DOI: https://doi.org/10.15407/pmach2022.01.040

UDC 532.5 + 536.24

MATHEMATICAL AND COMPUTER MODELING OF CONVECTIVE HEAT TRANSFER IN FUEL CARTRIDGES OF FUEL ELEMENTS WITH DIFFERENT SHAPES AND PACKING OF RODS

[1] **Kyrylo V. Maksymenko-Sheiko**
m-sh@ipmach.kharkov.ua
ORCID: 0000-0002-7064-2442

[1] **Tetiana I. Sheiko**
sheyko@ipmach.kharkov.ua
ORCID: 0000-0003-3295-5998

[2] **Denys O. Lisin**
d.lisin@karazin.ua
ORCID: 0000-0002-6718-7389

[2] **Timur B. Dudinov**
tima.dudinov@gmail.com
ORCID: 0000-0001-8365-0516

[1] A. Pidhornyi Institute of Mechanical Engineering Problems of NASU,
2/10, Pozharskyi str., Kharkiv, 61046, Ukraine

[2] V. N. Karazin Kharkiv National University,
4, Svobody sq., Kharkiv, 61022, Ukraine

The paper consists of three sections and is of an informational and generalizing nature, indicating promising areas for further research. The first section "R-functions method in mathematical modeling of convective heat transfer in fuel cartridges with fuel elements" is devoted to the use of new constructive tools of the R-functions method for mathematical and computer modeling of fuel elements packings with different types of symmetry, as well as the study of convective heat transfer in fuel elements grids and the effect of the type of packing on the distribution of velocity and temperature. An octahedral cartridge with 37 fuel elements packed according to three patterns (cyclic, checkerboard and in-line) is considered. It is noted that when constructing the equations of a cartridge with bundles of fuel elements using the new method, the number of R-operations and, accordingly, the calculation time are significantly reduced. An analysis of the obtained results allows to conclude that the maximum temperature is obtained with cyclic packing. The scheme of the reactor, the cartridges of which are hexagonal casings, with 91 fuel elements placed in each of them both with checkerboard and cyclic packing, is also considered. In the second section "Thermal-hydraulic calculation of fuel elements cartridges in case of violation of the rods packing symmetry", a hexagonal fuel cartridge with 169 fuel elements and checkerboard packing is considered. An increase in temperature is analyzed in case of violation of the packing symmetry while maintaining the parallelism of the rods, as well as in case of a curvature of one of them. The third section "R-functions, fuel element with polyzonal finning of the shell and heat transfer during fluid motion" is focused on the construction of equations for various finning surfaces of fuel elements and the study of hydrodynamic and temperature fields in case of polyzonal finning of the shell. At the same time, using the apparatus of tensor analysis, a transition to a curvilinear non-orthogonal (helical) coordinate system was made. It is noted that mathematical modeling and the associated computer experiment are indispensable in cases where a full-scale experiment is impossible or difficult to conduct for one reason or another. In addition, working with mathematical model of the process and the computational experiment make it possible to investigate the properties and behavior of the process in various situations relatively quickly and without significant expenses. The reliability of the methods, results and conclusions is confirmed by comparison with the information given in the references, the results of the analysis of the numerical convergence of solutions and the calculation of the residual.

Keywords: *nuclear reactor, cartridge, fuel element, R-function method, packing symmetry type, shell finning.*

Introduction

Currently, the world pays great attention to green energy, which is explained by the desire to replace dirty coal generation with renewable energy sources. It should be noted that many people consider nuclear energy to be the second component of replacement. With this in mind, Ukraine has set a goal for the next seven years – to increase nuclear capacity by three power units.

Due to the growth in the number of nuclear power plants, as well as in the number of models and modifications of nuclear reactors, the advantages of certain plants become of particular importance. At the same time, designers face a number of problems, the optimal solutions of which have not yet been found. There is the largest turnover of financial resources at nuclear power plants, and the slightest gain in efficiency brings huge profits, but one must not forget about the reliability and expenses during the construction of the plant. Naturally, this task, which is complex, is solved at the design stage. Calculation of the reactor at the design stage involves the determination of the main parameters of the core, temperature values, etc. In addition, one of the cornerstones in justifying the safe operation of a nuclear power plant is the thermal-hydraulic calculation of the reactor core. Calculation of coolant parameters and temperatures of fuel elements is carried out at all stages of designing and justifying the safety of nuclear power plants.

As is known, the processes of hydrodynamics and heat transfer in reactor fuel assemblies used in the power industry are complex, but their exact description is necessary to achieve optimal design characteristics under nominal operation modes and to analyze reliability in case of deviations from nominal modes and in emergency situations. Thus, an accurate calculation of the temperature regimes of fast reactor fuel elements makes it possible to refine the calculated estimates of the dimensions, swelling and bending of elements, etc., which determines the efficiency of the reactor core and the level of permissible fuel burnup. Increase of requirements for the thermal-hydraulic calculation of fuel elements cartridges, in turn, necessitated the development of new methods for the theoretical study of processes in rod bundles. The currently developed methods and programs [1–3] differ: in the initial equations that have to be solved and the methods for their solution; in taking into account various factors; in relations for the initial constants and, accordingly, in the accuracy of the calculation and the classes of problems that have to be solved. However, a common feature for all methods is a closed relation between the velocity and temperature fields. It is impossible to accurately calculate heat transfer without knowing the velocity field. For example, in [3], the conjugate problem of heat transfer in case of a rod flow of a coolant in fuel elements grids was reduced to a solution for one translational element. At the same time, the author considered the symmetry of the system. However, the analysis of the nature of the velocity and temperature distribution carried out in [4–6] allows to conclude that consideration of the velocity field for a cell, in case of its remoteness from the boundary, is appropriate. The temperature field in this case will be very far from reality, as evidenced by the results obtained for the cartridge as a whole. The papers [7–9] laid the foundations for new constructive tools of the R-functions theory for the analytical description of geometric objects with different types of symmetry.

The aim of the paper is:

– the use of new constructive tools of the R-function method for mathematical and computer modeling of fuel elements packings with checkerboard, in-line and cyclic symmetry, as well as the study of convective heat transfer in fuel elements grids and the effect of the packing type on the velocity and temperature distribution;

– the use of methods and the construction of equations for various surfaces of the fuel elements finning, as well as the study of hydrodynamic and temperature fields in case of polyzonal finning of the shell;

– study of the possibilities of joint application of the structural R-functions theory and variational methods for solving conjugate problems of convective heat transfer.

The paper is of an informational and generalizing nature; it indicates promising tasks for further research. At the same time, using the apparatus of tensor analysis, a transition to a curvilinear non-orthogonal (helical) coordinate system was made.

R-functions method in mathematical modeling of convective heat transfer in fuel cartridges with fuel elements

The reactor core, which is assembled from a large number of fuel cartridges, is considered [4, 5, 10]. An equation for a model octahedral fuel cartridge with 37 fuel elements packed according to three well-known patterns (cyclic, checkerboard, and in-line) is constructed. In this case, the octahedral cartridge was chosen due to convenience in the position of all types of fuel elements packings with the same number of the said packings, which is a necessary condition for assessing the effect of the type of packing on the velocity and temperature distribution.

To construct the equation for an octahedral casing, the method developed in [4, 5, 7, 8] is used. The equation of a straight line $\sigma \equiv R_v - x \geq 0$ and periodic function $\mu_v = \frac{8}{\pi n}\sum_k (-1)^{k+1} \frac{\sin\left[(2k-1)\frac{n\theta}{2}\right]}{(2k-1)^2}$ (n=8) are considered. As a result, we got $\omega_b \equiv R_v - r\cos\mu_v \geq 0$, where $r = \sqrt{x^2 + y^2}$, $\theta = arctg\frac{y}{x}$.

Let's dwell on the construction of the function $\omega(x,y)\equiv\omega_b\wedge_0\overline{\omega_{tv}}\geq 0$, when fuel element is translated with cyclic symmetry n_1 times along a circle of radius R_1, n_2 times along a circle of radius R_2 and n_3 times along a circle of radius R_3.

To construct the fuel element boundary equation, translated with cyclic symmetry n_1 times along the circle of radius R_1, the function $\omega_{o1}\equiv\frac{1}{2R_{tv}}\left(R_{tv}{}^2-(x-R_1)^2-y^2\right)$ and formula $\mu_1=\frac{8}{n_1\pi}\sum_k(-1)^{k+1}\frac{\sin\left[(2k-1)\frac{n_1\theta}{2}\right]}{(2k-1)^2}$ are used. As a result, we get $\omega_{tv1}\equiv\frac{1}{2R_{tv}}\left(R_{tv}{}^2-(r\cos\mu_1-R_1)^2-(r\sin\mu_1)^2\right)\geq 0$.

To construct the fuel element boundary equation, translated with cyclic symmetry n_2 times along the circle of radius R_2, the function $\omega_{o2}\equiv\frac{1}{2R_{tv}}\left(R_{tv}{}^2-(x-R_2)^2-y^2\right)$ and formula $\mu_2=\frac{8}{n_2\pi}\sum_k(-1)^{k+1}\frac{\sin\left[(2k-1)\frac{n_2\theta}{2}\right]}{(2k-1)^2}$ are used. Then $\omega_{tv2}\equiv\frac{1}{2R_{tv}}\left(R_{tv}{}^2-(r\cos\mu_2-R_2)^2-(r\sin\mu_2)^2\right)\geq 0$.

