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ELASTIC-PLASTIC PROBLEM FOR A STRINGER PLATE WITH A CIRCULAR HOLE

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When calculating the strength of machines, structures and buildings with technological holes, it is important to take into account the plastic zones that emerge around the holes. However, the unknown shape and size of the plastic zone complicate the solution of elastic-plastic problems. This paper gives an approximate method and solution of the plane elastic-plastic problem of the distribution of stresses in a thin plate, reinforced with a regular system of stiffeners (stringers). The stringer plate under consideration has a circular hole, which is completely surrounded by the zone of plastic deformation. At infinity, the plate is subjected to a uniform tension along the stiffeners. A constant normal load is applied to the contour of the hole. The plate and stringer materials are assumed to be isotropic. The loading conditions are assumed to be quasi-static. It is assumed that the plate is in the plane-stressed state. Taken as the plasticity condition in the plastic zone is the Tresca-Saint-Venant plasticity condition. Methods of perturbation theory, analytic function theory, and the least squares method are used. The solution to the stated elastic-plastic problem consists of two stages. At the first stage, the stress-strain state for the elastic zone is found, and then the unknown interface between the elastic and plastic zones is determined using the least squares method. A closed system of algebraic equations has been constructed in each approximation, the numerical solution of which makes it possible to study the stress-strain state of a stringer plate, with the hole entirely surrounded by the plastic zone, as well as to determine the magnitudes of the concentrated forces that replace the action of the stringers. The interface between the elastic and plastic deformations has been found. The presented solution technique can be developed to solve other elastic-plastic problems. The solution obtained in this paper makes it possible to consider elastic-plastic problems for a stringer plate with other plasticity criteria.

Keywords: *plate, stringers, elastic-plastic problem, interface between elastic and plastic deformations.*

Introduction

Elements of many structures and buildings have technological holes. During operation, due to the concentration of stresses around the holes, plastic zones emerge. Taking these zones into account is important when calculating a structure or a building for strength. Such problems are complicated by the need to determine the shape and size of the plastic zone, but they are of great interest [1–11]. An approximate method for solving plane elastic-plastic problems, based on N. I. Muskhelishvili's methods for solving the elastic problem and the method of least squares, is proposed in [1]. The elastic-plastic problem for an infinite plate weakened by two identical square holes with partially unknown boundaries is considered in [3]. With the help of the theory of functions of a complex variable and the theory of conformal mapping, the problem was reduced to a boundary-value problem of analytic function theory. A method for constructing an elastic-plastic boundary in the problem of stretching a plate weakened by two circular holes of different diameters is proposed in [4]. The proposed method is based on the use of conservation laws, and is applied to solve a similar problem in [10]. Using the conservation laws, the interface between the elastic and plastic zones was found in the course of solving the elastic-plastic problem of the stress state under complex shear conditions in a body weakened by a hole bounded by a piecewise-smooth contour [7]. In [6], an analytical method based on the Tresca yield criterion is shown for determining the elastic-plastic boundary around a circular hole in a plate subjected to biaxial tensile-compressive loads at infinity. In this case, two or four non-intersecting plastic zones emerge around the hole. In both cases, the conformal mapping method is used. In [9], the conformal mapping method and the differential evolution algorithm are used to determine the elastic-plastic boundaries in the case of two identical circular holes in an infinite plate.

In [2], a plane elastic-plastic problem is considered under the condition of the Coulomb limit equilibrium and various lateral thrust coefficients in the intact massif for elliptical, vaulted, square and polygonal cross-sectional workings, as well as mutually influencing workings. The solution to the problem is obtained by the small parameter method and the finite element method. An approximate method for solving the elastic-plastic problem for a rock mass, under the action of tectonic and gravitational forces, is given in [8]. It is believed that the working is entirely surrounded by the plastic zone, and the material of the massif obeys the plasticity condition of V. V. Sokolovsky.

