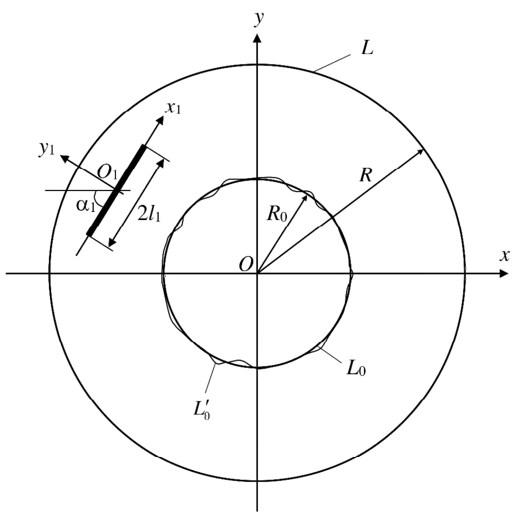

Fig. 1. A scheme of the wheeled vehicle brake mechanism drum with a pre-fracture zone

Consider some implementation of the rough inner surface of the drum. It is believed that the inner contour of the drum is close to circular. We assign the drum to the polar coordinate system r, θ with the origin at the center of the concentric circles L_0 and L with radii R_0 and R, respectively.

Imagine the boundary of the inner contour L_0' in the following form:

$$r = \rho(\theta) = R_0 + \varepsilon H(\theta).$$

Here, $\varepsilon = R_{\max}/R_0$ is a small parameter; $R_{\max}$ is the maximum height of the protrusion (trough) of the roughness of the inner drum surface; $H(\theta)$ is the function independent of the small parameter.

Consider the pre-fracture zone of length $2l_1$, located on the segment $|x_1| \le l_1$, $y_1=0$. Recall that l_1, the angle α_1, and the pre-fracture zone center $z_1^0 = x_1^0 + iy_1^0$ are not known in advance and must be determined.

The pre-fracture zone faces interact with each other. This interaction (interfacial bonds) restrains the initiation of a cohesive crack. For the mathematical description of the pre-fracture zone, it is accepted that there are bonds between them whose deformation law is given. During braking, as a result of the action of contact pressure and friction forces on the drum in the bonds that connect the pre-fracture zone faces, both the normal stresses $q_{y_1}(x_1)$ and tangential stresses $q_{x_1y_1}(x_1)$ will arise. Consequently, both the normal and tangential stresses, numerically equal to $q_{y_1}(x_1)$ and $q_{x_1y_1}(x_1)$, respectively, will be applied to the pre-fracture zone faces. These values are not known in advance.

The boundary conditions of the problem on the inner and outer brake drum contours of a braking wheeled vehicle will be

$$\sigma_n = -p(\theta), \quad \tau_{nt} = -fp(\theta) \quad \text{at } r = \rho(\theta) \quad \text{on the contact area;}$$

$$\sigma_n = 0, \quad \tau_{nt} = 0 \qquad \text{at } r = \rho(\theta) \quad \text{outside the contact area;} \tag{1}$$

$$\sigma_r = 0, \quad \tau_{r\theta} = 0 \quad \text{at } r=R;$$

$$\sigma_{y_1} = q_{y_1}, \quad \tau_{x_1y_1} = q_{x_1y_1} \quad \text{on the pre-fracture zone faces.} \tag{2}$$

Here, $p(\theta)$ is the contact pressure; f is the friction coefficient of the pair; σ_r, $\tau_{r\theta}$ are the components of the stress tensor in a polar coordinate system.

In the boundary conditions (1), it is assumed that in the contact zone there are both the contact pressure $p(\theta)$ and the tangential pressure connected therewith by the Amonton-Coulomb law.