To construct the fuel element boundary equation, translated with cyclic symmetry n_3 times along the circle of radius R_3, the function $\omega_{o3}\equiv\frac{1}{2R_{tv}}\left(R_{tv}{}^2-(x-R_3)^2-y^2\right)$ and formula $\mu_3=\frac{8}{n_3\pi}\sum_k(-1)^{k+1}\frac{\sin\left[(2k-1)\frac{n_3\theta}{2}\right]}{(2k-1)^2}$ are used. In this case $\omega_{tv3}\equiv\frac{1}{2R_{tv}}\left(R_{tv}{}^2-(r\cos\mu_3-R_3)^2-(r\sin\mu_3)^2\right)\geq 0$.

Thus, the boundary equation of the cartridge with 36 fuel elements has the form $\omega\equiv\left(\omega_b\wedge_0\overline{\omega_{tv1}\vee_0\omega_{tv2}\vee_0\omega_{tv3}}\right)\geq 0$, when n_1=18, R_1=4.2, n_2=12, R_2=2.8, n_3=6, R_3=1.5, R_{tv}=0.2 and belongs to a family of curves with seven parameters (n_1, n_2, n_3, R_1, R_2, R_3, R_{tv}) (Fig. 1, a).

It should be noted that here and below we used the simplest R-operations $f\wedge_0 g=f+g-\sqrt{f^2+g^2}$; $f\vee_0 g=f+g+\sqrt{f^2+g^2}$; $\bar{f}=-f$. Herewith, the equations $\omega(x,y)\geq 0$ constructed by this method are infinitely differentiable. In this case, R-operations were used only three times. With a central fuel element, we obtain $\omega\equiv\omega_b\wedge_0\omega_{tv1}\vee_0\omega_{tv2}\vee_0\omega_{tv3}\vee_0\frac{1}{2R_{tv}}\left(R_{tv}{}^2-x^2-y^2\right)\geq 0$, and the total number of fuel elements becomes equal to 37 (Fig. 1, a).

To construct a triangular (checkerboard) packing of fuel elements, we set $f_1=R_{tv}{}^2-\mu_x^2-\mu_y^2\geq 0$, where $\mu_x=\frac{4h_x}{\pi^2}\sum_k(-1)^{k+1}\frac{\sin\left[(2k-1)\frac{\pi x}{h_x}\right]}{(2k-1)^2}$, $\mu_y=\frac{4h_y}{\pi^2}\sum_k(-1)^{k+1}\frac{\sin\left[(2k-1)\frac{\pi x}{h_y}\right]}{(2k-1)^2}$, and $f_2=R_{tv}{}^2-\mu_{x1}^2-\mu_{y1}^2\geq 0$, where $\mu_{x1}=\frac{4h_x}{\pi^2}\sum_k(-1)^{k+1}\frac{\sin\left[\frac{(2k-1)\pi(x-h_x/2)}{h_x}\right]}{(2k-1)^2}$, $\mu_{y1}=\frac{4h_y}{\pi^2}\sum_k(-1)^{k+1}\frac{\sin\left[\frac{(2k-1)\pi(y-h_y/2)}{h_x}\right]}{(2k-1)^2}$.

Then the equation of the fuel cartridge has the form $\omega\equiv\omega_b\wedge_0\overline{\omega_{tv}}\geq 0, \omega_{tv}\equiv(f_1\vee_0 f_2)\geq 0$. The construction of the function $\omega(x,y)$ is performed with the following values of the literal parameters: R_{tv}=0.2, h_x=2.75, h_y=1.52 (Fig. 1, b). It should be noted that, during the construction of the cartridge equation with the new method, the R-operations were used only twice.

To construct a rectangular (in-line) packing of fuel elements, we set $f_1 = R_{tv}{}^2 - \mu_x^2 - \mu_y^2 \geq 0$, where $\mu_x = \frac{4h_x}{\pi^2} \sum_k (-1)^{k+1} \frac{\sin\left[(2k-1)\frac{\pi x}{h_x}\right]}{(2k-1)^2}$, $\mu_y = \frac{4h_y}{\pi^2} \sum_k (-1)^{k+1} \frac{\sin\left[(2k-1)\frac{\pi x}{h_y}\right]}{(2k-1)^2}$. In this case, the fuel cartridge equation has the form $\omega \equiv \omega_b \wedge_0 \overline{\omega_{tv}} \geq 0$, $\omega_{tv} \equiv f_1 \geq 0$. The construction of the function $\omega(x, y)$ is performed with the following values of the literal parameters: R_{tv}=0.2, h_x=h_y=1.5 (Fig. 1, c).

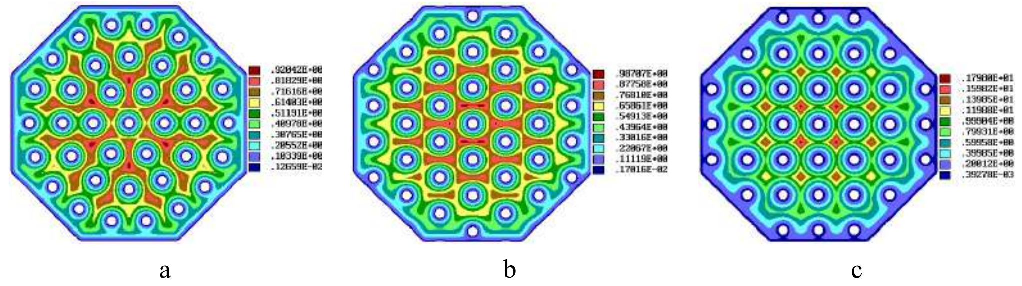


Fig. 1. Fuel cartridge with 37 fuel elements:
a – with cyclic symmetry and central fuel element; b – checkerboard pattern; c – in-line pattern

The main system of equations describing the process of heat transfer in a viscous fluid flow, with constant physical properties of the fluid and temperature, has the form

$$\begin{cases} \frac{DT}{D\tau} = a\Delta T + \frac{q_V}{\rho c_p} + \frac{\mu\Phi}{\rho c_p} \\ \frac{D\vec{V}}{D\tau} = -\frac{1}{\rho}\vec{\nabla} p + \nu\Delta\vec{V} \\ div\vec{V} = 0 \end{cases}, \quad (1)$$

where $\frac{D}{D\tau} = \frac{\partial}{\partial\tau} + (\vec{V} \cdot \vec{\nabla})$ is a substantial (or total) derivative; $\mu\Phi$ is a dissipative function; $a = \frac{\lambda}{\rho c_p}$ is a thermal diffusivity; c_p is a heat capacity of the medium; q_V is a power of internal heat sources.

In case of stationary processes, the body temperature does not depend on time, and the thermal conduction equation for longitudinal flow around fuel elements takes the form $V_z \frac{\partial T}{\partial z} = a\Delta T + \frac{q_V}{\rho c_p}$, and the mathematical model of the velocity field in laminar flow has the form $\Delta V_z = -\frac{\nabla P}{\mu l} = -C$, where ∇P is a constant pressure drop along the pipe in an arbitrarily chosen section of length l.

In the region of thermal stabilization, when $\frac{\partial T}{\partial z} = \text{const}$, we get $-div(\lambda \nabla T) = q_V - V_z C_1$. Thus, the mathematical model of heat transfer in case of laminar motion of fluid along the cartridge with fuel elements is reduced to the system of equations $\begin{cases} \Delta V_z = -C & в\ \Omega_b \cap \overline{\Omega_{tv}} \\ -div(\lambda_i \nabla T_i) = F_i & в\ \Omega_b \end{cases}$, where $\begin{cases} F_1 = -V_z & в\ \Omega_b \cap \overline{\Omega_{tv}} \\ F_2 = q_V & в\ \Omega_{tv} \end{cases}$, with boundary conditions of the form

$$V_z\big|_{\partial\Omega_b \cap \overline{\partial\Omega_{tv}}} = 0, \quad \frac{\partial T}{\partial n} + hT\Big|_{\Omega_b} = 0, \quad T_1\big|_{\partial\Omega_{tv}} = T_2\big|_{\partial\Omega_{tv}}, \quad \lambda_1 \frac{\partial T_1}{\partial n_1}\bigg|_{\partial\Omega_{tv}} = \lambda_2 \frac{\partial T_2}{\partial n_2}\bigg|_{\partial\Omega_{tv}} \quad (2)$$

Knowing the equations of the cartridge and fuel elements, we can rewrite the statement of problem (2) in the form $\begin{cases} \Delta V_z = -C \\ -div(\lambda \nabla T) = F \end{cases}$, with boundary conditions $V_z|_{\partial\Omega_b \cap \overline{\partial\Omega_{tv}}} = 0$, $\frac{\partial T}{\partial n} + hT|_{\Omega_b} = 0$, $T_1|_{\partial\Omega_{tv}} = T_2|_{\partial\Omega_{tv}}$, $\lambda_1 \frac{\partial T_1}{\partial n_1}\Big|_{\partial\Omega_{tv}} = \lambda_2 \frac{\partial T_2}{\partial n_2}\Big|_{\partial\Omega_{tv}}$, where $\lambda = \lambda_1 \frac{1 - sign\omega_{tv}}{2} + \lambda_2 \frac{1 + sign\omega_{tv}}{2}$, $F = -V_z \frac{1 - sign\omega_{tv}}{2} + q_V \frac{1 + sign\omega_{tv}}{2}$.