The plane elastic-plastic problem of stress distribution in a thin plate with a circular hole, with crack initiation and propagation in the elastic zone, was considered in [5]. It is assumed that the zone of plastic deformations completely surrounds the hole, and in the elastic zone, crack initiation and destruction of the plate material occur. The methods of perturbation theory and the theory of singular integral equations are used. The possibility of crack initiation in the elastic zone is taken into account when solving the plane elastic-plastic problem of stress distribution in a thin plate with a round hole in [11]. The interface between the elastic and plastic deformations was found, as well as the location and size of crack initiation zones.

In this paper, we consider a plane elastic-plastic problem of stress distribution in a thin plate reinforced by a regular system of stringers, the plate having a circular hole completely surrounded by the zone of plastic deformations.

Problem Formulation

Consider an infinite thin plate with a circular hole (Fig. 1). Elastic stiffeners (stringers) are attached to the plate symmetrically relative to the surface. The plate is subjected to a uniform tensile stress of $\sigma_y^\infty = \sigma_0$ along the stringers at infinity. Under the action of external tensile loads and internal pressure on the contour of the hole, there emerges a zone of plastic deformations around that hole, entirely surrounding it.

The plate material is assumed to be isotropic. The loading conditions are considered to be quasi-static. The plate is believed to be in the plane-stressed state. The hole contour is under constant normal load

$$\sigma_r = p\,,\ \tau_{r\theta} = 0\,.$$

Taken as the plasticity condition in the plastic zone is the Tresca-Saint-Venant plasticity condition. As is known [12–16], the plane problem of ideal plasticity is statically definable if the boundary conditions are specified in stresses.

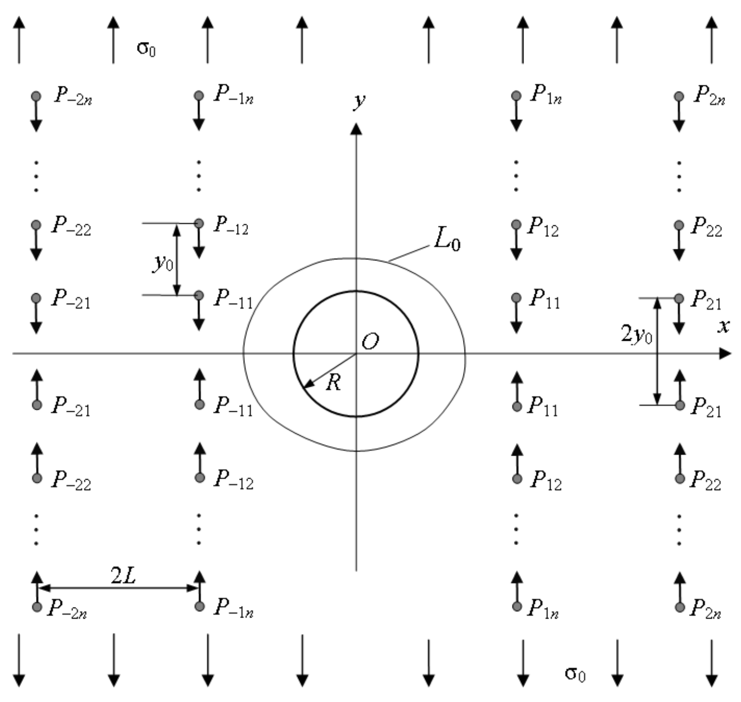

Fig. 1. Design diagram of the elastic-plastic problem for a stringer plate

The action of the stringers is replaced by the previously unknown equivalent concentrated forces P_{mn} applied at the points $z=\pm(2m+1)L\pm iky_0$ (m=0,1,2,…; k=1,2,…; $i=\sqrt{-1}$) where the stringers are attached to the plate.

It is necessary to determine the boundaries of the plastic deformation zone around the hole, the stress-strain state of the stringer plate, and the magnitude of the concentrated forces.