Conditions (1)–(2) must be supplemented by the relation connecting the opening of the pre-fracture zone faces with the stresses in the bonds

$$\left(v_1^+ - v_1^-\right) - i\left(u_1^+ - u_1^-\right) = \Pi_y\left(x_1, q_{y_1}\right) q_{y_1}(x_1) - i\Pi_x\left(x_1, q_{x_1 y_1}\right) q_{x_1 y_1}(x_1)\,. \quad (3)$$

Here, $\left(v_1^+ - v_1^-\right)$ is the normal and $\left(u_1^+ - u_1^-\right)$ is the tangent component of the opening of the pre-fracture zone faces; the functions $\Pi_y\left(x_1, q_{y_1}\right)$ and $\Pi_x\left(x_1, q_{x_1 y_1}\right)$ are the effective bond compliances depending on bond tension. With constant values of the functions Π_y and Π_x, in (3) we have the linear law of bond deformation. In the general case, the law of bond deformation is given and is nonlinear.

To find the contact pressure at which a cohesive crack initiates, the formulation of the problem must be supplemented by a cohesive crack initiation criterion (breaking of the interparticle bonds of the drum material). Such a criterion is the criterion of the critical opening of the faces of the zone of weakened interfacial bonds

$$\left|u_1^+(x_1,0) - u_1^-(x_1,0) - i\left(v_1^+(x_1,0) - v_1^-(x_1,0)\right)\right| = \delta_c\,, \quad (4)$$

where δ_c is crack resistance (a value characterizing the resistance of the drum material to crack initiation).

This additional condition allows us to find the parameters of the friction pair "drum-lining", at which a cohesive crack initiates in the drum.

Solution Method and Analysis

We seek stresses and displacements in the brake drum of a braking wheeled vehicle and other unknowns in the form of expansions in the small parameter ε. For simplicity, we neglect the terms containing ε with degree higher than 1

$$\sigma_r = \sigma_r^{(0)} + \varepsilon\sigma_r^{(1)} + \ldots, \quad \sigma_\theta = \sigma_\theta^{(0)} + \varepsilon\sigma_\theta^{(1)} + \ldots, \quad \tau_{r\theta} = \tau_{r\theta}^{(0)} + \varepsilon\tau_{r\theta}^{(1)} + \ldots,$$

$$\nu_r = \nu_r^{(0)} + \varepsilon\nu_r^{(1)} + \ldots, \quad \nu_\theta = \nu_\theta^{(0)} + \varepsilon\nu_\theta^{(1)} + \ldots$$

$$l_1 = l_1^0 + \varepsilon l_1^1 + \ldots, \quad \alpha_1 = \alpha_1^0 + \varepsilon\alpha_1^1 + \ldots,$$

$$q_{y1} = q_{y1}^{(0)} + \varepsilon q_{y1}^{(1)} + \ldots, \quad q_{x1y1} = q_{x1y1}^{(0)} + \varepsilon q_{x1y1}^{(1)} + \ldots.$$

We obtain the values of the stress tensor components at $r=\rho(\theta)$ by expanding in a series the expressions for stresses in the neighborhood at $r=R_0$.

Using the small parameter method for the boundary value problem of contact fracture mechanics, we obtain a sequence of boundary value problems for the brake drum with circular boundaries.

The boundary conditions of the problem have the form:

– in the zeroth approximation

$$\sigma_r^{(0)} = -p^{(0)}(\theta), \quad \tau_{r\theta}^{(0)} = -fp^{(0)}(\theta) \quad \text{at } r=R_0,\ |\theta| \le \theta_0 \quad \text{on the contact area;} \quad (5)$$

$$\sigma_r^{(0)} = 0, \quad \tau_{r\theta}^{(0)} = 0 \quad \text{at } r=R_0 \quad \text{outside the contact area;}$$

$$\sigma_r^{(0)} = 0, \quad \tau_{r\theta}^{(0)} = 0 \quad \text{at } r=R; \qquad \sigma_{y_1}^{(0)} = q_{y_1}^{(0)}, \quad \tau_{x_1 y_1}^{(0)} = q_{x_1 y_1}^{(0)} \quad \text{at } |x_1| = l_1^0; \quad (6)$$