The R-functions method was used in combination with the Ritz variational method to obtain the solution. The structure of the solution of the problem of laminar flow in case of longitudinal flow around fluid fuel elements has the form $V_z = \omega p_1$, where $\omega(x, y) \equiv \omega_b \wedge_0 \overline{\omega_{tv}} \geq 0$ is a boundary equation of the cartridge cross section, and an indefinite component $p_1 = \sum_{i=1}^{N} c_{ik} \varphi_{ik}(x, y)$ will be calculated by minimizing the functional $I = \int_{\Omega} \left((\nabla V_z)^2 - 2CV_z \right) d\Omega$. It should be noted that the solution V_z is obtained in an analytical form and used without any further processing (approximation, interpolation). Therefore, the resulting velocity distribution is substituted into the right side of the thermal conduction equation. The structure of the solution to the problem of determining the temperature field was used as exactly satisfying the boundary conditions on $\partial\Omega_b$ $u = p_2 + \omega_b(-D_1 p_2 + hp_2)$, as well as in the form $T = p_2$, where, as before, $p_2 = \sum_{i=1}^{N} d_{ik} \varphi_{ik}(x, y)$.

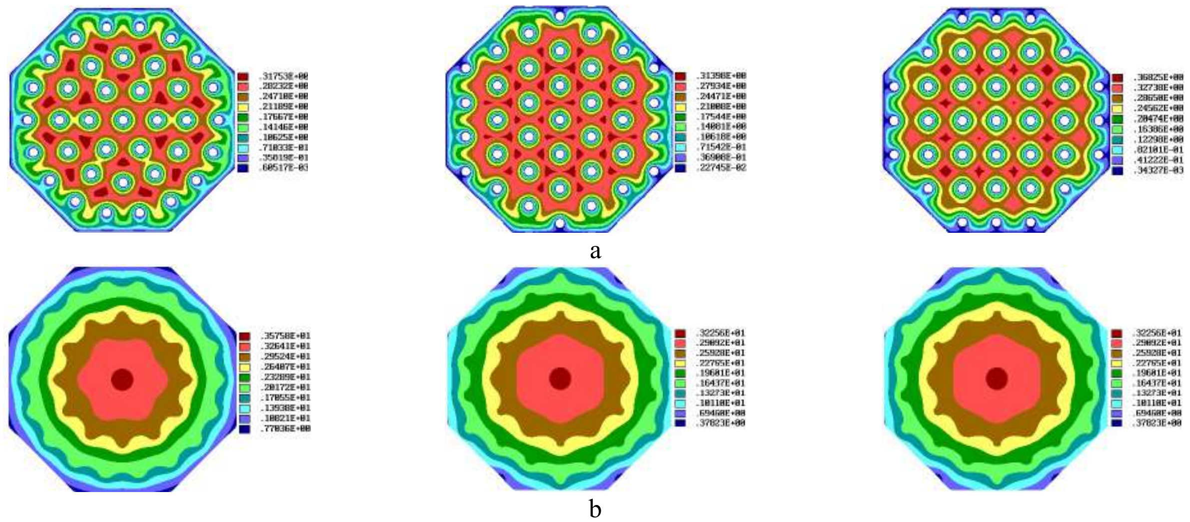


Fig. 2. Pictures of the distribution of the velocity (a) and temperature field (b)

It should be noted here that the boundary conditions $\frac{\partial T}{\partial n} + hT|_{\Omega_b} = 0$ and $\lambda_1 \frac{\partial T_1}{\partial n_1}\Big|_{\partial\Omega_{tv}} = \lambda_2 \frac{\partial T_2}{\partial n_2}\Big|_{\partial\Omega_{tv}}$ are natural and follow from the Ritz functional $I = \int_{\Omega} \left(\lambda(\nabla T)^2 - 2FT \right) d\Omega + \int_{\partial\Omega_b} hT^2 d\partial\Omega_b$. As approximation means $\varphi_{ik}(x, y)$, Schoenberg's cubic splines were used at N=6400, 10000. Computational experiments were carried out under the operating conditions of the POLYE system, developed in the Department of Applied Mathematics and Computational Methods of IPMach NAS of Ukraine. The results of studies for various packings of fuel elements are shown below (Fig. 2–3). Each packing contains 37 rods, with all other conditions being equal λ_1=1, λ_1=10, h=1, q_V=10. By changing the values of the literal parameters, it is possible to obtain different distributions of the studied fields.

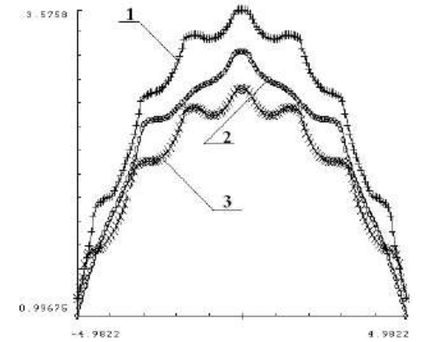
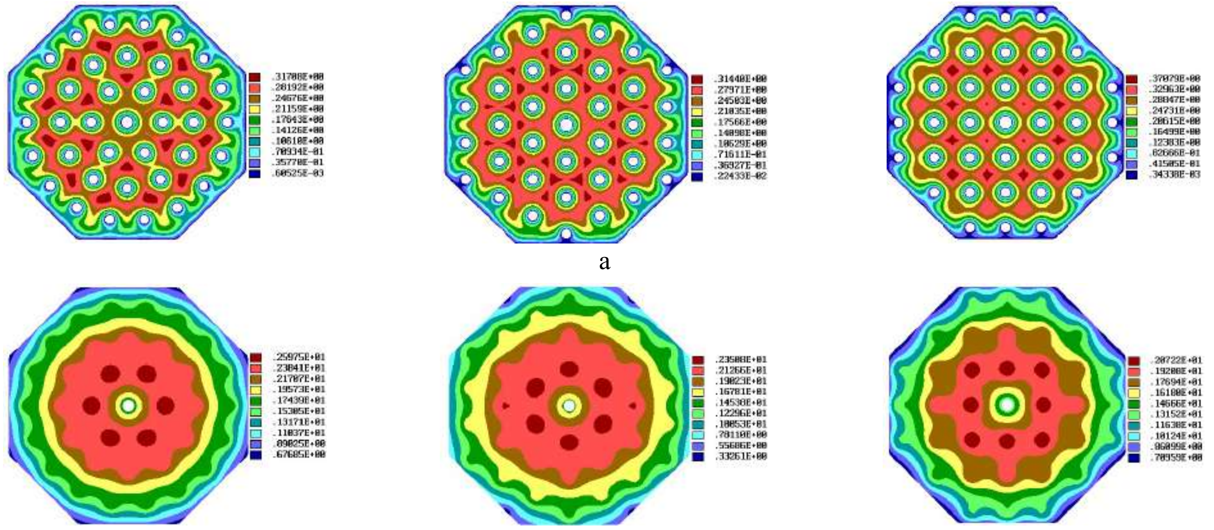


Fig. 3. Graphs of temperature distribution in the section x=0:
1 – cyclic, 2 – checkerboard,
3 – in-line packings

By the analysis of the obtained results, one can conclude that with cyclic packing, the maximum temperature is 9.8% higher than with checkerboard packing, and 18.6% higher than with in-line packing.

In real structures, a fastening in the form of a rod is located in place of the central fuel element. Then $\omega_b = (R_v - r\cos\mu_v) \wedge_0 (x^2 + y^2 - R_s^2) \geq 0$, $(R_s = R_{tv} + 0.05)$, and all calculations are performed as before. The results are shown in Figs. 4–5.

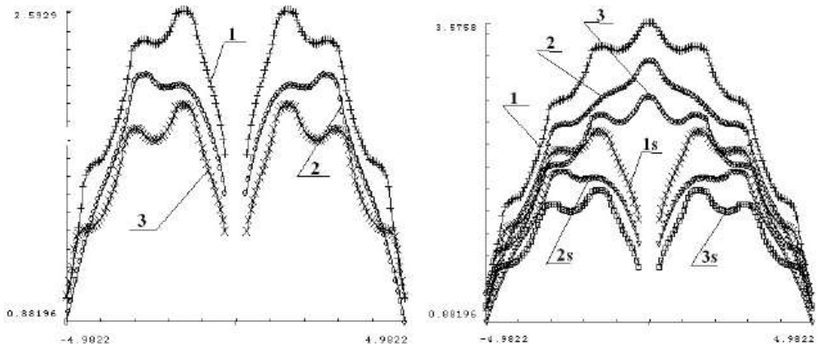


a

b

Fig. 4. Pictures of the velocity (a) and temperature (b) field distribution with a central fastening

In this case, with cyclic packing, the maximum temperature is 9% higher than with checkerboard packing, and 20% higher than with in-line packing. If we compare the results of the solution with central fuel element and central rod fastening, then during cyclic packing, the maximum temperature drops by 27%, during checkerboard packing it drops by 27%, and with in-line packing – by 29%.