Let there be the inequality $\sigma_\theta \geq \sigma_r > 0$ in the plastic zone. In this case [16], the characteristics in the plastic zone will be radial straight lines, and the stresses will be determined by the formulas

$$\sigma_r^p = \sigma_s + (p - \sigma_s)\frac{R}{r}\,, \qquad \sigma_\theta^p = \sigma_s\,, \qquad \tau_{r\theta}^p = 0\,,$$

where σ_s is the tensile yield stress of the plate material; R is the radius of the hole.

For the inequality $\sigma_\theta \geq \sigma_r > 0$ to hold, it is obvious that the load p must satisfy the condition $0 \leq p \leq \sigma_s$.

On the unknown contour L_0 separating the elastic and plastic zones, all the stresses are continuous. The boundary conditions on L_0 have the form

$$\sigma_r^e = \sigma_r^p, \qquad \sigma_\theta^e = \sigma_\theta^p, \qquad \tau_{r\theta}^e = \tau_{r\theta}^p.$$

Therefore, to determine the stress state in the elastic zone of the plate, we have the following boundary conditions:

$$\sigma_r^e - i\tau_{r\theta}^e = \sigma_r^p - i\tau_{r\theta}^p \quad \text{on} \quad L_0. \tag{1}$$

To find the boundary of L_0, we have the condition

$$\sigma_\theta^e = \sigma_\theta^p \quad \text{on} \quad L_0. \tag{2}$$

Solution of the Boundary Value Problem

We will search for the unknown contour L_0 in the class of contours close to circular. We represent it in the form

$$r = \rho(\theta) = c_0 + \varepsilon H(\theta),$$

where the function $H(\theta)$ is to be determined; $\varepsilon = R_0/c_0$ is a small parameter. Here, R_0 is the maximum height of the deflection of L_0 from the circle $r=c_0$.

Without losing the generality of the problem under consideration, we assume that the required function $H(\theta)$ is symmetric with respect to the coordinate axes, and can be represented as a Fourier series $H(\theta) = \sum_{k=1}^{\infty} c_{2k} \cos 2k\theta$.

Stresses and displacements in the elastic zone, as well as concentrated forces, are sought in the form of small-parameter expansions, in which, for simplicity, the terms containing ε powers above the first one are discarded

$$\sigma_n = \sigma_n^{(0)} + \varepsilon\sigma_n^{(1)} + \ldots, \qquad \sigma_t = \sigma_t^{(0)} + \varepsilon\sigma_t^{(1)} + \ldots, \qquad \tau_{nt} = \tau_{nt}^{(0)} + \varepsilon\tau_{nt}^{(1)} + \ldots,$$

$$u_n = u_n^{(0)} + \varepsilon u_n^{(1)} + \ldots, \qquad v_n = v_n^{(0)} + \varepsilon v_n^{(1)} + \ldots,$$

$$P_{mn} = P_{mn}^{(0)} + \varepsilon P_{mn}^{(1)} + \ldots.$$

Each of the approximations satisfies the system of differential equations of the plane problem of the theory of elasticity. The components of the stress tensor at $r=\rho(\theta)$ are found by expanding the expressions for the stresses in the vicinity $r=c_0$ in a series. In each approximation, the solution is found using the analytic function theory.

Using the well-known formulas [17] for the stress components σ_n and τ_{nt}, we write the boundary conditions of problem (1) – (2) on the contour $r=c_0$ in the following form:

– for the zeroth approximation

$$\sigma_r^{(0)} = \sigma_r^p, \qquad \tau_{r\theta}^{(0)} = \tau_{r\theta}^p; \tag{3}$$

– for the first approximation

$$\sigma_r^{(1)} = N, \qquad \tau_{r\theta}^{(1)} = 0. \tag{4}$$

Here, $N = 2\frac{\tau_{r\theta}^{(0)}}{c_0}\frac{dH(\theta)}{d\theta} - H(\theta)\frac{\partial\sigma_r^{(0)}}{\partial r}$ at $r=c_0$.