– in the first approximation

$$\sigma_r^{(1)} = N - p^{(1)}(\theta), \ \tau_{r\theta}^{(1)} = T - fp^{(1)}(\theta) \quad \text{at } r=R_0,\ |\theta| \le \theta_0 \quad \text{on the contact area;}$$

$$\sigma_r^{(1)} = N, \quad \tau_{r\theta}^{(1)} = T \quad \text{at } r=R_0 \quad \text{outside the contact area;} \quad (7)$$

$$\sigma_r^{(1)} = 0, \quad \tau_{r\theta}^{(1)} = 0 \quad \text{at } r=R; \qquad \sigma_{y_1}^{(1)} = q_{y_1}^{(1)}, \quad \tau_{x_1 y_1}^{(1)} = q_{x_1 y_1}^{(1)} \quad \text{at } |x_1| = l_1^1, \quad (8)$$

where θ_0 is the half contact angle;

$$N = -H(\theta)\frac{\partial\sigma_r^{(0)}}{\partial r} + 2\tau_{r\theta}^{(0)}\frac{1}{R_0}\frac{dH(\theta)}{d\theta} \quad \text{at } r=R_0, \quad T = (\sigma_\theta^{(0)} - \sigma_r^{(0)})\frac{1}{R_0}\frac{dH(\theta)}{d\theta} - H(\theta)\frac{\partial\tau_{r\theta}^{(0)}}{\partial r}\,. \quad (9)$$

Using the Kolosov-Muskhelishvili formulas [7], the boundary conditions of problem (5)–(6) can be written as a boundary value problem for finding two complex potentials, $\Phi^{(0)}(z)$ and $\Psi^{(0)}(z)$.

We seek the complex potentials describing the stress-strain state in the drum in the following form:

$$\Phi^{(0)}(z)=\Phi_0^{(0)}(z)+\Phi_1^{(0)}(z)+\Phi_2^{(0)}(z)\,,\quad \Psi^{(0)}(z)=\Psi_0^{(0)}(z)+\Psi_1^{(0)}(z)+\Psi_2^{(0)}(z)\,. \tag{10}$$

Here,

$$\Phi_0^{(0)}(z)=\sum_{k=-\infty}^{\infty}a_k z^k\,,\qquad \Psi_0^{(0)}(z)=\sum_{k=-\infty}^{\infty}b_k z^k\,; \tag{11}$$

$$\Phi_1^{(0)}(z)=\frac{1}{2\pi}\int_{-l_1^0}^{l_1^0}\frac{g_1^0(t)dt}{t-z_1}\,,\qquad \Psi_1^{(0)}(z)=\frac{1}{2\pi}e^{-2i\alpha_1^0}\int_{-l_1^0}^{l_1^0}\left[\frac{\overline{g_1^0(t)}}{t-z_1}-\frac{\overline{T}_1 e^{i\alpha_1^0}}{(t-z_1)^2}g_1^0(t)\right]dt\,;$$

$$\Phi_2^{(0)}(z)=\frac{1}{2\pi}\int_{-l_1^0}^{l_1^0}\left[\frac{1-T_1\overline{T}_1}{\overline{T}_1T_2^2}e^{-i\alpha_1^0}\overline{g_1^0(t)}-\frac{1}{zT_2}e^{i\alpha_1^0}g_1^0(t)\right]dt\,, \tag{12}$$

$$\Psi_2^{(0)}(z)=\frac{1}{2\pi z}\int_{-l_1^0}^{l_1^0}\left\{\left[\frac{1}{zT_1}-\frac{1}{z^2}-\frac{1}{z^2T_2}+\frac{\overline{T}_1^2}{T_2^2}\right]e^{i\alpha_1^0}g_1^0(t)+\left[\frac{1-T_1\overline{T}_1}{z\overline{T}_1T_2^2}-\frac{1}{1-zT_1}-\frac{2\left(1-T_1\overline{T}_1\right)}{T_2^3}\right]e^{-i\alpha_1^0}\overline{g_1^0(t)}\right\}dt\,.$$

Here, $g_1^{(0)}(x_1)=\dfrac{2G}{i(1+\kappa)}\dfrac{\partial}{\partial x_1}\left\{u_1^{0+}(x_1,0)-u_1^{0-}(x_1,0)-i\left(v_1^{0+}(x_1,0)-v_1^{0-}(x_1,0)\right)\right\}$; G is the shear modulus of the drum material; κ=3–4μ; μ is Poisson's ratio of the drum material; $T_1=te^{i\alpha_1^0}+z_1^0$; $z_1=e^{-i\alpha_1^0}\left(z-z_1^0\right)$; $T_2=1-z\overline{T}_1$.