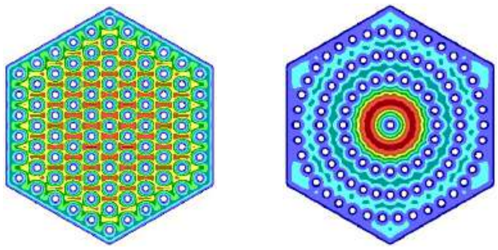


a b

Fig. 5. Graphs of temperature distribution in the section:
a – with central fastening, where 1 – cyclic, 2 – checkerboard, 3 – in-line packing;
b – general graph, where 1s is cyclic, 2s is checkerboard,
3s is in-line packing with central fastening

In a similar setting and using the same methodology, we will consider a typical structural diagram of a reactor whose cartridges are hexagonal casings, with 91 fuel elements are placed in each of them, both with an expanded triangular packing, which is sometimes called a checkerboard one, and during translation with cyclic symmetry.

Fig. 6, a shows the pattern of the level lines of the function $\omega(x, y)$ with 91 fuel elements arranged in a checkerboard pattern. Fig. 6, b shows the pattern of the level lines of the function $\omega(x, y)$ with 91 fuel elements during translation with cyclic symmetry.



a b

Fig. 6. Fuel cartridge with 91 fuel elements arranged in a checkerboard pattern (a) and during translation with cyclic symmetry (b)

Computational experiments, as in the previous problem, were carried out under the operating conditions of the POLYE system. Schoenberg's cubic splines were used as approximation means $\varphi_{ik}(x, y)$ with the dimension of the approximation space N=10000–40000. The results of studies for various fuel elements packings are shown in Figs. 7–8.

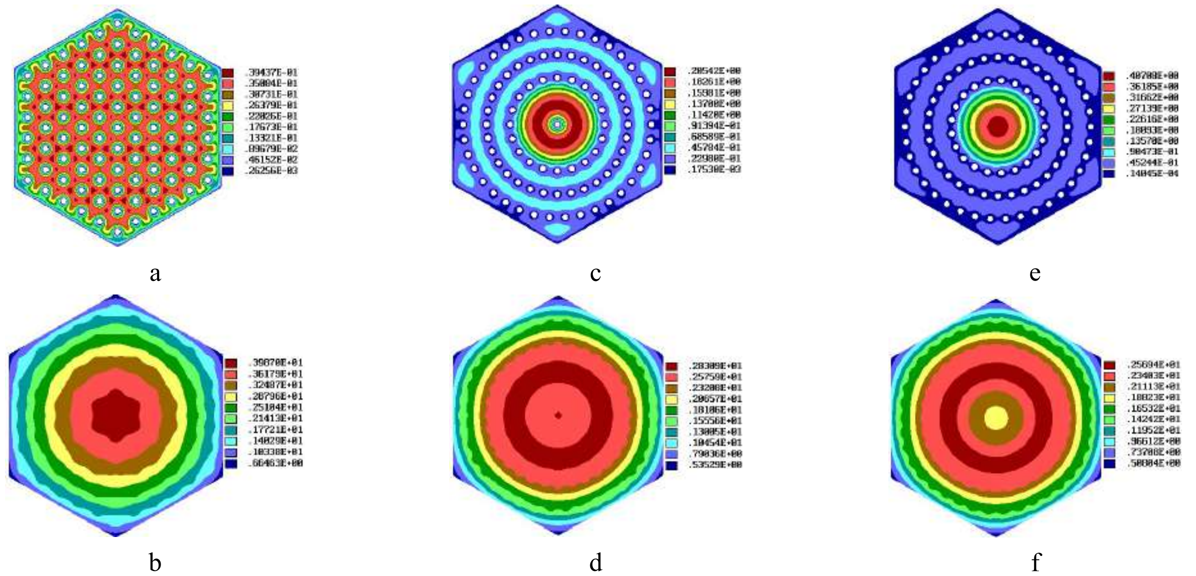


Fig. 7. Pictures of the velocity (a, c, e) and temperature (b, d, f) distribution in the cartridge with fuel elements arranged in a checkerboard pattern (a, b), and with cyclically arranged fuel elements and central fuel element (c, d) and with cyclically arranged fuel elements without central fuel element (e, f)

Analyzing the results, one can conclude that the presence of rods in the central zone leads to a higher temperature. Therefore, by changing the structural nature of the packing and the types of symmetry, it is possible to control the nature of the flow and the temperature distribution over the cartridge, achieving the required value, conditioned by the technical task.

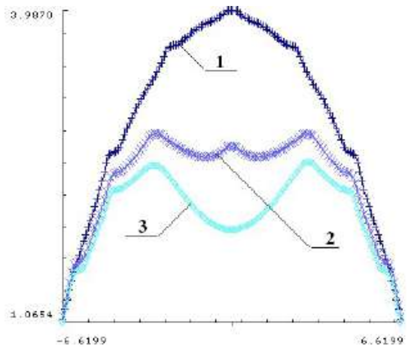


Fig. 8. Graphs of the temperature field for various packings of fuel elements in the section y=0:
1 – checkerboard pattern; 2 – cyclic symmetry with central fuel element; 3 – cyclic symmetry without central fuel element

Thermal-hydraulic calculation of fuel elements cartridges in case of violation of the rods packing symmetry

A typical structural diagram of the reactor, the active zone of which is assembled from a large number of fuel cartridges, is considered [2, 4]. Cartridges are hexagonal casings in which fuel elements are placed. To construct the equation for a hexagonal fuel cartridge with 169 fuel elements and an expanded triangular (checkerboard) packing, we will use the methodology from the previous paragraph.

As a result, we get

$$\omega_b \equiv R_v - r\cos\mu_v \geq 0,$$

where $\mu_v = \dfrac{4}{3\pi}\sum_k (-1)^{k+1}\dfrac{\sin[(2k-1)3\theta]}{(2k-1)^2}$, $r = \sqrt{x^2 + y^2}$, $\theta = arctg\dfrac{y}{x}$.

$$\omega_{tv} \equiv (f_1 \vee_0 f_2) \geq 0,$$

where $f_1 = R^2 - \mu_x^2 - \mu_y^2 \geq 0$, $f_2 = R^2 - \mu_{x1}^2 - \mu_{y1}^2 \geq 0$,

$$\mu_x = \frac{4h_x}{\pi^2}\sum_k (-1)^{k+1}\frac{\sin\left[(2k-1)\dfrac{\pi x}{h_x}\right]}{(2k-1)^2},\ \mu_y = \frac{4h_y}{\pi^2}\sum_k (-1)^{k+1}\frac{\sin\left[(2k-1)\dfrac{\pi x}{h_y}\right]}{(2k-1)^2},$$

$$\mu_{x1} = \frac{4h_x}{\pi^2}\sum_k (-1)^{k+1}\frac{\sin\left[\dfrac{(2k-1)\pi(x-h_x/2)}{h_x}\right]}{(2k-1)^2},\ \mu_{y1} = \frac{4h_y}{\pi^2}\sum_k (-1)^{k+1}\frac{\sin\left[\dfrac{(2k-1)\pi(y-h_y/2)}{h_y}\right]}{(2k-1)^2}.$$

Then the equation of the fuel cartridge has the form: $\omega \equiv \omega_b \wedge_0 \overline{\omega_{tv}} \geq 0$. The construction of the function $\omega(x,y)$ is performed with the following values of the literal parameters: R=1.542, h_x=11.9248, h_y=6.939, n_0=6, r_k=46.

The construction of displaced fuel elements was carried out as follows [11]. For displacement in the central zone

$$fs = \frac{R^2 - \left(x - \dfrac{h_x}{3}\right)^2 - y^2}{2R} \geq 0;\ f_p = \frac{(R+\delta)^2 - (x-h_x)^2 - y^2}{2R} \geq 0;\ \omega_{tv} \equiv \left((f_1 \vee_0 f_2) \wedge_0 \overline{f_p}\right) \vee_0 fs \geq 0.$$

For displacement in the far zone

$$fsd = \frac{R^2 - \left(x - \left(3h_x - \dfrac{h_x}{3}\right)\right)^2 - y^2}{2R} \geq 0;\ f_{pd} = \frac{(R+\delta)^2 - (x-3h_x)^2 - y^2}{2R} \geq 0;$$