Based on the Kolosov-Muskhelishvili formulas [17]

$$\sigma_x^e + \sigma_y^e = \sigma_r^e + \sigma_\theta^e = 2\left[\Phi(z) + \overline{\Phi(z)}\right],$$

$$\sigma_y^e - \sigma_x^e + 2i\tau_{xy}^e = e^{-2i\theta}(\sigma_\theta^e - \sigma_r^e + 2i\tau_{r\theta}^e) = 2\left[\bar{z}\Phi'(z) + \Psi(z)\right],$$

$$2\mu(u^e + iv^e) = 2\mu e^{i\theta}(u_r + iv_\theta) = \kappa\varphi(z) - z\overline{\Phi(z)} - \overline{\psi(z)}$$

and boundary conditions (3) on the contours of the holes, the problem in the zeroth approximation is reduced to determining two analytic functions, $\Phi^{(0)}(z)$ and $\Psi^{(0)}(z)$, from the boundary condition

$$\Phi_0(z) + \overline{\Phi_0(z)} - e^{2i\theta}\left[\bar{z}\Phi_0'(z) + \Psi_0(z)\right] = \sigma_r^p - i\tau_{r\theta}^p. \tag{5}$$

Here, $z=c_0e^{i\theta}$; μ is the shear modulus of the plate material; $\varphi'(z)=\Phi(z)$; $\psi'(z)=\Psi(z)$; $\kappa=(3-\nu)/(1+\nu)$; ν is Poisson's ratio of the material of the medium.

We seek the solution to boundary-value problem (5) in the form (k=0)

$$\Phi^{(k)}(z)=\Phi_0^{(k)}(z)+\Phi_1^{(k)}(z),\quad \Psi^{(k)}(z)=\Psi_0^{(k)}(z)+\Psi_1^{(k)}(z). \tag{6}$$

Here, the potentials $\Phi_0^{(0)}(z)$ and $\Psi_0^{(0)}(z)$ describe the field of stresses and strains in a solid plate under the action of a system of the concentrated forces $P_{mn}^{(0)}$ and σ_0, and are determined by the following formulas:

$$\Phi_0^{(0)}(z)=\frac{1}{4}\sigma_0-\frac{i}{2\pi h(1+\kappa)}\sum_{m,n}{}' P_{mn}^{(0)}\left[\frac{1}{z-mL+iny_0}-\frac{1}{z-mL-iny_0}\right], \tag{7}$$

$$\Psi_0^{(0)}(z)=\frac{1}{2}\sigma_0-\frac{i\kappa}{2\pi h(1+\kappa)}\sum_{m,n}{}' P_{mn}^{(0)}\left[\frac{1}{z-mL+iny_0}-\frac{1}{z-mL-iny_0}\right]+$$

$$+\frac{i}{2\pi h(1+\kappa)}\sum_{m,n}{}' P_{mn}^{(0)}\left[\frac{mL-iny_0}{(z-mL+iny_0)^2}-\frac{mL+iny_0}{(z-mL-iny_0)^2}\right],$$

where h is the thickness of the plate; the prime at the summation sign indicates that during summation the index $m=n=0$ is excluded.

To determine the functions $\Phi_1^{(0)}(z)$ and $\Psi_1^{(0)}(z)$, we modify boundary condition (5), and, for it to be solved, apply the N. I. Muskhelishvili method [17]. As a result, we find

$$\Phi_1^{(0)}(z)=\frac{\sigma_0}{2z^2}-iA\sum_{m,n}{}' P_{mn}^{(0)}\left\{\frac{A_1A_2-1}{A_1(zA_1-1)^2}-\frac{A_2A_1-1}{A_2(zA_2-1)^2}\right\}+iA\kappa\sum_{m,n}{}' P_{mn}^{(0)}\left[\frac{1}{z(zA_1-1)}-\frac{1}{z(zA_2-1)}\right], \tag{8}$$

$$\Psi_1^{(0)}(z)=\frac{\sigma_0}{2z^2}+\frac{iA}{z}\sum_{m,n}{}' P_{mn}^{(0)}\left[\frac{1}{zA_1-1}-\frac{1}{zA_2-1}+\frac{1}{zA_1}-\frac{1}{zA_2}\right]+\frac{\Phi_0^{(0)}(z)}{z^2}-\frac{\Phi_0^{(0)\prime}(z)}{z}.$$

Here, $A=\dfrac{1}{2\pi h(1+\kappa)}$, $A_1=mL-iny_0$, $A_2=mL+iny_0$.