Satisfying the boundary conditions on the pre-fracture band in the zeroth approximation by functions (10) to (12), we obtain a complex singular integral equation [8] with respect to the unknown function $g_1^{(0)}(x_1)$ [8]

$$\int_{-l_1^0}^{l_1^0}\left[R_{11}^0(t,x_1)g_1^{(0)}(t)+S_{11}^0(t,x_1)\overline{g_1^{(0)}(t)}\right]dt=\pi f^0(x_1)\quad |x_1|=l_1^0\,, \tag{13}$$

where $f^0(x_1)=-\left(q_{y_1}^{(0)}-iq_{x_1y_1}^{(0)}\right)-\left[\Phi_0^{(0)}(x_1)+\overline{\Phi_0^{(0)}(x_1)}+x_1\overline{\Phi_0'^{(0)}(x_1)}+\overline{\Psi_0^{(0)}(x_1)}\right]$.

Satisfying the boundary conditions (5) written in terms of complex potentials by functions (10) to (12), after some transformations, we obtain an infinite system of algebraic equations with respect to the coefficients a_k, b_k of the potentials $\Phi_0^{(0)}(z)$ and $\Psi_0^{(0)}(z)$.

For the internal pre-fracture zone, the singular integral equation must be supplemented with the condition

$$\int_{-l_1^0}^{l_1^0}g_1^{(0)}(t)dt=0\,. \tag{14}$$

This condition ensures the uniqueness of displacements in bypassing the pre-fracture zone contour.

The singular integral equation (13) under condition (14), using the algebraization procedure [8, 9], reduces to a system of M algebraic equations for determining the M unknowns $g_1^{(0)}(t_m)$ (m=1, 2, …, M)

$$\frac{1}{M}\sum_{k=1}^{M}l_1^0\left[g_1^{(0)}(t_m)R_{11}^0(l_1^0t_m,l_1^0x_r)+\overline{g_1^{(0)}(t_m)}S_{11}^0(l_1^0t_m,l_1^0x_r)\right]=f^0(x_r)\,, \tag{15}$$

$$r=1,2,\ldots,M-1,\quad \sum_{m=1}^{M}g_1^{(0)}(t_m)=0\,,$$

where $t_m=\cos\dfrac{2m-1}{2M}\pi$ (m=1 ,2, …, M); $x_r=\cos\dfrac{\pi r}{M}$ (r=1, 2, …, M–1).

If in system (15) we proceed to complex conjugate quantities, we will obtain more M algebraic equations.

The right-hand sides of system (15) include unknown values of the normal stresses $q_{y_1}^{(0)}(x_1)$ and tangential stresses $q_{x_1y_1}^{(0)}(x_1)$ at the nodal points of the pre-fracture zone partition. The condition that determines the unknown stresses in the bonds between the pre-fracture zone faces is the additional relation (3) in the zeroth approximation. Using the resulting solution in the zeroth approximation, we can write

$$g_1^{(0)}(x_1)=\frac{2G}{1+\kappa}\frac{d}{dx_1}\left[\Pi_y\left(x_1,q_{y_1}^{(0)}(x_1)\right)q_{y_1}^{(0)}(x_1)-i\Pi_x\left(x_1,q_{x_1y_1}^{(0)}(x_1)\right)q_{x_1y_1}^{(0)}(x_1)\right]. \qquad (16)$$

This complex differential equation serves to find the stresses $q_{y_1}^{(0)}(x_1)$ and $q_{x_1y_1}^{(0)}(x_1)$ in the bonds between the pre-fracture zone faces.

To construct the missing algebraic equations for finding the approximate nodal stresses $q_{y_1}^{(0)}(t_m)$ and $q_{x_1y_1}^{(0)}(t_m)$, we require that conditions (16) be satisfied at the points t_m (m=1, 2, …, M) contained in the pre-fracture zone. In this case, the finite-difference method is used. As a result, we obtain a complex algebraic system of M equations for determining the approximate values $q_{y_1}^{(0)}(t_m)$ and $q_{x_1y_1}^{(0)}(t_m)$ at the nodal points of the pre-fracture zone.