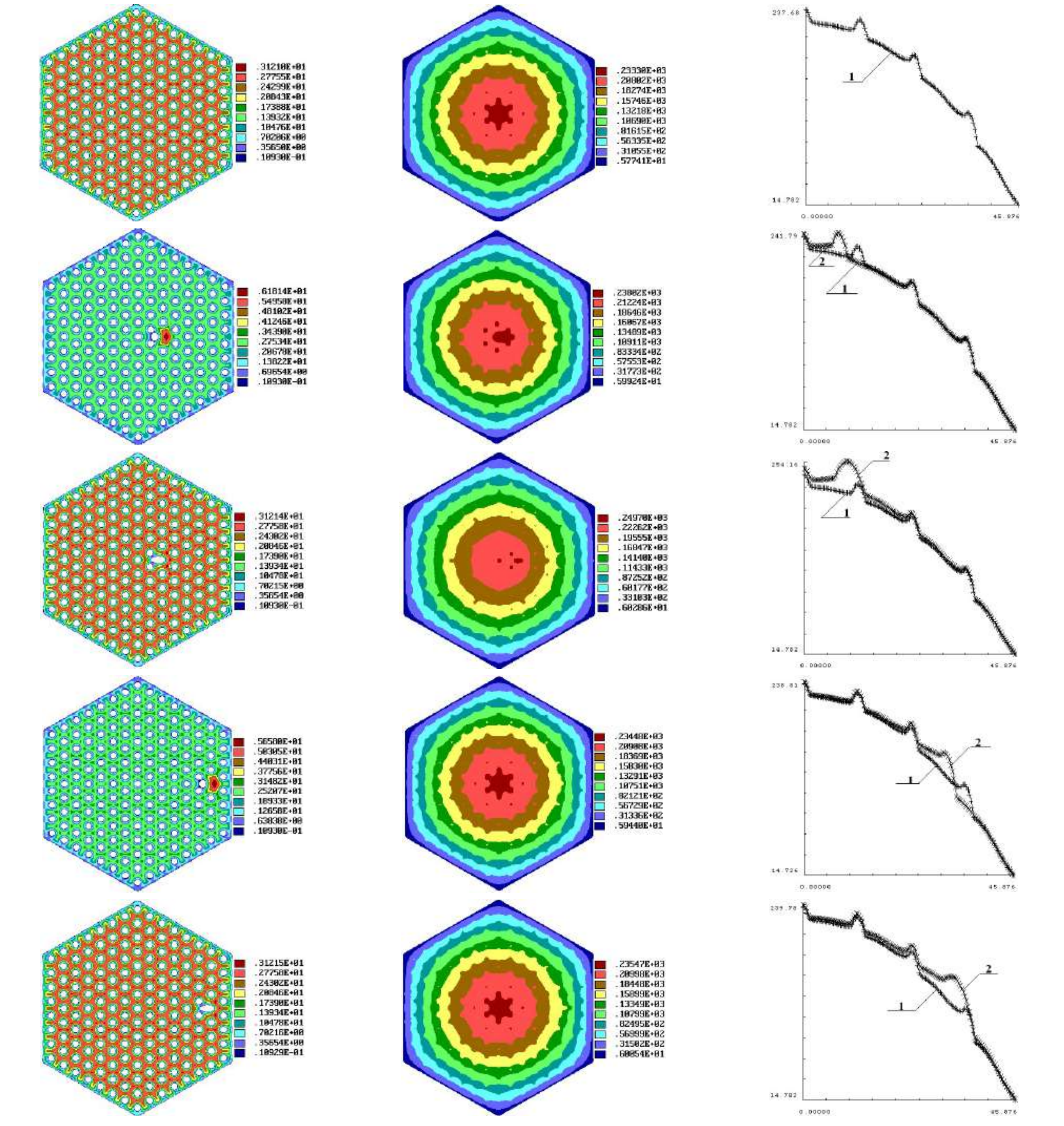
$$\omega_{tv} \equiv \left((f_1 \vee_0 f_2) \wedge_0 \overline{f_{pd}}\right) \vee_0 fsd \geq 0.$$

The statement of the problem and the solution method are similar to the previous ones. The results of studies for symmetrical packing, packing with symmetry violation of a straight and curved fuel element in the central zone and on the periphery are shown in Fig. 9. Each packing contains 169 rods. From the analysis of the obtained results, it follows that in case of the packing symmetry violation, while maintaining the parallelism of the rods, the local temperature increases by 2%. In case of rod bending, the local temperature increases by 7%.

In this case, only one rod, which violates the packing symmetry, is considered. With several "non-standard" rods, the calculation of the temperature field for the cartridge as a whole is all the more important. Mathematical modeling and the associated computer experiment are indispensable in cases where a full-scale experiment is impossible or difficult to conduct for one reason or another. In addition, working with mathematical model of the process and the computer experiment make it possible to investigate the properties and behavior of the process in various situations freely, relatively quickly and without significant expenses. At the same time, computational experiments with object models make it possible to study them in detail and in depth, based on modern numerical methods. The reliability of the analytical identification of the cartridges was confirmed by their visualization. The reliability of the calculation methods, results and conclusions is confirmed by the analysis of the numerical convergence of solutions and the calculation of the residual.

R-functions, fuel element with polyzonal finning of the shell and heat transfer during fluid motion

Finning of heat transfer surfaces is widely used to increase the effective heat transfer coefficient in engineering. The shape of the edge is very diverse. Longitudinal, transverse, spiral edges, edges in the form of spikes, etc. are used. Detailed information about various methods of finning and technology for manufacturing finned surfaces can be found in [10, 12]. The finning not only increases the heat exchange surface, but also has a great influence on the hydrodynamics of the flow, and thus on the heat transfer coefficient. In the course of experiments with various methods of fuel elements shells finning, more advantageous forms of finning, the so-called polyzonal and chevron ones, were developed. With the latter, for example, the entire surface of the shell is divided into four, six, or eight sectors, and on neighboring sectors, the spirals are arranged symmetrically about the longitudinal axis.



a b c

Fig. 9. Pictures of the velocity field (a) and temperature field (b) distribution, including in case of violation of the rods packing symmetry, and a comparative analysis of graphs in the section $0 \le x \le R_v$, $y=0$ (c)

The problem of heat transfer in the laminar fluid flow for the fuel element with polyzonal shell finning is considered. Let's build a mathematical model of such a fuel element shell (Fig. 10) [12–14].

$$\omega = \left(1 - \frac{(sx-2)^2}{0{,}56^2} - \frac{sy^2}{0{,}2^2} \vee_0 4 - x^2 - y^2\right) \wedge_0 7{,}5^2 - z^2 \geq 0\,;\; \begin{cases} sx = rs\cos\mu s \\ sy = rs\sin\mu s \end{cases};$$

$$\mu s = \frac{8}{\pi no1}\sum_k (-1)^{k-1}\frac{\sin(2k-1)\frac{\theta sno1}{2}}{(2k-1)^2}\,;\; rs = \sqrt{xs^2 + ys^2}\,;\; \theta s = arctg\frac{ys}{xs}\,;$$

$$\begin{cases} xs = x\cos\frac{fi}{n} + y\sin\frac{fi}{n} \\ ys = y\cos\frac{fi}{n} - x\sin\frac{fi}{n} \end{cases};\; fi = \frac{2\pi z}{10}\,;\; n = 3\,.$$

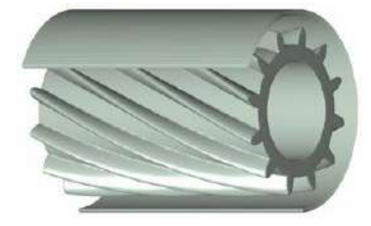


Fig. 10. Fuel element with polyzonal shell finning

The main system of equations describing the process of heat transfer in the viscous fluid flow, with constant physical properties of the fluid and temperature, has the form (1).

The invariant form of writing equations (1) allows, using the apparatus of tensor analysis, to move to a new coordinate system, in our case, to a curvilinear non-orthogonal system

$$\begin{cases} x = \hat{x}\cos\alpha z - \hat{y}\sin\alpha z \\ y = \hat{x}\sin\alpha z + \hat{y}\cos\alpha z\,. \\ z = z \end{cases}$$

In the helical coordinate system, we obtain (1) in the form [8]

$$\frac{\partial T}{\partial \tau} + (\vec{V}\nabla)T = a\Delta T + \frac{q_V}{\rho c_p} + \frac{\mu\Phi}{\rho c_p};$$

$$\frac{\partial V^1}{\partial \tau} + (\vec{V}\nabla)V^1 - 2\alpha V^2\frac{V^3}{\sqrt{f}} - \alpha^2\hat{x}\frac{V^{3^2}}{f} = \frac{1}{\rho}\left(-(1+\alpha^2\hat{y}^2)\frac{\partial p}{\partial \hat{x}} + \alpha^2\hat{x}\hat{y}\frac{\partial p}{\partial \hat{y}} - \alpha\hat{y}\frac{\partial p}{\partial z}\right) +$$

$$+\nu\left[\Delta V^1 - 2\alpha^2\left(\hat{y}\frac{\partial V^2}{\partial \hat{x}} - \hat{x}\frac{\partial V^2}{\partial \hat{y}}\right) - 2\alpha\left(\frac{1}{\sqrt{f}}\frac{\partial V^3}{\partial \hat{y}} + \frac{\partial V^2}{\partial z}\right) - \alpha^2 V^1 + 2\frac{\alpha^3\hat{y}V^3}{f^{3/2}}\right];$$

$$\frac{\partial V^2}{\partial \tau} + (\vec{V}\nabla)V^2 + 2\alpha V^1\frac{V^3}{\sqrt{f}} - \alpha^2\hat{y}\frac{V^{3^2}}{f} = \frac{1}{\rho}\left(\alpha^2\hat{x}\hat{y}\frac{\partial p}{\partial \hat{x}} - (1+\alpha^2\hat{x}^2)\frac{\partial p}{\partial \hat{y}} + \alpha\hat{x}\frac{\partial p}{\partial z}\right) +$$