In formulas (8), all the linear dimensions are referred to radius R.

The magnitude of the concentrated forces is determined using Hooke's law. According to this law, the magnitude of the concentrated force $P_{mn}^{(0)}$ acting on each attachment point from the side of the stringer,

$$P_{mn}^{(0)}=\frac{E_sA_s}{2y_0n}\Delta v_{mn}^{(0)}\qquad (m,\ n=1,2,\ldots), \tag{9}$$

where E_s is Young's modulus of the stringer material; $2y_0n$ is the distance between the attachment points; $\Delta v_{mn}^{(0)}$ is the mutual displacement of the considered attachment points, equal to the elongation of the corresponding section of the stringer.

It is believed that stringers work only in tension (they do not undergo bending), their thickness remains unchanged during deformation, and the stress state is uniaxial. It is assumed that the truss-type stringer system does not weaken the stringers due to the setting of the attachment points. The attachment points (adhesion areas) are the same, with a radius of a_0, which is small compared to their step of $2L$ and other typical dimensions. The plate and stringers interact with each other in the same plane and only at the attachment points.

Let us accept [18] the natural assumption that the mutual elastic displacement of the points $z=mL+i(ny_0-a_0)$ and $z=mL-i(ny_0-a_0)$ in the problem under consideration is equal to the mutual displacement $\Delta v_{mn}^{(0)}$ of the attachment points. This additional condition for the compatibility of the displacements makes it possible to find a solution to the problem.

Using the Kolosov-Muskhelishvili formulas [17], relations (6)–(8), after performing elementary calculations, we find the mutual displacement $\Delta v_{mn}^{(0)}$ of the attachment points

$$\Delta v_{kr}^{(0)} = \Delta v_{kr}^{0} + \Delta v_{kr}^{1}\,, \tag{10}$$

$$\Delta v_{kr}^{0} = \frac{\sigma_0}{4\mu}(1+\kappa)d_1 + \frac{1}{2\pi(1+\kappa)\mu h}\sum_{m,n}{}' P_{mn}^{(0)}\left\{\kappa \ln\frac{d_3+a^2}{d_3+b^2} + \frac{2d_0 d_1\left[2k(k-m)L^2+ab\right]}{(d_3+b^2)(d_3+a^2)}\right\},$$

$$\Delta v_{kr}^{1} = \frac{\sigma_0}{2\mu}\left\{\frac{(1+\kappa)d_1}{B_2} + \frac{2d_1 kL}{B_2^2}\left(\frac{kL}{B_2} - kL\right) + \frac{k^2L^2 - d_1^2}{B_2^2}\left(\frac{d_1}{B_2} - d_1\right)\right\} + + \frac{1}{2\pi(1+\kappa)\mu h}\sum_{m,n}{}' P_{mn}^{(0)}\Pi\,,$$

where $b=(r-n)y_0-a$; $d_0=b+a$; $d_1=ry_0-a$; $d_3=(k-m)^2L^2$;

$$\Pi = \kappa\left\{\frac{C_4(DkL - D_1 d_1) - C_2^*(D_1 kL - d_1 D)}{(D_1^2+D^2)B_2} - \frac{C_4(D_3 kL - D_2 d_1) - C_2^*(D_2 kL - d_1 D_3)}{(D_3^2+D_2^2)B_2}\right\} -$$