For the isolation of the obtained algebraic equations, we miss two complex equations that determine the size of the pre-fracture zone.

Since the solution to the integral equation (13) is sought in the class of everywhere bounded functions (stresses), the resulting system (15) must be supplemented with the boundedness conditions of stresses at the pre-fracture zone ends $x_1=\pm l_1^0$. These conditions in the zeroth approximation have the form

$$\sum_{m=1}^{M}(-1)^{M+m}g_1^{(0)}(t_m)\,\mathrm{tg}\frac{2m-1}{4M}\pi=0\,,\ \sum_{m=1}^{M}(-1)^{m}g_1^{(0)}(t_m)\,\mathrm{ctg}\frac{2m-1}{4M}\pi=0\,. \qquad (17)$$

Due to the fact that the location and size of the pre-fracture zone are unknown, the resulting algebraic system is non-linear. Its numerical solution allows us to find both the coordinates of the vertices (location) and size of the pre-fracture zone, as well as the stress-strain state of the brake drum in the zeroth approximation.

Due to the unknown quantities l_1^0, α_1^0, z_1^0, the combined system of equations turns out to be nonlinear even with linearly elastic bonds. To solve it, the method of successive approximations was used. The united system was solved at some certain values l_1^{0*}, α_1^{0*}, $z_1^{0*}=x_1^{0*}+iy_1^{0*}$ with respect to the remaining unknowns a_k, b_k, $g_1^{(0)}(t_m)$, $q_{y_1}^{(0)}(t_m)$ and $q_{x_1y_1}^{(0)}(t_m)$. In the case of linearly elastic bonds, these unknowns enter the system in a linear manner. To solve this system of equations, the Gauss method with the choice of the main element is used. The values l_1^{0*}, α_1^{0*}, z_1^{0*} and the corresponding values of the remaining unknowns will not, in the general case, satisfy equations (17). Then, using the Newton method, we find corrections to the solution of equations (17). Selecting the values l_1^{0*}, α_1^{0*}, z_1^{0*}, we will repeatedly conduct calculations until equations (17) are satisfied with the given accuracy.

In the case of the nonlinear law of deformation of material bonds, to determine the forces in the pre-fracture zone, an iterative algorithm similar to A. A. Ilyushin's method of elastic solutions was used. The calculation of effective compliance was carried out similar to finding the secant modulus in the method of variable elastic parameters. The nonlinear part of the bond deformation curve was taken as a bilinear dependence, with the ascending section corresponding to the elastic deformation of the bonds $0<V(x_1)<V_*$ with their maximum tension. For $V(x_1)>V_*$, the deformation law was described by the nonlinear dependence determined by the points (V_*, σ_*), (δ_c, σ_c). For $\sigma_c\geq\sigma_*$, an increasing linear dependence occurs (linear hardening corresponding to the elastic-plastic deformation of the bonds).

The process of successive approximations ends as soon as the forces in the pre-fracture zone bonds, obtained at two successive steps, differ little.

The obtained systems of equations with regard to a_k, b_k, $g_1^{(0)}(t_m)$ (m=1, 2, …, M) allow us, for a given external load, to find the stress-strain state of the brake drum in the presence of a pre-fracture zone.

After solving the combined algebraic system, we proceed to constructing a solution to the problem in the first approximation. For $r=R_0$, we find the functions N and T according to formulas (9). Using the Kolosov-Muskhelishvili formulas [7], the boundary conditions of the problem in the first approximation (7)–(8) can be written in the form of a boundary-value problem for determining the complex potentials $\Phi^{(1)}(z)$ and $\Psi^{(1)}(z)$. The functions $\Phi^{(1)}(z)$ and $\Psi^{(1)}(z)$ are sought in the form similar to (10)–(12) with obvious changes. The further process of solving the problem is similar to the zeroth approximation.

After determining the required values for predicting the limit value of the contact pressure in the brake drum of a wheeled vehicle, at which a cohesive crack may initiate, the criterion of the critical opening of pre-fracture zone faces was used (4).