$$+\nu\left[\Delta V^2 + 2\alpha^2\left(\hat{y}\frac{\partial V^1}{\partial \hat{x}} - \hat{x}\frac{\partial V^1}{\partial \hat{y}}\right) + 2\alpha\left(\frac{1}{\sqrt{f}}\frac{\partial V^3}{\partial \hat{x}} + \frac{\partial V^1}{\partial z}\right) - \alpha^2 V^2 - 2\frac{\alpha^3\hat{x}V^3}{f^{3/2}}\right];$$

$$\frac{\partial V^3}{\partial \tau} + (\vec{V}\nabla)V^3 - \frac{\alpha^2 V^3}{f}(\hat{x}V^1 + \hat{y}V^2) = \frac{\sqrt{f}}{\rho}\left(-\alpha\hat{y}\frac{\partial p}{\partial \hat{x}} + \alpha\hat{x}\frac{\partial p}{\partial \hat{y}} - \frac{\partial p}{\partial z}\right) +$$

$$+\nu\left[\Delta V^3 - \frac{2\alpha^2}{f}\left(\hat{x}\frac{\partial V^3}{\partial \hat{x}} + \hat{y}\frac{\partial V^3}{\partial \hat{y}}\right) - \frac{\alpha^2(2 - \alpha^2\hat{x}^2 - \alpha^2\hat{y}^2)}{f^2}V^3\right];$$

$$\frac{\partial V^1}{\partial \hat{x}} + \frac{\partial V^2}{\partial \hat{y}} + \frac{1}{\sqrt{f}}\frac{\partial V^3}{\partial z} = 0\,,$$

where $f = 1 + \alpha^2\hat{x}^2 + \alpha^2\hat{y}^2$, $(\vec{V}\nabla) = V^1\frac{\partial}{\partial \hat{x}} + V^2\frac{\partial}{\partial \hat{y}} + \frac{V^3}{\sqrt{f}}\frac{\partial}{\partial z}$,

$$\Delta = (1+\alpha^2\hat{y}^2)\frac{\partial^2}{\partial \hat{x}^2} + (1+\alpha^2\hat{x}^2)\frac{\partial^2}{\partial \hat{y}^2} + \frac{\partial^2}{\partial z^2} - 2\alpha^2\hat{x}\hat{y}\frac{\partial^2}{\partial \hat{x}\partial \hat{y}} + 2\alpha\hat{y}\frac{\partial^2}{\partial \hat{x}\partial z} - 2\alpha\hat{x}\frac{\partial^2}{\partial \hat{y}\partial z} - \alpha^2\left(\hat{x}\frac{\partial}{\partial \hat{x}} + \hat{y}\frac{\partial}{\partial \hat{y}}\right).$$

The boundary condition for the velocity is formulated as the condition of adhesion of fluid particles to a solid wall: $\vec{V}\big|_{\partial\Omega} = 0$, and for the temperature field on the wall it can be specified in different ways:

$$T\big|_{\partial\Omega_i} = T_i\,;\ \ \frac{\partial T}{\partial n}\bigg|_{\partial\Omega} = -\frac{q}{\lambda}\,;\ \ \left(\frac{\partial T}{\partial n} + hT\right)\bigg|_{\partial\Omega} = q_c\,;\ \ \begin{cases} T\big|_{\partial\Omega+0} = T\big|_{\partial\Omega-0}; \\ \lambda_{\text{ж}}\dfrac{\partial T}{\partial n}\bigg|_{\partial\Omega+0} = \lambda_{cm}\dfrac{\partial T}{\partial n}\bigg|_{\partial\Omega-0} \end{cases}.$$

In the curvilinear non-orthogonal coordinate system, the normal derivatives included in the boundary conditions, taking into account the normalization of the function $\omega_1(\hat{x}_1, \hat{x}_2)$, have the form

$$\frac{\partial T}{\partial n}\bigg|_{\partial\Omega} = \left(1 + \alpha^2 \hat{x}_2{}^2\right)\frac{\partial u}{\partial \hat{x}_1}\frac{\partial \omega_1}{\partial \hat{x}_1} + \left(1 + \alpha^2 \hat{x}_1{}^2\right)\frac{\partial u}{\partial \hat{x}_2}\frac{\partial \omega_1}{\partial \hat{x}_2} - \alpha^2 \hat{x}_1 \hat{x}_2 \left(\frac{\partial u}{\partial \hat{x}_1}\frac{\partial \omega_1}{\partial \hat{x}_2} + \frac{\partial u}{\partial \hat{x}_2}\frac{\partial \omega_1}{\partial \hat{x}_1}\right)\bigg|_{\partial\Omega}, \tag{3}$$

and the equations obtained in [8] take the following form

$$\left(1+\alpha^2\hat{y}^2\right)\frac{\partial^2 V^3}{\partial \hat{x}^2} + \left(1+\alpha^2\hat{x}^2\right)\frac{\partial^2 V^3}{\partial \hat{y}^2} - 2\alpha^2\hat{x}\hat{y}\frac{\partial^2 V^3}{\partial \hat{x}\partial \hat{y}} - \alpha^2\left(\hat{x}\frac{\partial V^3}{\partial \hat{x}} + \hat{y}\frac{\partial V^3}{\partial \hat{y}}\right) - \frac{\alpha^2\left(2+\alpha^2 x^2+\alpha^2 y^2\right)}{f^2}V^3 = -\frac{1}{\mu\sqrt{f}}\frac{\partial p}{\partial z}; \tag{4}$$

$$-\left[\left(1+\alpha^2\hat{y}^2\right)\frac{\partial^2 T}{\partial \hat{x}^2} + \left(1+\alpha^2\hat{x}^2\right)\frac{\partial^2 T}{\partial \hat{y}^2} - 2\alpha^2\hat{x}\hat{y}\frac{\partial^2 T}{\partial \hat{x}\partial \hat{y}} - \alpha^2\left(\hat{x}\frac{\partial T}{\partial \hat{x}} + \hat{y}\frac{\partial T}{\partial \hat{y}}\right)\right] = -\frac{V^3}{a\sqrt{f}}C. \tag{5}$$

Thus, from three-dimensional boundary problems we come to two-dimensional problems, for which the Ritz method in combination with the R-functions method can be applied [7, 8]. The positive definiteness of the operators of problems (5) was proved in [8].

We minimize the functionals equivalent to the boundary problems (4) and (5),

$$I_1 = \int\limits_{\Omega}\left[(1+\alpha^2\hat{y}^2)\left(\frac{\partial V}{\partial \hat{x}}\right)^2 + (1+\alpha^2\hat{x}^2)\left(\frac{\partial V}{\partial \hat{y}}\right)^2 - 2\alpha^2\hat{x}\hat{y}\frac{\partial V}{\partial \hat{x}}\frac{\partial V}{\partial \hat{y}} + KV^2 - 2FV\right]d\hat{x}d\hat{y},$$

where $K = \dfrac{\alpha^2\left(2+\alpha^2\hat{x}^2+\alpha^2 y^2\right)}{f^2}$; $F = -\dfrac{1}{\mu\sqrt{f}}\dfrac{\partial p}{\partial z}$; $f = \sqrt{1+\alpha^2\hat{x}^2+\alpha^2\hat{y}^2}$;

$$I_2 = \int\limits_{\Omega}\left[(1+\alpha^2\hat{y}^2)\left(\frac{\partial T1}{\partial \hat{x}}\right)^2 + (1+\alpha^2\hat{x}^2)\left(\frac{\partial T1}{\partial \hat{y}}\right)^2 - 2\alpha^2\hat{x}\hat{y}\frac{\partial T1}{\partial \hat{x}}\frac{\partial T1}{\partial \hat{y}} - 2GT1\right]d\hat{x}d\hat{y} +$$

$$+2\int\limits_{\Omega}\left[(1+\alpha^2\hat{y}^2)\frac{\partial T1}{\partial \hat{x}}\frac{\partial T0}{\partial \hat{x}} + (1+\alpha^2\hat{x}^2)\frac{\partial T1}{\partial \hat{y}}\frac{\partial T0}{\partial \hat{y}} - \alpha^2\hat{x}\hat{y}\left(\frac{\partial T0}{\partial \hat{x}}\frac{\partial T1}{\partial \hat{y}} + \frac{\partial T1}{\partial \hat{x}}\frac{\partial T0}{\partial \hat{y}}\right)\right]d\hat{x}d\hat{y},$$

where $G = -\dfrac{V^3}{a\sqrt{f}}C$. On the outer wall ($f_1 = \left(R^2 - x^2 - y^2\right)/2R = 0$) $T\big|_{\partial\Omega_1} = 0$, on the inside wall – ($\omega = 0$) $T\big|_{\partial\Omega_2} = 1$. The structure of the solution to problem (4) has the form $V^3 = \omega_1\Phi_1$, and to problem (5) it has the form $T = T0 + T1$; $T0 = \dfrac{f_1}{f_1 + \omega_2}$; $T1 = \omega_1\Phi_2$, where $\omega_1 = f_1 \wedge_0 \omega_2$. For different values of the swirl parameter α, we obtain the results shown in Fig. 11.

The results shown on Fig. 11 indicate that with an increase in the value of the swirl parameter α, heating in the intercostal zone increases.

The conducted studies make it possible in the future to effectively continue work related to mathematical and computer modeling of convective heat transfer in fuel cartridges of fuel elements with various shapes and packing of rods, as, for example, in the following problems.