$$-\kappa(d_5-1)\left\{\frac{D_3 d_6 - 2ny_0 mLD_2}{(D_3^2+D_2^2)D_4} - \frac{Dd_6 - 2ny_0 mLD_1}{(D^2+D_1^2)D_4}\right\} + \frac{1+\kappa}{2}\ln\frac{D^2+D_1^2}{D_3^2+D_2^2} + \frac{2ny_0(ny_0-a)}{d_5 B_2} -$$

$$-(d_5-1)\left\{\frac{C_4\left[mL(D^2-D_1^2)+2ny_0 DD_1\right] - C_2^*\left[2mLD_1 D - ny_0(D^2-D_1^2)\right]}{(D^2+D_1^2)^2 d_5} -\right.$$

$$\left. - \frac{C_4\left[mL(D_3^2-D_2^2) - 2ny_0 D_2 D_3\right] - C_2^*\left[ny_0(D_3^2-D_2^2) + 2mLD_2 D_3\right]}{(D_3^2+D_2^2)d_5}\right\};$$

$$d_5 = m^2L^2 + n^2y_0^2;\quad d_6 = m^2L^2 - n^2y_0^2;\quad B_2 = k^2L^2 + d_1^2;\; C_4 = kL + \frac{kL}{B_2};\quad C_2^* = d_1\left(1+\frac{d_1}{B_2}\right);$$

$$D = kmL^2 + ny_0 d_1 - 1;\; D_1 = d_1 mL - ny_0 kL;\quad D_2 = d_1 mL + ny_0 kL;$$

$$D_3 = kmL^2 - ny_0 d_1 - 1;\; D_4 = d_6^2 + 4y_0^2 L^2 m^2\,.$$

Solving system (9)–(10), we determine the magnitude of the concentrated forces $P_{mn}^{(0)}$, and thereby the complex potentials of the zeroth approximation. According to the Kolosov-Muskhelishvili formulas and (6), the stress components in the reinforced plate are found in the zeroth approximation. Knowing the stress state in the zeroth approximation, we find the functions *N*.

After finding the solution in the zeroth approximation, we turn to the solution of the problem in the first approximation (4).

The boundary conditions of the problem for the first approximation are written in the form

$$\Phi^{(1)}(\tau) + \overline{\Phi^{(1)}(\tau)} - e^{2i\theta}\left[\bar{\tau}\Phi^{(1)\prime}(\tau) + \Psi^{(1)}(\tau)\right] = N \quad \text{at } \tau = c_0 e^{i\theta}. \tag{11}$$

The solution of the boundary-value problem (11), similarly to the zeroth approximation, is sought in the form (6) at k=1, where the potentials $\Phi_0^{(1)}(z)$ and $\Psi_0^{(1)}(z)$ describe the field of stresses and strains under the action of a system of the concentrated forces $P_{mn}^{(1)}$, and are determined by formulas similar to (7), in which we should set σ_0 equal to zero, replace $P_{mn}^{(0)}$ with $P_{mn}^{(1)}$.

To determine the potentials $\Phi_1^{(1)}(z)$ and $\Psi_1^{(1)}(z)$ from the boundary condition (11), we again use the method of N.I. Muskhelishvili. As a result, we find

$$\Phi_1^{(1)}(z) = \Phi_{1*}^{(1)}(z) + \sum_{k=0}^{\infty} a_{2k} z^{-2k}\,, \qquad \Psi_1^{(1)}(z) = \Psi_{1*}^{(1)}(z) + \sum_{k=0}^{\infty} b_{2k} z^{-2k}\,.$$

Here, $\Phi_{1*}^{(1)}(z)$ and $\Psi_{1*}^{(1)}(z)$ are determined by formulas similar to (8), in which σ_0 should be set equal to zero, $P_{mn}^{(0)}$ be replaced with $P_{mn}^{(1)}$. The coefficients a_{2k} and b_{2k} are found using the formulas

$$a_{2n} = C'_{2n} R^{2n} \quad (n=1,2,\ldots), \quad a_0=0,$$

$$b_{2n} = (2n-1) R^2 a_{2n-2} - R^{2n} a_{-2n+2} \quad (n\geq 2), \quad b_0 = 0, \quad b_2 = -C'_0 R^2, \quad N = \sum_{k=-\infty}^{\infty} C'_{2k} e^{2ik\theta}. \tag{12}$$