Using the obtained solution, we find the limiting condition under which a cohesive crack will initiate in the drum:

$$\frac{1+\kappa}{2G}\frac{\pi l_1}{M}\sqrt{A_1^2+B_1^2}=\delta_c\,, \tag{18}$$

Here, $A_1=\sum_{m=1}^{M_1}\left[v_1^0(t_m)+\varepsilon v_1^1(t_m)\right]$, $B_1=\sum_{m=1}^{M_1}\left[u_1^0(t_m)+\varepsilon u_1^1(t_m)\right]$, M_1 is the number of nodal points in the segment $[-l_1, x_1^0)$.

A joint solution of the obtained equations with condition (18) makes it possible, for the given characteristics of the brake drum material, to predict the critical value of the contact pressure and the dimensions of the pre-fracture zone for the state of ultimate equilibrium.

The resulting systems were solved for the values M=20 and 40, which corresponds to dividing the integration interval [–1, 1] by 20; 40 Chebyshev nodes. The calculations were performed for the KamAZ-5320 brake drums.

Fig. 2 shows graphs of the dependence of the pre-fracture zone length $l_1/(R–R_0)$ on the contact pressure p_0/σ_* (p_0 is the force factor). Curve *1* corresponds to the rough friction surface of the brake drum; curve *2*, to the smooth one. Fig. 3 shows the distribution of the normal forces q_{y_1}/q_* in the pre-fracture zone, and Fig. 4, the tangents $q_{x_1y_1}/q_*$. Curve *1* in both figures corresponds to the linear law of bond deformation; curve *2*, to the bilinear one.

The compliances of the bonds in the normal and tangential directions were taken equal and constant along the pre-fracture zone. The law of change in the tangential stresses along the pre-fracture zone is similar to a change in the normal forces with the difference that the absolute values of the tangential forces are much smaller. In this case, the maximum values of the tangential stresses are achieved at smaller sizes of the pre-fracture zone. For this case, it was found that α_1=36°, $z_1^0=1{,}27R_0e^{i\pi/13}$.

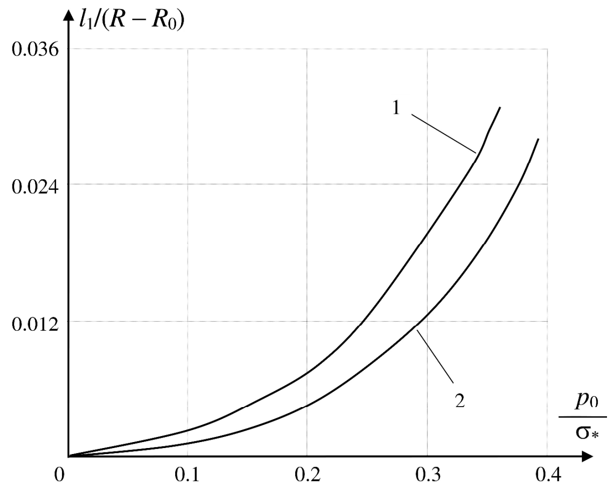

Fig. 2. The dependence of the pre-fracture zone length on contact pressure

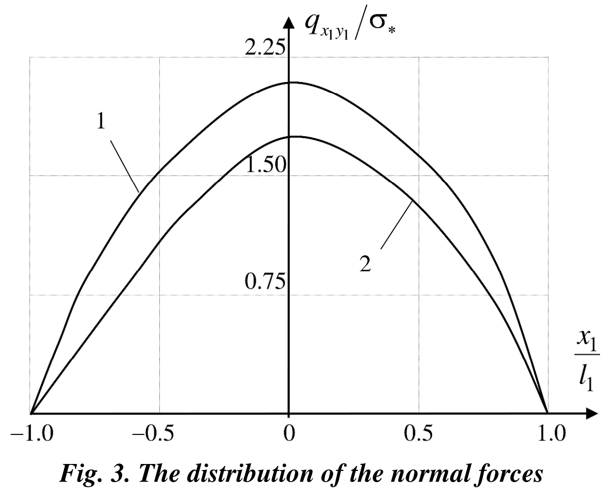

Fig. 3. The distribution of the normal forces in the pre-fracture zone

Conclusion

We note that with the proposed mathematical model of cohesive crack initiation in the brake drum, it is possible, at the design stage, to establish the maximum values of workloads and the limit level of the brake drum defectiveness, at which a sufficient strength margin is maintained, as well as determine the optimal material for the brake drum and its safe life. The parameters of the brake drum at the design stage must be selected in accordance with the condition

$$p_{\max} < p_{cr},$$

where $p_{\max}$ is the design maximum contact pressure in the braking mechanism, p_{cr} is the critical contact pressure.