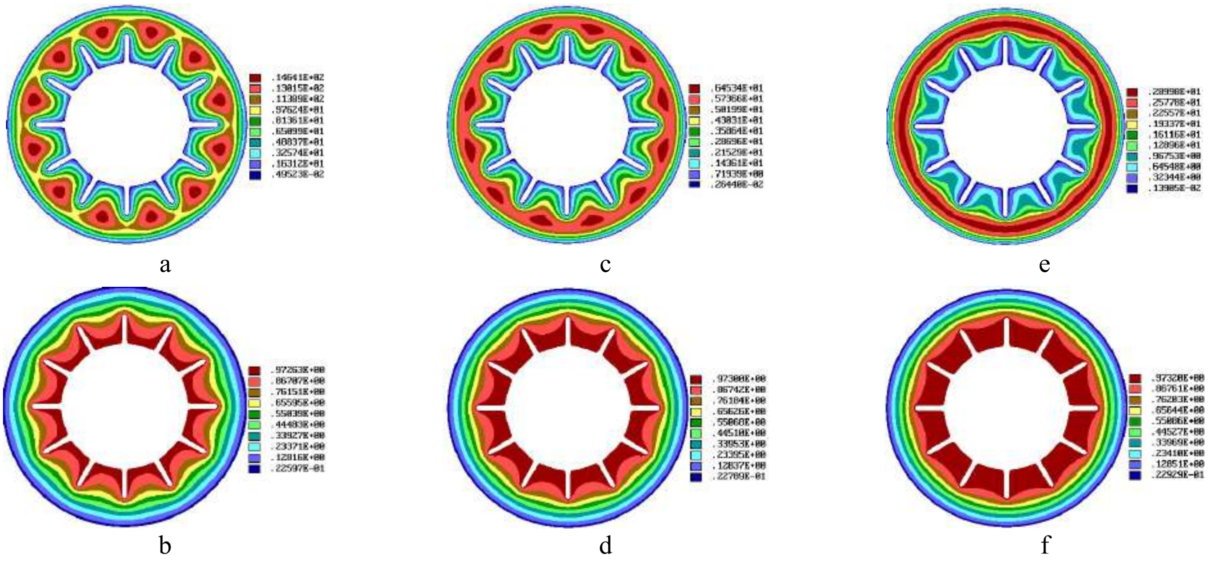


Fig. 11. Distributions of the velocity (a, c, e) and temperature (, d, f) field for various values of the swirl parameter:
a, b – α=0; c, d – α=π/2; e, f – α=π

At present, conceptual designs for a supercritical water reactor (SCWR) have been implemented in various countries. In this case, both square and hexagonal grids of fuel elements with cavities filled with a moderator, for example, water or a solid moderator, are used. With the help of R-functions, it has already been possible to develop a methodology and construct an equation for a bundle of fuel elements with hexagonal grid (Fig. 12) [15].

In addition, it is noted that finning of heat transfer surfaces is widely used in engineering. It was possible to build a number of mathematical models of multi-zone fuel elements with longitudinal finning and helical plates (Fig. 13) [15].

The obtained mathematical models can be used by designers in research work during computational experiments.

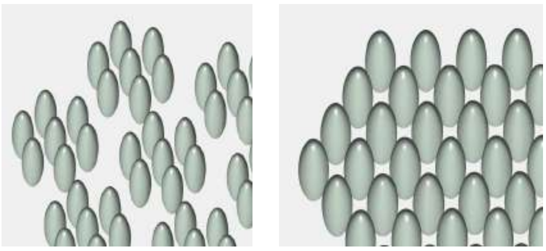


Fig. 12. Fuel elements with hexagonal grid

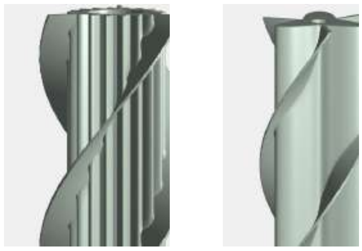


Fig. 13. Fuel elements with longitudinal finning and helical plates

Conclusions

It has been established that the R-functions method is an effective method for solving problems of calculating physical fields in structural elements of nuclear power plants of complex shape. The developed constructive tools for constructing equations of the boundaries of regions with translational and cyclic types of symmetry made it possible to significantly reduce the number of operations with subsequent automation of this process, and, consequently, reduce the time for solving problems. The conducted studies allow designers to choose certain types of packing depending on technical requirements. The paper considers only one rod that breaks the symmetry of the packing. With several "non-standard" rods, the calculation of the temperature field for the cartridge as a whole is all the more important. The application of the R-functions theory to mathematical and computer modeling of heat transfer during fluid flow for the fuel element with polyzonal finning of the shell and the analytical recording of the designed objects makes it possible to use alphabetic geometric parameters, complex superpositions of functions, which, in turn, allows to quickly change their structural elements. Reducing the pitch of the helical spiral leads to an increase in resistance, outstripping the increase in heat transfer. It is noted that an increase in the thickness of the edges increases their efficiency. In the future, it is of interest to study the influence of the edge thickness and channel length on heat transfer and hydrodynamic resistance. Mathematical modeling and the computer experiment associated with it are irreplaceable in cases where a full-scale experiment is impossible or difficult to conduct for one reason or another. In addition, working with the mathematical model of the process and the computational experiment make it possible to investigate the properties and behavior of the process in various situations freely, relatively quickly and without significant expenses. At the same time, computational experiments with object models make it possible to study them in detail and in depth, based on modern numerical methods. The reliability of the analytical identification of cartridges is confirmed by their visualization, and the calculation methods, results and conclusions are confirmed by the analysis of the numerical convergence of solutions and the calculation of the residual.

References

1. Tarapon, A. G. (2006). *Prichina avarii na Chernobylskoy AES. Modelirovaniye protsessov razrusheniya reaktora i teploprovodnosti v obyekte «Ukrytiye»* [Cause of the accident at the Chernobyl nuclear power plant. Modeling of the processes of destruction of the reactor and heat conduction in the object "Shelter"]. Kyiv: Institute for Modeling Problems in Energy. G.E. Pukhov National Academy of Sciences of Ukraine, 183 p. (in Russian).
2. Petukhov, B. S., Genin, L. G., & Kovalev, S. A. (1974). *Teploobmen v yadernykh energeticheskikh ustanovkakh* [Heat exchange in nuclear power plants]. Moscow: Atomizdat, 367 p. (in Russian).
3. Slesarenko, A. P. & Kotulskiy, D. A. (2000). *Regionalno-analiticheskiy i variatsionnyye metody v reshenii sopryazhennykh zadach konvektivnogo teploobmena* [Regional-analytical and variational methods in solving conjugate problems of convective heat transfer]. *Heat and Mass Transfer MMF-2000*: Proceedings of the IV Minsk International Forum (Belarus, Minsk, May 2000). Minsk: ITMO Academy of Sciences of Belarus, vol. 3, pp. 135–142 (in Russian).
4. Maksimenko-Sheyko, K. V., Sheyko, T. I., & Uvarov, R. A. (2013). The R-functions method in mathematical modeling of convective heat transfer in fuel cartridge with fuel rods. *Problems of atomic science and technology. Series: Nuclear Physics Investigations*, vol. 60, no. 3 (85), pp. 205–209.
5. Maksimenko-Sheyko, K. V., Tolok, A. V., Sheyko, T. I. (2013). *Sopryazhennaya zadacha konvektivnogo teploobmena v toplivnoy kassete TVELov* [Adjoint problem of convective heat transfer in a fuel cassette of fuel elements]. *Informatsionnyye tekhnologii – Information Technology*, no. 11, pp. 32–36 (in Russian).
6. Kolyada, R. A., Maksymenko-Sheiko, K. V., & Sheyko, T. I. (2019). R-functions method in the mathematical modeling of convective heat exchange in an octahedral fuel assembly with 37 fuel elements. *Journal of Mathematical Sciences*, vol. 238, iss. 2, pp. 154–164. https://doi.org/10.1007/s10958-019-04225-w.
7. Rvachev, V. L. (1982). *Teoriya R-funktsiy i nekotoryye yeye prilozheniya* [The R-functions theory and some of its applications]. Kyiv: Naukova dumka, 552 p. (in Russian).
8. Maksimenko-Sheyko, K. V. (2009). *R-funktsii v matematicheskom modelirovanii geometricheskikh obyektov i fizicheskikh poley* [R-functions in the mathematical modeling of geometric objects and physical fields]. Kharkiv: IPMash NAS of Ukraine, 306 p. (in Russian).
9. Maksymenko-Sheiko, K. V. & Sheiko T. I. (2008). R-functions in mathematical modeling of geometric objects with symmetry. *Cybernetics and Systems Analysis*, vol. 44, iss. 6, pp. 855–862. https://doi.org/10.1007/s10559-008-9061-5.
10. Andreyev, P. A., Gremilov, D. I., & Fedorovich, Ye. D. (1969). *Teploobmennyye apparaty yadernykh energeticheskikh ustanovok* [Heat exchangers of nuclear power plants]. Leningrad: Sudostroyeniye, 352 p. (in Russian).
11. Sheyko, T. I., Maksymenko-Sheiko, K. V., Uvarov, R. A., & Khazhmuradov, M. A. (2019). The thermal-hydraulic calculation in a fuel cartridge when the symmetry of fuel rods packing is broken. *Problems of Atomic Science and Technology. Series: Nuclear Physics Investigations*, vol. 71, no. 3 (121), pp. 74–79. https://doi.org/10.46813/2019-121-074.
12. Antufyev, V. M. (1966). *Effektivnost razlichnykh form konvektivnykh poverkhnostey nagreva* [Efficiency of various forms of convective heating surfaces]. Moscow, Leningrad: Energiya, 184 p. (in Russian).
13. Maksimenko-Sheyko, K. V. & Sheyko, T. I. (2017). *R-funktsii, TVEL s polizonalnym orebreniyem obolochki i teploobmen pri dvizhenii zhidkosti* [R-functions, TVEL with polyzonal finning of the shell and heat transfer during fluid motion]. *Vestnik Zaporozhskogo natsional'nogo universiteta. Fiziko-matematicheskiye nauki – Visnyk of Zaporizhzhia National University. Physical and Mathematical Sciences*, no. 1, pp. 277–285 (in Russian).
14. Maksymenko-Sheiko, K. V., Litvinova, Yu. S., Sheyko, T. I., & Khazhmuradov, M. A. (2017). *Matematicheskoye modelirovaniye teploobmena pri techenii zhidkosti dlya TVELa s polizonalnym orebreniyem obolochki* [Mathematical simulation of heat transfer during fluid flow for a fuel element with a polyzonal finned shell]. *Problemy mashinostroyeniya – Journal of Mechanical Engineering – Problemy Mashinobuduvannia*, vol. 20, no. 4, pp. 58–63 (in Russian). https://doi.org/10.15407/pmach2017.04.058.