For the concentrated forces $P_{mn}^{(1)}$, we have

$$P_{mn}^{(1)} = \frac{E_s A_s}{2 y_0 n} \Delta v_{mn}^{(1)}. \tag{13}$$

The mutual displacement $\Delta v_{mn}^{(1)}$ is determined in the same way as the zeroth approximation.

The solutions of the problem for the elastic zone in subsequent approximations can be constructed in a similar way.

The resulting systems of equations of the first approximation are not yet closed, since the right-hand sides of these systems include the coefficients c_{2k} of the expansion of the function $H(\theta)$ in a Fourier series. To construct the missing equations, we use the boundary condition (2). Using the obtained solution, we find $\sigma_\theta^e(\theta)$ ($r=\rho(\theta)$) up to first-order quantities with respect to the small parameter ε

$$\sigma_\theta^e = \sigma_\theta^{e(0)}(\theta)_{|r=c_0} + \varepsilon \left[H(\theta) \frac{\partial \sigma_\theta^{e(0)}}{\partial r} + \sigma_\theta^{e(1)}(\theta) \right]_{|r=c_0}.$$

The stresses $\sigma_\theta^e(\theta)$ depend on the coefficients of the Fourier series of the required function $H(\theta)$. To construct the missing equations that allow determining these coefficients, we demand that the condition (2) be satisfied. To satisfy the condition (2), we use the least squares method.

The stress σ_θ^e on L_0 is a function of the independent variable of the polar angle θ and $(m_1 + 1)$ of the parameters c_0, c_{2k} ($k=1,2,\ldots,m_1$). The unknown parameters c_0, c_{2k} are constant and must be determined. To determine them, we carry out a number of calculations. We divide the segment $[0, 2\pi]$ of the θ change into M_1 parts, where

$$\theta_j = \theta_1 + j\Delta\theta \quad (j=0,1,2,\ldots,M_1-1), \quad \Delta\theta = \frac{2\pi}{M_1}, \qquad M_1 > m_1+1.$$

Calculate the normal tangential stress σ_θ^e at the points of division

$$\sigma_\theta^e(\theta_j) = F(\theta_j, c_0, c_{2k}) \qquad (j=0,1,2,\ldots,M_1-1).$$

Therefore, it is required to find such values of the unknown parameters c_0, c_{2k}, which will provide the values σ_θ^p for the values of the circumferential stress function σ_θ^e on L_0 in the best way.

The Least Squares Principle states that the most likely parameter values will be those at which the sum of the squares of the deviations is smallest

$$U = \sum_{j=0}^{M_1-1} \left[F(\theta_j, c_0, c_{2k}) - \sigma_s \right]^2 \to \min.$$

Writing the necessary condition for the extremum of the function U, we obtain (m_1+1) equations with (m_1+1) unknowns

$$\frac{\partial U_0}{\partial c_0} = 0, \quad \frac{\partial U_0}{\partial c_{2k}} = 0, \quad (k=1,2,\ldots,m_1). \tag{14}$$

The system of equations (14) closes the obtained algebraic systems of problem (12), (13). The listed systems must be solved simultaneously.

Note that the issues of convergence of the perturbation method in solving elastic-plastic problems and the theory of inhomogeneous elasticity were discussed in detail in monographs [19, 20].

It should be remembered that the complex potentials of the first approximation depend on the components of the zero-order stresses and the coefficients c_0, c_{2k}. To determine the stresses in the first approximation (see boundary condition (11)), the coefficients c_0, c_{2k} are used. In this case, it is necessary to simultaneously solve the systems of equations (12)–(14).