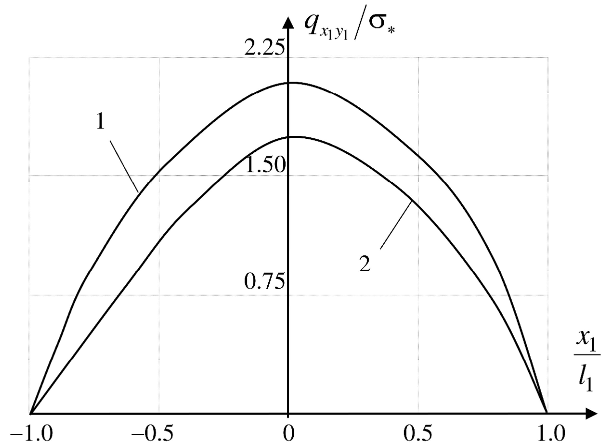

Fig. 4. The distribution of the tangential forces in the pre-fracture zone

Knowing the basic values of the critical fracture (crack initiation) parameters and the influence on them of the properties of the drum material, the class of technological treatment of the drum and lining surfaces, we can reasonably control the process of crack initiation in the brake drum by means of design-technological solutions at the design stage.

References

1. Petrik, A. A., Volchenko, A. I., Volchenko, N. A., & Volchenko, D. A. (2006). *Barabanno-kolodochnyye tormoznyye ustroystva* [Drum-shoe braking devices]. Krasnodar: Kuban State Technological University, 264 p. (in Russian).
2. Volchenko, A. I., Kindrachuk, M. V., Bekish, I. O., Malyk, V. Ya., & Snurnikov, V. I. (2015). *Termicheskiye napryazheniya v obodakh tormoznykh barabanov avtotransportnykh sredstv* [Thermal stresses of the rims of brake drums in vehicles]. *Problemy treniya i iznashivaniya – Problems of friction and wear*, no. 4 (69), pp. 28–37 (in Russian).
3. Mirsalimov, V. M., Hasanov, Sh. G., & Geydarov, Sh. G. (2018). *Iznosokontaktnaya zadacha o vdavlivanii kolodki s friktsionnoj nakladkoj v poverkhnost barabana* [Wear-contact problem of pressing brake shoe with friction lining into drum surface]. XII International scientific conference "Tribology for Mechanical Engineering" dedicated to the 80th anniversary of IMASH RAS, Moscow, 19–21 November 2018, pp. 342–344 (in Russian).
4. Mirsalimov, V. M. (2007). The solution of a problem in contact fracture mechanics on the nucleation and development of a bridged crack in the hub of a friction pair. *Journal of Applied Mathematics and Mechanics*, vol. 71, iss. 1, pp. 120–136. https://doi.org/10.1016/j.jappmathmech.2007.03.003.
5. Demkin, N. B. & Ryzhov, E. V. (1981). *Kachestvo poverkhnosti i kontakt detalej mashin* [Surface quality and contact of machine parts]. Moscow: Mashinostroyeniye, 244 p. (in Russian).
6. Thomas, T. R. (1982). Rough surfaces. London: Longman, 387 p.
7. Muskhelishvili, N. I. (1977). Some basic problems of the mathematical theory of elasticity. Dordrecht: Springer, 732 p. https://doi.org/10.1007/978-94-017-3034-1.
8. Panasyuk, V. V., Savruk, M. P., & Datsyshin, A. P. (1976). *Raspredeleniye napryazhenij okolo treshchin v plastinakh i obolochkakh* [Stress distribution near cracks in plates and shells]. Kiev: Naukova Dumka, 443 p. (in Russian).
9. Mirsalimov, V. M. (1987). *Neodnomernyye uprugoplasticheskiye zadachi* [Non-one-dimensional elastoplastic problems]. Moscow: Nauka, 256 p. (in Russian).