15. Lisin, D. O. (2012). *Kompiuterna prohrama "Systema vizualizatsii ta pobudovy sitky na poverkhni heometrychnykh obiektiv, yaki opysani za dopomohoiu matematychnykh zasobiv teorii R-funktsii "RFPreview"* [Computer program "System of visualization and construction of a grid on the surface of geometric objects, which are described by mathematical means of the theory of R-functions "RFPreview"]: Certificate of registration of copyright to the work no. 45951 (in Ukrainian).

Received 21 February 2022

Математичне й комп'ютерне моделювання конвективного теплообміну в паливних касетах ТВЕЛів за різної форми та упаковки стрижнів

[1] К. В. Максименко-Шейко, [1] Т. І. Шейко, [2] Д.О. Лісін, [2] Т. Б. Дудінов

[1] Інститут проблем машинобудування ім. А. М. Підгорного НАН України, 61046, Україна, м. Харків, вул. Пожарського, 2/10

[2] Харківський національний університет імені В. Н. Каразіна, 61022, Україна, м. Харків, майдан Свободи, 4

Робота складається з трьох розділів і має інформаційно-узагальнюючий характер із зазначенням перспективних напрямів подальших досліджень. Перший розділ «Метод R-функцій у математичному моделюванні конвективного теплообміну в паливних касетах з ТВЕЛами» присвячено застосуванню нових конструктивних засобів методу R-функцій для математичного й комп'ютерного моделювання упаковок ТВЕЛів із різними типами симетрії, а також дослідженню конвективного теплообміну в ґратках ТВЕЛів і впливу виду упаковки на розподіл швидкості й температури. Розглянуто восьмигранну касету з 37 ТВЕЛами, упакованими за трьома схемами: циклічною, шаховою й коридорною. Зазначено, що при побудові рівнянь касети з пучками ТВЕЛів за новою методикою кількість R-операцій і, відповідно, час розрахунку істотно зменшилися. Спираючись на отримані результати, зроблено висновок, що при циклічній упаковці отримуємо максимальну температуру. Розглянуто також схему реактора, касети якого являють собою шестигранні кожухи, де в кожному розміщено по 91 ТВЕЛу як з шаховою, так і циклічною упаковкою. У другому розділі «Теплогідравлічний розрахунок касет ТВЕЛів при порушенні симетрії упаковки стрижнів» розглянуто шестигранна паливна касета із 169 ТВЕЛами й шаховою упаковкою, а також підвищення температури у разі порушення симетрії упаковки і при збереженні паралельності стрижнів, і у разі викривлення одного з них. Третій розділ «R-функції, ТВЕЛ з полізональним оребренням оболонки і теплообмін при русі рідини» присвячено побудові рівнянь різних поверхонь оребрення ТВЕЛів і дослідженню гідродинамічних і температурних полів при полізональному оребренні оболонки. При цьому завдяки використанню апарату тензорного аналізу здійснено перехід у криволінійну неортогональну (гвинтову) систему координат. Зосереджено увагу на тому, що математичне моделювання й пов'язаний із ним комп'ютерний експеримент незамінні в тих випадках, коли натурний експеримент неможливий або ускладнений з тих чи інших причин. Крім того, робота з математичною моделлю процесу і проведення обчислювального експерименту дають змогу відносно швидко і без істотних витрат досліджувати властивості й поведінку процесу в різних ситуаціях. Достовірність методів, результатів і висновків підтверджена порівнянням із відомостями, раніше наведеними в літературі, результатами аналізу чисельної збіжності розв'язків й обчисленням нев'язки.

Ключові слова: *ядерний реактор, касета, ТВЕЛ, метод R-функцій, тип симетрії упаковки, оребрення оболонки.*

Література

1. Тарапон А. Г. Причина аварии на Чернобыльской АЭС. Моделирование процессов разрушения реактора и теплопроводности в объекте «Укрытие». Киев: Институт проблем моделирования в энергетике им. Г. Е. Пухова НАН Украины, 2006. 183 с.
2. Петухов Б. С., Генин Л. Г., Ковалев С. А. Теплообмен в ядерных энергетических установках. Москва: Атомиздат, 1974. 367 с.
3. Слесаренко А. П., Котульский Д. А. Регионально-аналитический и вариационные методы в решении сопряженных задач конвективного теплообмена. *Тепломассообмен ММФ-2000.* Труды IV Минского международного форума (Беларусь, Минск, май 2000). Минск: ИТМО АН Беларуси. 2000. Т. 3. С. 135–142.
4. Maksimenko-Sheyko K.V., Sheyko T. I., Uvarov R. A. The R-functions method in mathematical modeling of convective heat transfer in fuel cartridge with fuel rods. *Problems of atomic science and technology. Series: Nuclear Physics Investigations*. 2013. Vol. 60. No. 3 (85). P. 205–209.

5. Максименко-Шейко К. В., Толок А. В., Шейко Т. И. Сопряженная задача конвективного теплообмена в топливной кассете ТВЭЛов. *Информационные технологии.* 2013. № 11. С. 32–36.
6. Kolyada R. A., Maksymenko-Sheiko K. V., Sheyko T. I. R-functions method in the mathematical modeling of convective heat exchange in an octahedral fuel assembly with 37 fuel elements. *Journal of Mathematical Sciences*. 2019. Vol. 238. Iss. 2. P. 154–164. https://doi.org/10.1007/s10958-019-04225-w.
7. Рвачев В. Л. Теория R-функций и некоторые ее приложения. Киев: Наук. думка, 1982. 552 с.
8. Максименко-Шейко К. В. R-функции в математическом моделировании геометрических объектов и физических полей. Харьков: ИПМаш НАН Украины, 2009. 306 с.
9. Maksymenko-Sheiko K. V., Sheiko T. I. R-functions in mathematical modeling of geometric objects with symmetry. *Cybernetics and Systems Analysis*. 2008. Vol. 44. Iss. 6. P. 855–862. https://doi.org/10.1007/s10559-008-9061-5.
10. Андреев П. А., Гремилов Д. И., Федорович Е. Д. Теплообменные аппараты ядерных энергетических установок / под ред. Н. М. Синева. Ленинград: Судостроение, 1969. 352 с.
11. Sheyko T. I., Maksymenko-Sheiko K. V., Uvarov R. A., Khazhmuradov M. A. The thermal-hydraulic calculation in a fuel cartridge when the symmetry of fuel rods packing is broken. *Problems of Atomic Science and Technology. Series: Nuclear Physics Investigations*. 2019. Vol. 71. No. 3 (121). P. 74–79. https://doi.org/10.46813/2019-121-074.
12. Антуфьев В. М. Эффективность различных форм конвективных поверхностей нагрева. Москва-Ленинград: Энергия, 1966. 184 с.
13. Максименко-Шейко К. В., Шейко Т. И. R-функции, ТВЭЛ с полизональным оребрением оболочки и теплообмен при движении жидкости. *Вестник Запорожского национального университета. Физико-математические науки*. 2017. № 1. С. 277–285.
14. Максименко-Шейко К. В., Литвинова Ю. С., Шейко Т. И., Хажмурадов М. А. Математическое моделирование теплообмена при течении жидкости для ТВЭЛА с полизональным оребрением оболочки. *Проблемы машиностроения*. 2017. Т. 20. № 4. С. 58–63. https://doi.org/10.15407/pmach2017.04.058.
15. Лісін Д. О. Свідоцтво про реєстрацію авторського права на твір № 45951. Комп’ютерна програма «Система візуалізації та побудови сітки на поверхні геометричних об’єктів, які описані за допомогою математичних засобів теорії R-функцій «RFPreview». 2012.