Results Analysis

A simultaneous numerical solution to the obtained systems of algebraic equations (12)–(14) makes it possible, for a given external load, to find concentrated forces, complex potentials, and the coefficients c_0, c_{2k} ($k = 1,2,\ldots,m_1$).

The calculations were carried out for the following values of free parameters: a_0/L=0.01; y_0/L=0.25. It was considered that the stringers were made of Al-steel composite, the plate was made of the B95 alloy, E=7.1·10^4 MPa, E_s=11.5·10^4 MPa. For simplicity, it was assumed that A_s/y_0h=1. It was assumed that the number of stringers and attachment points is finite (14), and M_1=72. It was assumed that p/σ_s=0.6; σ_0/σ_s=0.75. The results of calculating the expansion coefficients of the function $H(\theta)$ are given below.

c_0	c_2	c_4	c_6	c_8	c_{10}	c_{12}
1.903	0.798	0.511	0.384	0.267	0.188	0.139

Conclusions

A solution has been found for the plane elastic-plastic problem on the stress distribution around a circular hole in the stringer plate being stretched, the hole being entirely surrounded by the zone of plastic deformations. A closed system of algebraic equations has been obtained, the numerical solution of which makes it possible to study the stress state of a stringer plate with a hole entirely surrounded by the plastic zone. The constructed equations allow us, for the given mechanical and geometric characteristics of the stringer plate, to use numerical calculations to find the interface between the elastic and plastic deformations.

The proposed method for solving the elastic-plastic problem for a stringer plate can be developed for solving other elastic-plastic problems. The solution obtained in this paper makes it possible to consider problems with other plasticity criteria, as well as especially complex elastic-plastic problems, taking into account the initiation and development of cracks in the elastic zone. A stringer plate can have not one, but several holes or a whole system, which is why research in this direction can be continued.

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Пружно-пластична задача для стрингерної пластини з круговим отвором

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При розрахунку на міцність машин, конструкцій і споруд, що мають технологічні отвори, важливо враховувати пластичні області, що виникають навколо отворів. Однак невідомі форма і розміри пластичної області ускладнюють розв’язання пружно-пластичних задач. У даній роботі дається наближений метод і розв’язок плоскої пружно-пластичної задачі про розподіл напружень в тонкій пластині, підкріпленої регулярною системою ребер жорсткості (стрингерів). Вже згадана стрингерна пластина має круговий отвір, який цілком охоплюється зоною пластичних деформацій. На нескінченності пластина схильна до однорідного розтягування уздовж ребер жорсткості. До контуру кругового отвору прикладена постійне нормальне навантаження. Матеріали пластини і стрингерів прийняті ізотропними. Умови навантаження припускаються квазістатичними. Прийнято, що пластина знаходиться в плоско-напруженому стані. Як умова пластичності в пластичній зоні приймається умова пластичності Треска-Сен-Венана. Використовуються методи теорії збурень, теорії аналітичних функцій і метод найменших квадратів. Розв’язок поставленої пружно-пластичної задачі складається з двох етапів. На першому етапі знаходиться напружено-деформований стан для пружної зони, а потім за допомогою методу найменших квадратів визначається невідома межа розділу пружною і пластичної зон. Побудована в кожному наближенні замкнута система алгебраїчних рівнянь, числовий розв’язок якої дозволяє досліджувати напружено-деформований стан стрингерної пластини з повним охопленням отвору пластичної зони, а також визначити величини зосереджених сил, які замінюють дію стрингерів. Знайдена межа розділу пружних і пластичних деформацій. Наведена методика розв’язання може бути розвинена для розв’язання інших пружно-пластичних задач. Отриманий в роботі розв’язок дає можливість розглядати пружно-пластичну задачу для стрингерної пластини з іншими критеріями пластичності.

Ключові слова*: пластина, стрингери, пружно-пластична задача, межа розділу пружних і пластичних деформацій.*

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