Received 01 April 2020

Розрахунок гальмівного барабана колісної машини за критеріями тріщиностійкості

С. А. Аскеров

Азербайджанський технічний університет,
AZ1073, Азербайджан, м. Баку, пр. Г. Джавіда, 25

Для створення умов безпеки транспортних засобів на стадії проектування особливе значення має розробка математичної моделі, у рамках якої можливо ефективно прогнозувати тріщиноутворення у барабані гальмівного механізму в процесі гальмування колісної машини. Розглядається задача механіки контактного руйнування про зародження когезійної тріщини у барабані гальмівного механізму колісної машини. Вважається, що за багаторазового гальмування колісної машини відбувається руйнування матеріалу під час тертя, викликаного контактною взаємодією. Враховується, що реальна поверхня гальмівного барабана не буває абсолютно гладкою, але має мікро- або макроскопічні нерівності технологічного характеру, що утворюють шорсткість. Запропонована математична модель, у рамках якої описується зародження тріщини в гальмівному барабані під час гальмування колісної машини. Зона зародження тріщини моделюється як область ослаблених міжчасткових зв'язків матеріалу (зона передруйнування). Місце розташування і розмір зони передруйнування заздалегідь невідомі і мають бути визначені в процесі розв'язання задачі. Використовується метод збурень і апарат теорії сингулярних інтегральних рівнянь. Задача про рівновагу гальмівного барабана колісної машини із зародковою тріщиною зводиться до розв'язання в кожному наближенні нелінійного інтегродиференційного рівняння типу Коші. За використання колокаційної схеми розв'язання в кожному наближенні сингулярне інтегральне рівняння зводиться до системи нелінійних рівнянь алгебри. Для їх розв'язання використовується метод послідовних наближень та ітераційний алгоритм пружних розв'язків. З розв'язку отриманої системи рівнянь знайдена нормальна і дотична напруга в зоні передруйнування. Умова появи когезійної тріщини в гальмівному барабані формулюється з урахуванням критерію граничної витяжки зв'язків матеріалу.

***Ключові слова**: гальмівний барабан, зона передруйнування, зародження тріщини, шорстка поверхня.*

Література

1. Петрик А. А., Вольченко А. И., Вольченко Н. А., Вольченко Д. А. Барабанно-колодочные тормозные устройства. Краснодар: Кубан. технолог. ун-т, 2006. 264 с.
2. Вольченко А. И., Киндрачук М. В., Бекиш И. О., Малык В. Я., Снурников В. И. Термические напряжения в ободах тормозных барабанов автотранспортных средств. *Пробл. трения и изнашивания*. 2015. № 4 (69). С. 28–37.
3. Мирсалимов В. М., Гасанов Ш. Г., Гейдаров Ш. Г. Износоконтактная задача о вдавливании колодки с фрикционной накладкой в поверхность барабана *Трибология – машиностроению*: тр. XII междунар. науч.-техн. конф., посв. 80-летию ИМАШ РАН (Москва, 19–21 нояб. 2018 г.). Ижевск: Ин-т компьютерных исследований, 2018. С. 342–344.
4. Мирсалимов В. М. К решению задачи механики контактного разрушения о зарождении и развитии трещины со связями между берегами во втулке фрикционной пары. *Прикл. математика и механика*. 2007. Т. 71. № 1. С. 132–151.
5. Демкин Н. Б., Рыжов Э. В. Качество поверхности и контакт деталей машин. М.: Машиностроение, 1981. 244 с.
6. Thomas T. R. Rough surfaces. London: Longman, 1982. 387 p.
7. Мусхелишвили Н. И. Некоторые основные задачи математической теории упругости. М.: Наука, 1966. 707 с.
8. Панасюк В. В., Саврук М. П., Дацышин А. П. Распределение напряжений около трещин в пластинах и оболочках. Киев: Наук. думка, 1976. 443 с.
9. Мирсалимов В. М. Неодномерные упругопластические задачи. М.: Наука, 1987, 256 с.